ANALYTICAL ESTIMATES OF BEAM INTENSITY LIMITATIONS IN THE EIC BEAM ACCUMULATOR RING

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Abstract

We have done analytical estimates of beam intensity limitations in the Beam Accumulator Ring (BAR), a part of the Electron-Ion Collider injector complex under development at Brookhaven National Laboratory. Analytical models of longitudinal and transverse collective effects are developed using a broadband impedance model calibrated with the beam-based measurement results from NSLS VUV. The study evaluates the impact of resistive-wall and geometric impedances, assessing microwave and transverse mode coupling instabilities, bunch lengthening, and beam-induced heating.

INTRODUCTION

The Electron-Ion Collider (EIC) project is under development at Brookhaven National Laboratory to explore the fundamental structure of matter. The Beam Accumulator Ring (BAR) [1] is a part of the EIC injector. It is dedicated to accumulating a polarized electron beam with the high single-bunch intensity required by the Electron Storage Ring (ESR). The beam with a charge of about 1 nC is injected from the linac into BAR with a repetition rate of 30 Hz. Once the single-bunch charge of 28 nC is achieved, the beam is extracted from the BAR and injected into the Rapid Cycling Synchrotron (RCS), where it is accelerated up to the full energy of 5 to 18 GeV for injection into the ESR.

To provide a cost-efficient solution, the ring circumference was reduced from 50 m of the National Synchrotron Light Source Vacuum Ultraviolet (NSLS VUV) ring to 36 m, and the lattice design evolved from four Double-Bend Achromat cells to two Four-Bend Achromat cells. The BAR beam parameters related to the collective effects are summarized in Table 1, its schematic layout is shown in Fig. 1.

IMPEDANCE MODEL

To estimate the collective effects of beam dynamics, we assume a vacuum chamber with a rectangular cross-section 80 mm × 40 mm (Fig. 2) made of aluminum as a basic option and also consider a 316L stainless steel as a cost-saving option.

The resistive-wall impedance is calculated by analytical equations for a rectangular vacuum chamber [2]. The geometric impedance is approximated by an equivalent broadband resonator (BBR) model, which is adequate to estimate single-bunch effects of low-frequency beam-impedance interaction, such as bunch lengthening and transverse head-tail mode coupling. To determine the resonance frequency and shunt impedance of the BBR model, we fit the intensity-dependent bunch lengthening with experimental data [3] measured at the NSLS VUV. The total longitudinal impedance consisting of the BBR and resistive-wall parts is shown in Fig. 3, assuming the aluminum chamber.

Table 1: Lattice and Beam Parameters

Parameter	Value
Energy (GeV)	0.75
Circumference (m)	36.52
Hor. tune	2.42
Ver. tune	1.46
Natural hor. chromaticity	-3.8
Natural ver. chromaticity	-1.6
Momentum compaction	0.084
Hor. damping time (s)	0.0073
Ver. damping time (s)	0.012
Long. damping time (s)	0.0096
Energy spread	5.78×10^{-4}
Energy loss per turn (keV)	14.66
Revolution frequency (MHz)	8.2089
Aver. hor. beta (m)	7.2
Aver. ver. beta (m)	4.7
Aver. hor. dispersion (m)	0.72
RF voltage (kV)	100
RF frequency (MHz)	98.5062
RF harmonic	12
Synchrotron tune	0.0046
Bunch length (mm)	61.5
RF acceptance	0.008

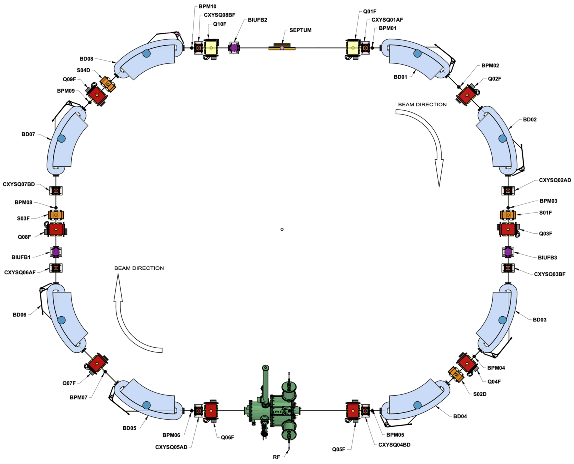

Figure 1: BAR layout.

The transverse resistive-wall impedance is also calculated by the analytical equations [2], and the transverse geometric impedance is modeled by an equivalent broadband resonator with the same resonance frequency and quality factor as

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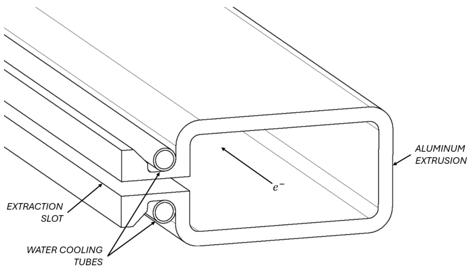

Figure 2: Vacuum chamber cross section.

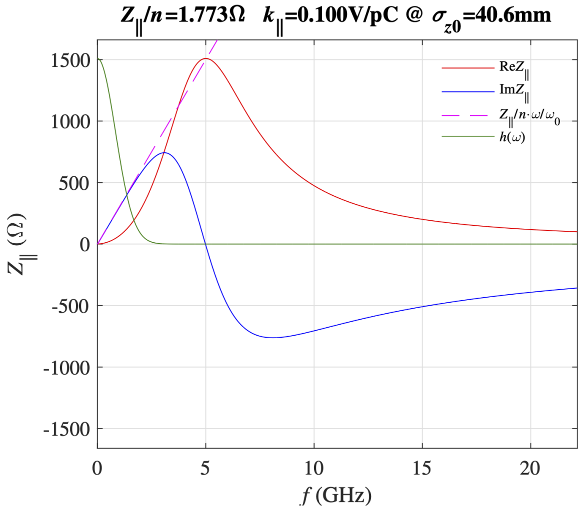

Figure 3: Model longitudinal broadband impedance.

for the longitudinal one. The horizontal and vertical shunt impedances are scaled from the longitudinal one following the Panofsky-Wenzel theorem $R_{x,y} = R_\parallel \frac{2c}{A_{x,y}\omega_r}$, where ω_r is the resonance frequency and $A_{x,y}$ is the width/height of the chamber.

LONGITUDINAL COLLECTIVE EFFECTS

If the bunch current I_b does not exceed the threshold of microwave instability (MWI), the beam interaction with a broadband longitudinal impedance $Z_\parallel$ results in the intensity-dependent growth of the bunch length σ_s caused by the potential well distortion [4]:

$$\left(\frac{\sigma_s}{\sigma_{s0}}\right)^3 - \frac{\sigma_s}{\sigma_{s0}} = \frac{I_b\,\alpha_c}{4\sqrt{\pi}\,\nu_s^2\,E/e}\,\frac{R_{\mathrm{av}}^3}{\sigma_{s0}^3}\,\mathrm{Im}\left(\frac{Z_{\parallel\mathrm{eff}}}{n}\right), \qquad (1)$$

where $\nu_s = \omega_s/\omega_0$ is the synchrotron tune; σ_{s0} is the bunch length at zero intensity, α_c is the momentum compaction factor, E is the beam energy, $n = \omega/\omega_0$, R_{av} is the average machine radius. The efficient impedance $Z_{\parallel\mathrm{eff}}$ is the impedance weighted with the bunch power spectrum.

Above the threshold, the MWI results in the intensity-dependent increase of the beam energy spread σ_δ

$$\sigma_\delta^3 = \frac{\nu_s}{\sqrt{2\pi}\,\alpha_c^2\,E/e}\,I_b\left|\frac{Z_{\parallel\mathrm{eff}}}{n}\right|, \qquad (2)$$

causing the turbulent bunch lengthening [5]:

$$\left(\frac{\sigma_s}{R_{\mathrm{av}}}\right)^3 = \frac{I_b\,\alpha_c}{\sqrt{2\pi}\,\nu_s^2\,E/e}\left(\left|\frac{Z_\parallel}{n}\right|_{\mathrm{cr}} + \frac{Z_{\parallel\mathrm{eff}}}{n}\right), \qquad (3)$$

where $\left|\frac{Z_\parallel}{n}\right|_{\mathrm{cr}}$ is the total impedance at the critical frequency $\omega_{\mathrm{cr}} = c/\sigma_s$ determined by the bunch length.

Using Eqs. (1)–(3) and the model impedance shown in Fig. 3, the intensity-dependent bunch length and energy spread were calculated. We also carried out numerical simulations using the ELEGANT [6] code. Figure 4 shows the current-dependent bunch length calculated analytically and numerically compared with the data measured at NSLS VUV [3]. As one can see, the calculated and measured data show quite a good agreement.

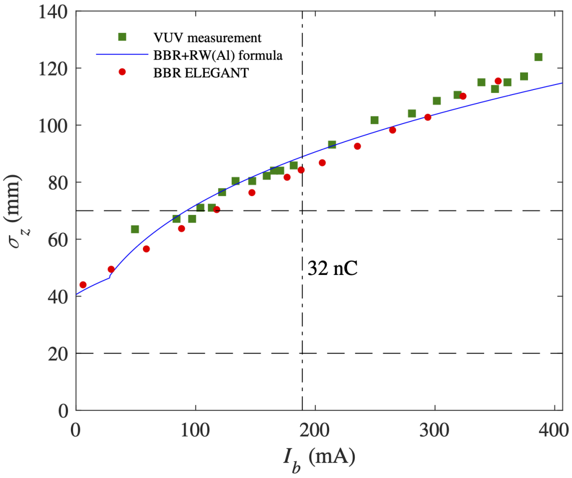

Figure 4: Intensity-dependent bunch length.

Although no measured intensity-dependent energy spread is available, the analytical and numerical results are consistent, as shown in Fig. 5.

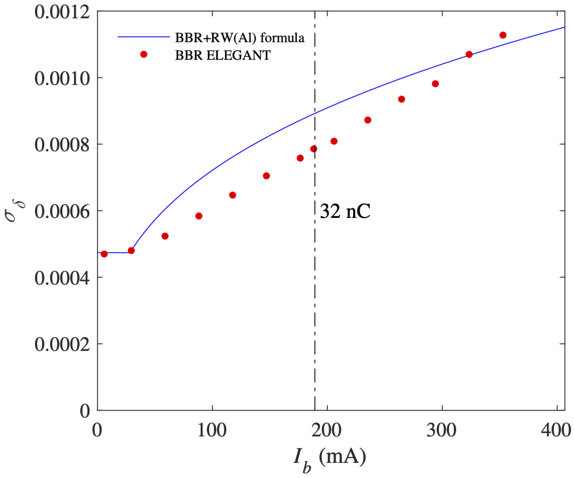

Figure 5: Intensity-dependent energy spread.

The interaction of a single-bunch beam with the real part of the longitudinal impedance results in the vacuum chamber heating by the dissipated power

$$P = k_\parallel \frac{I_b^2}{f_0}, \qquad (4)$$

where f_0 is the revolution frequency, $k_\parallel$ is the longitudinal loss factor. Taking into account the BAR duty cycle, the average beam-induced power is about 30 W for aluminum and about 50 W for stainless steel, and this level of power is not a matter of concern.

TRANSVERSE COLLECTIVE EFFECTS

Transverse mode coupling instability (TMCI) caused by the beam interaction with the transverse broadband impedance is one of the main factors limiting single-bunch beam intensity. Using the model impedance including resistive-wall and geometric contributions, we calculated the current-dependent shift of betatron tunes:

$$\frac{d\nu_{x,y}}{dI_b} = \frac{\left\langle \beta_{x,y} \right\rangle k_{x,y}}{2\omega_0 E/e}\,, \qquad (5)$$

where $\left\langle \beta_{x,y} \right\rangle$ is the average horizontal (x) or vertical (y) beta function, and $k_{x,y}$ is the kick factor. Figure 6 shows the horizontal and vertical tune shifts calculated with the bunch lengthening (Fig 4) taken into account. Since the vertical aperture of the vacuum chamber is smaller and the impedance is higher, the TMCI instability occurs if the vertical current-dependent tune shift is approaching the synchrotron tune ν_s.

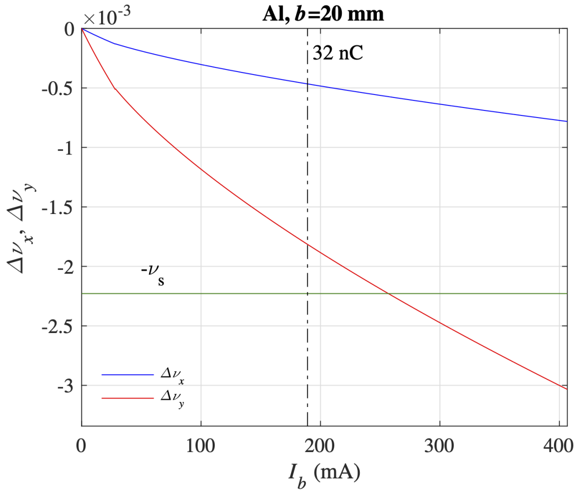

Figure 6: Intensity-dependent shifts of the betatron tunes for aluminum vacuum chamber.

The mode coupling theory [7, 8] allows us to carry out a more accurate analysis. For a Gaussian beam interacting with a broadband transverse impedance, the complex frequency $\Omega = \omega + i/\tau$ of l-th head-tail mode can be found by solving the eigenvalue problem

$$\det\left[\left(\frac{\Omega - \omega_\beta}{\omega_s} - l\right)\mathbf{I} - \mathbf{M}\right] = 0\,, \qquad (6)$$

here ω_β is the unperturbed betatron frequency, τ is the rising/damping time. The matrix elements are

$$M_{kk'} = I_b \frac{\beta}{2\nu_s E/e} \sum_{p=-\infty}^{\infty} Z_\perp(\omega') g_{lk}(\omega' - \omega_\xi) g_{lk'}(\omega' - \omega_\xi)\,, \qquad (7)$$

where β is the average beta function, $\omega' = p\omega_0 + \omega_\beta + l\omega_s$, $\omega_\xi = \xi\omega_0/\alpha_c$ is the chromatic frequency (ξ is the chromaticity). The functions characterizing oscillation modes of the Gaussian bunch are:

$$g_{lk}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}}\left(\frac{\omega\sigma_t}{\sqrt{2}}\right)^{|l|+2k} \exp\left(-\frac{\omega^2\sigma_t^2}{2}\right)\,. \qquad (8)$$

We solve the eigenvalue problem in Eq. (6) using the total vertical impedance to obtain the intensity-dependent shift of the betatron frequency and the damping/growth rate, assuming zero chromaticity and taking the intensity-dependent bunch lengthening into account. This approach has been benchmarked with beam-based measurements of the head-tail mode coupling [9], the measured results showed a good agreement with the theory. The result is presented in Fig. 7, the instability occurs when the head-tail mode 0 (center-of-mass motion) approaches mode -1. As one can see, the beam is stable at the BAR maximum charge of 32 nC with a decent margin.

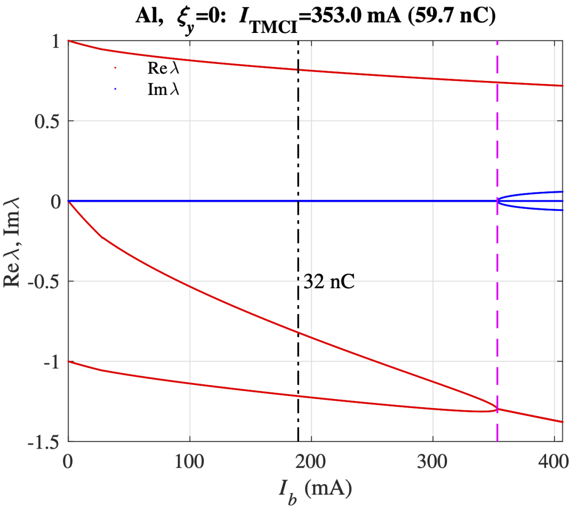

Figure 7: Intensity-dependent head-tail mode coupling.

CONCLUSION

We report an assessment of collective effects in the Beam Accumulator Ring of the EIC injector, focusing on impedance-driven instabilities and beam-induced heating. Using a broadband impedance model calibrated with empirical data from the NSLS VUV ring, the analysis demonstrates that the BAR can accommodate the design goal of a 32 nC single-bunch charge. With zero chromaticity, the transverse mode coupling instability threshold is higher than the design beam current. Although the threshold of the longitudinal microwave instability is quite low, this instability does not limit the beam intensity but results in significant bunch lengthening and energy spread growth. These results support the feasibility of the BAR design while highlighting critical constraints for future design iterations.

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