APPLICATION OF THE NONLINEAR OPTICS FROM OFF-ENERGY ORBITS METHOD AT THE SIRIUS STORAGE RING

M. M. S. Velloso*[1], X. R. Resende, F. H. de Sá
Brazilian Synchrotron Light Laboratory, Campinas, Brazil
[1]also at Gleb Wataghin Institute of Physics, University of Campinas, Campinas, Brazil

Abstract

The applicability of the Nonlinear Optics from Off-Energy Closed Orbits (NOECO) method to the SIRIUS storage ring is investigated. Off-energy orbit response matrices (OEORMs) were measured and used for sextupole strength calibration through a Levenberg-Marquardt (LM) fitting procedure with Tikhonov regularization. The fitted strengths reproduced the measured chromaticities with reasonable accuracy and were consistent with control system estimates. Analysis of the OEORM Jacobian revealed strong correlations among sextupole family signatures, indicating significant quasi-degeneracies in the inverse problem. Machine experiments and model-based simulations showed that localized perturbations are redistributed across correlated families, especially in the presence of small optics mismatches. The results indicate that OEORM-based calibration is feasible at SIRIUS, although its resolving power is fundamentally limited by parameter correlations and model imperfections.

INTRODUCTION

NOECO [1] is a calibration method that fits chromatic sextupole strengths to best reproduce the measured OEORM, in a least-squares sense. The OEORM is defined as the numerical derivative of the orbit response matrix (ORM) with respect to relative energy deviations δ:

$$\mathrm{OEORM} = \frac{\mathrm{ORM}_{+\delta} - \mathrm{ORM}_{-\delta}}{2\delta}, \qquad (1)$$

where $\mathrm{ORM}_{\pm\delta}$ corresponds to the ORM measured at energy deviation $\pm\delta$ and ORM elements read

$$\mathrm{ORM}_{ij} = \frac{\Delta u_i}{\Delta \theta_j}, \qquad (2)$$

with $\mathbf{u} = \begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix}^\top$ and $\Delta\theta = \begin{bmatrix}\theta_x & \theta_y\end{bmatrix}^\top$. At SIRIUS, like the ORM, the OEORM is a 320×281 matrix, with the dispersive orbit response as the last column.

Regarding SIRIUS sextupole lattice, there are 6 achromatic sextupole families in the straight sections and 15 chromatic families in the arcs [2]. The NOECO scheme purports to calibrate the chromatic lattice only.

OEORM MEASUREMENTS AT SIRIUS

OEORM measurements at the SIRIUS storage ring are carried out according to the following procedure. Energy deviations of $\delta = \pm 0.5\%, \pm 0.25\%$ are introduced into the beam by adjusting the RF master oscillator frequency around the nominal operating frequency f_0. The required frequency shifts Δf are calculated as $\Delta f = -\alpha f_0 \delta$, where δ is the desired energy shift and $\alpha = 1.636 \times 10^{-4}$ is the lattice momentum compaction factor at $\delta = 0$. At each energy offset, a fast "AC" ORM measurement [3] is performed. Using the ORMs measured over the energy shift range together with the ORM at $\delta = 0$, the data are fitted with a linear model of ORM versus energy deviation. The OEORM corresponds to the angular coefficient of the fit.

THE FITTING SETUP

The objective function to be minimized is

$$\chi^2 = \sum_{ij} \frac{|\mathrm{OEORM}^{\mathrm{meas}}_{ij} - \mathrm{OEORM}^{\mathrm{model}}_{ij}|^2}{w_{ij}^2}, \qquad (3)$$

where $\mathrm{OEORM}^{\mathrm{meas}}$ is the measured OEORM and $\mathrm{OEORM}^{\mathrm{model}}$ the model prediction. The weights w_{ij} represent the OEORM experimental noise.

The fitting parameters are the chromatic sextupole strengths (15 knobs) together with calibration gains for BPM readings (320) and calibration gains for corrector strengths (280). The gains enter the minimization as linear transformations applied to the measured OEORM:

$$\mathrm{OEORM}'^{\mathrm{meas}} = G^{\mathrm{BPM}}\, \mathrm{OEORM}^{\mathrm{meas}}\, G^{\mathrm{corr}}. \qquad (4)$$

The parameter vector to be calibrated is organized as

$$\mathbf{P} = \begin{bmatrix}\mathbf{S} & \mathbf{G}^{\mathrm{corr}} & \mathbf{G}^{\mathrm{BPM}}\end{bmatrix}^\top,$$

with dimensions $15 + (120 + 160) + (2 \times 160) = 615$. Off-diagonal elements of the OEORM are not considered in the fitting, eliminating the need of couplings among the horizontal and vertical BPMs signals as additional parameters. The vector $\mathbf{P}$ minimizing the least-squares problem is iteratively calculated until convergence as $\mathbf{P}^{(n+1)} \to \mathbf{P}^{(n)} + \Delta\mathbf{P}^{(n)}$, with Levenberg-Macquardt (LM) steps

$$\Delta\mathbf{P}^{(n)} = -\left[J^\top J + \lambda \,\mathrm{diag}(J^\top J)\right]^{-1} J^\top \mathbf{r}^{(n-1)}, \qquad (5)$$

with $\mathbf{r}^{(n)}$ being the residuals vector at the n-th iteration. The residuals elements $r_k = \left(\mathbf{r}^{(n)}\right)_k$ are obtained from the vectorization of the weighted OEORM residue ath the n-th iteration

$$r_k = \frac{\mathrm{OEORM}'^{\mathrm{meas}}_{ij} - \mathrm{OEORM}^{\mathrm{model}}_{ij}}{w_{ij}}, \qquad (6)$$

* matheus.velloso@lnls.br

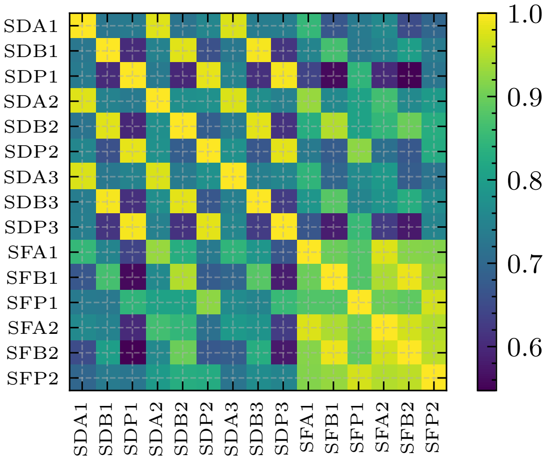

Figure 1: Jacobian columns pairwise correlation, or cosine distance. Columns are highly correlated, meaning many sextupole families display similar OEORM signatures.

with $k = 281i + j, \quad 0 \le i \le 319, \quad 0 \le j \le 280$. J is the Jacobian matrix:

$$J_{kl} = \frac{\partial r_k}{\partial P_l}. \tag{7}$$

The Jacobian columns corresponding to the sextupole knobs are obtained numerically using OEORM calculations in the model. The columns corresponding to correctors and BPMs gains can be calculated analytically.

The LM constant λ is updated adaptively depending on whether χ^2 improves or worsens. Additional Tikhonov/Ridge regularization of the form $\chi^2 \to \chi^2 + \|W^\top \Delta \mathbf{P}\|^2$, with $W = \alpha I_{15}$, is applied to mitigate degeneracies and prevent excessive excursions of the sextupole strengths. With the regularization, the normal equation reads $\Delta \mathbf{P}^{(n)} = -A^{-1} J^\top \mathbf{r}^{(n-1)}$, where

$$A = J^\top J + \lambda \operatorname{diag}(J^\top J) + W^\top W. \tag{8}$$

DEGENERACIES AND PARAMETERS SELECTION

The pairwise correlations between Jacobian columns i and j, defined as $C_{ij} = \frac{J_i \cdot J_j}{\|J_i\|\|J_j\|}$ and shown in Fig. 1, highlight strong co-linearities among the family signatures, particularly within a highly correlated cluster formed by the focusing sextupole families. This indicates that several knobs produce similar signatures in the off-energy orbit responses, limiting the identifiability of individual knobs and potentially leading to an ill-posed optimization problem.

The singular value decomposition of the matrix A from Eq. (8) reveals a condition number of 10^6, which can be improved through an appropriate choice of the LM parameter λ and the Tikhonov regularization parameter α. In our tests, the values $\lambda = 10^{-3}$ and $\alpha = 10^2$ raised the singular value spectrum, improved the conditioning, attenuated degeneracies while still provided an acceptable convergence time.

FITTING RESULTS

Error Identification in the Machine

Our initial test of the NOECO scheme consisted on benchmarking the method's ability to identify intentionally introduced perturbations. Three OEORMs were measured under three sextupoles strength configurations. The first corresponds to the current operational strengths (OS), defined in 2023 through online optimization of injection efficiency and dynamic aperture [4]. The other two measurements use the same operational settings with additional point perturbations of +1% and +2% applied to the SDP2 sextupole family. We denote these measurements as OEORM0 (OS), OEORM1 (+1% perturbation), and OEORM2 (+2% perturbation).

An ORM measurement at operation sextupoles strengths and nominal energy is used to fit a LOCO model which in turn was used both to calculate the NOECO Jacobian and to perform all OEORM evaluations. The NOECO fit free parameters consisted solely of the sextupole family strengths, since BPM and steerer gains were not included in the fit and their values obtained from the LOCO model were used instead. The LM parameter λ was initialized at 10^{-3} and updated by a factor of 10 each iteration, depending on whether χ^2 improved or worsened. A fixed Tikhonov regularization of $\alpha = 10^2$ was applied to sextupole knobs.

Figure 2-a) shows the evolution of χ^2/dof for each OEORM measured, while in Fig. 2-b) the fitted strengths are shown as relative variations with respect to the model nominal strengths. The blue curves correspond to the fit to the OS, whereas the green and orange curves correspond to the +1% and +2% perturbations, respectively. The black curve represents the control system strength readings, which are strength estimates from the magnet excitation curves and the design beam energy. Although χ^2/dof decreases throughout the minimization, its final value for OEORMs 1 and 2 remains above unity, as values close to one would be expected for a representative model and weithing w_{ij}. This may indicate incomplete modeling.

The strengths fitted to the operation strengths resemble the control system estimates and yield model chromaticities $(\xi_x, \xi_y) = (3.17, 2.91)$, in reasonable agreement with the measured values of $(3.11, 2.67)$. Without machine measurements of nonlinear optics functions, however, it is difficult to assess whether the model with NOECO-fitted strengths also reproduce the machine's nonlinear optics, which would serve as an additional validation of the calibration. The method's ability to fit the nonlinear optics and provide a representative sextupole lattice model can still be validated with iterative correction or symmetrization, which is an upcoming effort.

Regarding the introduced perturbations, NOECO was not able isolate them within the SDP2 family alone, highlighting the issues with identifiability anticipated by the analysis of the Jacobian columns co-linearities.

Error Identification in the Model

Given the method's difficulty in identifying isolated perturbations, model-based simulations were carried out to assess

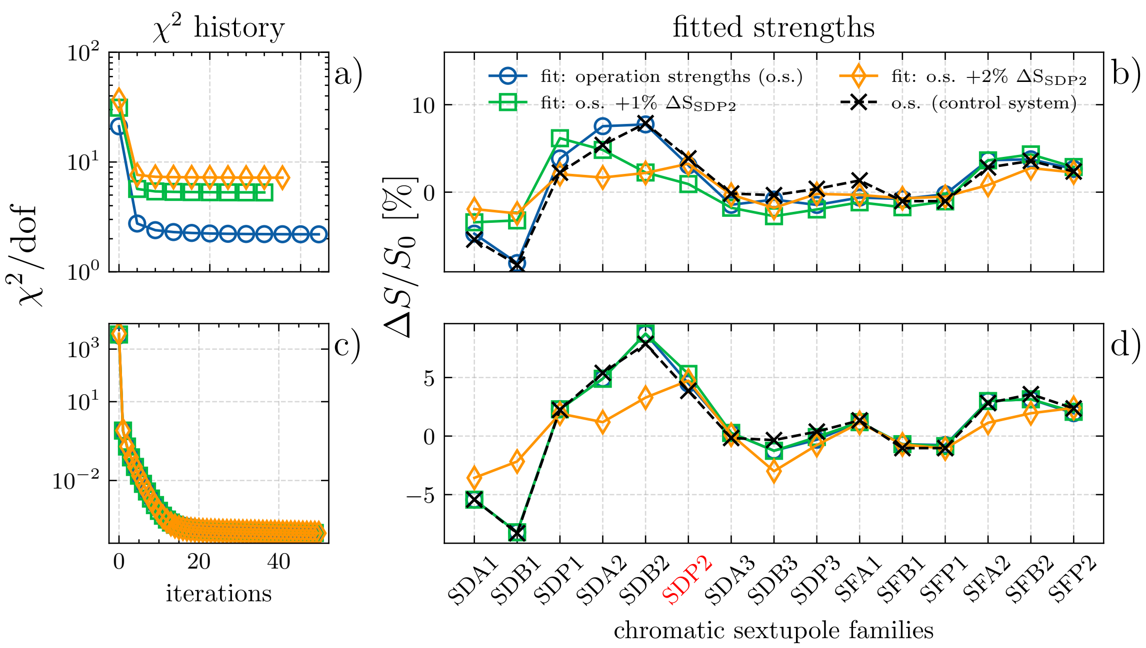

Figure 2: History of objective function throughout the NOECO fittings for machine measured data (a) and model data (c). Relative sextupole strengths variations required to explain measured OEORMs (b) and simulated OEORMs (d). Strength perturbations are inserted to the SDP2 family (red label). Black curve

the extent to which identifiability is feasible. In the tests, when a given machine model is perturbed solely in the sextupole lattice, the NOECO fitting is able to reproduce the perturbed strengths, both for isolated perturbations and for distributed perturbations across all families. However, if additional optics mismatches are present, the correct identification of perturbations is no longer guaranteed. A small perturbation of 0.3% in a single chromatic quadrupole, introducing a slight tune variation at the third decimal place and a sub-1% beta-beating, is already sufficient to limit the identifiability of isolated perturbations. This is a typical magnitude of quadrupole gradient variation that can be left uncorrected in the machine during LOCO optics symmetrization.

The simulations proceeded as follows. In the machine model, a small quadrupole variation is introduced together with the nonlinear lattice strengths that are intended to be identified. We used the same OS from the control system readings as the reference strengths. The OEORM is then calculated for this perturbed model and used as the target OEORM to be reproduced. The nominal, unperturbed model used internally by NOECO to calculate the OEORM of candidate strengths and the Jacobian has its tunes and chromaticities corrected to match those of the perturbed model being fitted. The fit to the reference strengths and isolated perturbations of +1% and +2% to SDP2 are then performed using the same hyperparameters and setup adopted in the machine OEORM fits.

Figure 2-d) presents the fitted strengths for the model tests. The reference strengths and the +1% perturbation are correctly identified, while the +2% perturbation is jointly attributed to several correlated families. We speculate that, due to the strong co-linearities among the OEORM signatures, even slight optics mismatches introduce additional OEORM components that cannot be fully represented by the nominal model. These incompatible signatures may then be projected onto particular strength signatures within highly correlated families that can partially reproduce the observed OEORM variation. Since small optics mismatches between the machine and the model are unavoidable, the ability of NOECO to identify isolated perturbations in the SIRIUS case appears to be fundamentally limited.

CONCLUSIONS

An initial assessment of NOECO applicability to the SIRIUS storage ring has been presented. The method was applied both to fit the operational sextupole strengths and to benchmark its ability to identify intentionally introduced perturbations. The fitted strengths reproduced the measured chromaticities with reasonable accuracy and showed good agreement with control system estimates.

When isolated perturbations were introduced into the operational strengths, however, NOECO redistributed them across correlated families rather than identifying them individually. Model-based simulations also showed limited identifiability, particularly in the presence of optics mismatches. This behavior reflects the quasi-degeneracy intrinsic to the SIRIUS sextupole, corrector, and BPM layout, as evidenced by the strong correlations in the OEORM Jacobian.

Overall, the results indicate that OEORM-based calibration may be feasible at SIRIUS, although its resolving power is fundamentally limited by strong parameter correlations. Overcoming these limitations may require additional or alternative observables capable of breaking the degeneracies.

REFERENCES

[1] D. K. Olsson, Å. Andersson, M. Sjöström, "Nonlinear optics from off-energy closed orbits", *Phys. Rev. Accel. Beams*, vol. 23, p. 102803, 2020. doi:10.1103/PhysRevAccelBeams.23.102803

[2] L. Liu, X. R. Resende, and F. H. de Sá, "A new optics for Sirius", in *Proc. IPAC'16*, Busan, Korea, May 2016, pp. 3413–3416. doi:10.18429/JACoW-IPAC2016-THPMR013

[3] M. M. S. Velloso, M. B. Alves, and F. H. de Sá, "Fast orbit response matrix measurement via sine-wave Excitation of correctors at Sirius", in *Proc. IPAC'22*, Bangkok, Thailand, Jun. 2022, pp. 425–428. doi:10.18429/JACoW-IPAC2022-MOPOTK002

[4] M. M. S. Velloso, M. B. Alves, L. Liu, X. Huang, X. R. Resende and F. H. de Sá, "Online optimization of SIRIUS nonlinear optics", in *Proc. IPAC'23*, Venice, Italy, May 2023, pp. 3302–3305. doi:10.18429/JACoW-IPAC2023-WEPL087