MUON PRODUCTION WITH TWO-BEAM FFA COLLIDER FOR MuCF

Y. Mori*, T. Uesugi,Y. Ishi, KURNS, Kyoto University, Osaka, Japan

Abstract

Muon-catalyzed fusion (MuCF), facilitated by negative muons, is regarded as a promising candidate for future energy generation. To realize this potential, it is imperative to produce negative muons with maximal energy efficiency. In this study, we propose a hadron beam collider based on a two-beam Fixed-Field Alternating Gradient (FFAG) accelerator as a viable approach to achieve this objective, and we examine its feasibility.

INTRODUCTION

The muon, a fundamental lepton, serves as a versatile probe in numerous scientific domains. Among its prospective applications, muon-catalyzed fusion (MuCF) stands out as a promising candidate for future energy generation [1, 2]. This process is initiated when a negative muon is captured by a deuterium-tritium (DT) molecule, forming a muonic DT complex. Owing to the muon's mass approximately 200 times greater than that of the electron, the internuclear separation between the deuteron and triton is reduced by a comparable factor relative to an ordinary DT molecule, thereby greatly enhancing the probability of fusion.

Experimental investigations [1] have demonstrated that a single negative muon can catalyze on the order of γ_f=150-200 successive fusion reactions before the catalytic cycle(γ_f) terminates, typically when the muon becomes bound to an alpha particle (helium nucleus) produced in the reaction. Despite its conceptual appeal, this mechanism encounters significant challenges with respect to practical energy production.

According to the study by Jandel et al. [3], an effective thermal energy released per fusion E_f is given by,

$$E_f = 25MeV + E_\mu(MeV)/\gamma_f. \quad (1)$$

Here, E_μ is the energy cost per muon, and the released energy, 25 MeV, is not simply the $d + t \rightarrow \alpha + n$ fusion yield, 17.6MeV, but contains the energy gain from the breeding of tritium by neutron break up at of ^6Li. The last term of Eq.(1) must be less than 15% of E_f for a viable energy reactor.

For MuCF to be economically viable, the energy expenditure required to generate a single negative muon, E_μ, must be smaller than 1GeV or less. In realistic energy production scenarios, an even lower production energy would be necessary.

At present, the most efficient method for generating negative muons is via the decay of negative pions $\pi^- \rightarrow \mu^- + \bar{\nu}_e$. These pions are produced through the excitation of NΔ resonances in nuclear interactions, typically arising when high-energy hadrons such as deuterons collide with neutron-rich targets. However, prior studies [3] indicate that fixed-target approaches have thus far failed to produce negative muons at energies below 2.5 GeV, thereby precluding attainment of the break-even condition.

As a consequence, muon-catalyzed fusion has often been regarded as impractical, leading to a decline in sustained research interest. However, some studies have been conducted to enhance the fusion catalytic cycles(γ_f) to overcome this difficulty [4].

The center-of-mass energy required to excite the NΔ resonance in nucleon-nucleon interactions is approximately $\sqrt{s}$=2.34 GeV/u [5], corresponding to about 1.0 GeV per nucleon in the laboratory frame. This implies that, in head-on nucleon collisions, each nucleon must possess kinetic energy exceeding roughly 230 MeV per nucleon. Under such conditions, the energy cost associated with muon production can be substantially reduced to less than 1 GeV. Accordingly, efficient MuCF implementation would benefit from head-on collisions involving neutron-rich projectiles, such as tritons.

To address these limitations, this work proposes a two-beam Fixed-Field Alternating Gradient (FFA) collider. The concept of counter-rotating beams of identical charge states within a single storage ring was originally introduced by Ohkawa in 1958 [6], forming the basis of a two-beam scaling FFA configuration. In this arrangement, oppositely circulating beams of same charge state particles effectively reduce the net beam current, thereby mitigating wakefield induced instabilities and enabling enhanced luminosity.

Within this framework, the effective energy cost of muon production can be reduced to less than 30% of the energy yield per muon in MuCF. Moreover, a muon production rate exceeding $2\times10^{15}\,\mu^-$/s becomes feasible, corresponding to an energy output on the order of 1 MW, thereby significantly advancing the practical prospects of muon-catalyzed fusion.

TWO-BEAM FFA COLLIDER

In comparison with a conventional two-ring collider, a two-beam FFA collider exhibits several distinctive characteristics.

(1) The beam optics of a two-beam FFA collider adhere to a scaling law under which the chromaticity is strictly zero. Consequently, multiple colliding beam orbits with slightly differing momenta can coexist within a single ring.

(2) The system accommodates a large number of interaction points, with two collision points situated within each FODO cell.

(3) The net circulating beam current is significantly reduced—approaching zero—due to the counter-rotation of beams with identical charge states within the ring. This characteristic effectively mitigates beam instabilities induced by wakefield effects.

* mori.yoshiharu.4w@kyoto-u.ac.jp

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In the following sections, these features are examined in detail, and the feasibility of a two-beam FFA collider for muon production in muon-catalyzed fusion is systematically evaluated.

Beam Optics of Linear Model

The scaling-type FFA accelerator exhibits a distinctive characteristic in its beam optics when compared with other strong-focusing ring accelerators [7]. Specifically, the chromaticity is identically zero, as the closed orbits corresponding to different beam energies are geometrically similar.

Accordingly, it is essential that orbit similarity be maintained for all closed trajectories with varying momenta, alongside a constant geometrical magnetic field index.

Under these conditions, the scaling FFA ring satisfies the following fundamental principles governing beam optics.

$$\frac{d\rho}{dp} = 0, \quad and \quad \frac{dk}{dp} = 0. \tag{2}$$

Here ρ is the curvature of the closed orbit and k is the geometrIcal field index. From these two equations, the shape of the magnetic field that constitutes the FFA ring is given by the following equation:

$$B_z(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k f(\theta). \tag{3}$$

Strong AG focusing can be achieved by combining normal(F) and reverse field(D) magnets.

Figure 1(a) shows a half cell of two-beam FFA lattice schematically. Here, ρ_f and ρ_d are orbit curvatures of the focusing and defocusing magnets, respectively.

The optical stability region, as constrained by the two-beam scaling law, is determined by the geometrical field index (k), the number of cells (N), and the θ_f/θ_d (f/d) ratio of a ring composed of FD doublets, analyzed within the linear approximation. Figure 2 illustrates the optical stability region for n_c and k, assuming a magnet packing factor of 0.75, where this factor represents the fraction of the cell orbit occupied by the F and D magnets. In Fig.3, the acceptance of the ring for horizontal and vertical directions is presented respectively.

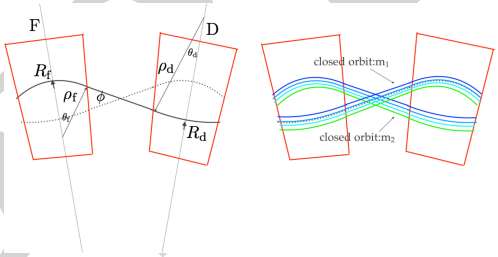

Figure 1: (a) left:FODO two-beam parameters presented in Eq.1 (b) right:Closed orbits scaled to various beam momenta.

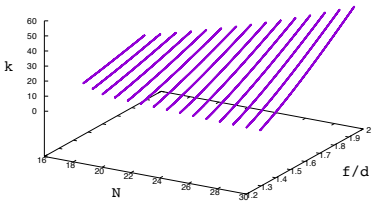

Figure 2: A stable area for the number of cells(N), k-value(k) and $\theta_f/\theta_d(f/d)$.

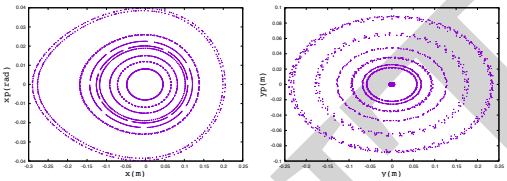

Figure 3: Acceptance (left:horizontal,right:vertical)

Intersecting Angle

The beam dynamics of a two-beam FFA collider were analyzed in detail by Ohkawa. Although the magnetic field described in Eq. (3) contains higher-order harmonics, their amplitudes are relatively small. Therefore, assuming a purely sinusoidal magnetic field configuration for a singlet FODO lattice represents a reasonable approximation. The colliding half-angle at the beam intersection is approximately given by:

$$\phi \simeq \left(\frac{2}{k+1}\right)^{\frac{1}{2}}. \tag{4}$$

Typical parameters and characteristics of the two-beam FFAG collider ring are summarized in Table 1. The ring is composed of 24 singlet FD cells and the average radius is 5m.

Table 1: Specification and Parameters of Two-beam FFA Collider for MuCF

Specification	**Parameter**
particle	triton(Z=1,A=3)
kinetic energy	230 MeV/u
$\sqrt{s}$	2.34 GeV/u
$\Delta p/p$	0.1
ring radius	5 m
number of cells	24
k	20
β_f	3.5 deg
β_d	3.5 deg
β_s	1 deg
half collision angle(ϕ)	0.308 rad

Luminosity

Generally, a particle collider designed to collide particles of the same charge state employs two separate storage rings,

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in which the beams intersect at a specific collision angle (2ϕ). This configuration is referred to as an intersecting storage-ring type collider.

The event rate per unit volume is expressed as [8]:

$$\frac{d^2N}{dtdV} = \sigma n_1 n_2 \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{(\vec{v}_1 \times \vec{v}_2)^2}{c^2}}. \quad (5)$$

Here, σ is the reaction cross section, c the light velocity and dV the volume of the interaction region (two-beam overlapping region). The particle density and velocity of each beam are n_1 and n_2, and $\vec{v}_1$ and $\vec{v}_2$, respectively.

The luminosity,L, is defined as, $dN/dt = \sigma L$. In a coasting beam collider, the luminosity at a collision point is given by,

$$L = \frac{\lambda_1 \lambda_2 \beta c \sqrt{1 - \beta^2 \sin\phi^2}}{\tan\phi \times h_{eff}}. \quad (6)$$

Here, λ_1 and λ_2 are the particle line densities and heff is an effective beam height, respectively, and $h_{eff} = 2\sqrt{\pi}\sigma_y$ where σ_y is the rms beam size.

In the case of the proposed two-beam FFA collider, the total number of beam collision points equals twice the number of beam cells, i.e., $2N_c$.

The number of particles that can be stored in the collider ring is limited by the effects of space-charge forces, beam-beam tune shifts, and incoherent space-charge tune shifts. Under the weak-strong approximation at non-relativistic energies, these effects are fundamentally equivalent. Accordingly, the particle population can be estimated based on the incoherent space-charge effect.

In an ordinary two-ring collider, β_y^* is designed to be as small as possible in order to improve the beam luminosity. In this case, when the beam-beam tune shift is less than $\Delta Q = -0.05$ that is about 1/5 of the cell tune, the particle line density λ becomes,

$$\lambda = 3.62 \times 10^{13} [cm^{-1}]. \quad (7)$$

The total luminosity of the ring becomes,

$$L = 1.05 \times 10^{40} [cm^{-2} s^{-1}]. \quad (8)$$

The $N\Delta$ resonance process [5] [9] provides an estimate of the negative muon production cross section of the reaction at triton−triton collision, which is approximately 2×10^{-25} cm^2. Thus, the negative muon production yield becomes approximately $2\times 10^{15}/sec$, which can generate about 1MW output power.

In a two-beam scaling-type FFAG ring, if the particle populations in both beams are equal, the net circulating current approaches zero, as beams of identical charge states travel in opposite directions. This configuration effectively mitigates beam instabilities in the coasting beam, including negative-mass and microwave instabilities induced by wakefield effects. According to the Keil-Schnell-Boussard criterion, the net beam current must remain below 10 kA to prevent microwave instabilities arising from a space-charge impedance of approximately 200Ω. This condition can be readily achieved in a two-way FFAG collider.

SUMMARY

A new scheme of negative muon production with two-beam FFAG collider is proposed. The colliding particles in the ring are the triton and the center-of-mass of beam energy is $\sqrt{s}$=2.15GeV/u which is about a kinetic energy of 142MeV/u for each colliding particle. A beam luminosity of more than 10^{40} $cm^{-2}s^{-1}$, generating more than 1MW output power, could be reachable. Although the circulating beam current for each beam is more than 100kA, the beam direction is opposite so that the net beam current can be negligible and the beam wake instabilities could be negligible.

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