ACCELERATING THROUGH PARTIAL SNAKE RESONANCES WITH BETATRON COUPLING SPIN RESONANCE COMPENSATION

V. Schoefer*, L. Hajdu, K. Hock, H. Huang, J. Laster, W. Lin, K. Zeno
Brookhaven National Laboratory, Upton,NY, USA
E. Hamwi, Cornell University (CLASSE), Ithaca, NY, USA

Abstract

Partial Siberian snakes are used at the Brookhaven Alternating Gradient Synchrotron (AGS) to avoid strong vertical spin resonances during acceleration of polarized proton beam from 2.5 to 23 GeV. An unfortunate side effect is that these snake magnets excite numerous weak resonances associated with the horizontal betatron motion. We present experimental demonstration and operational experience of the compensation of these resonances during acceleration by exciting betatron coupling with a set of 15 pulsed skew quadrupoles.

POLARIZED BEAM ACCELERATION IN AGS

The AGS at Brookhaven National Laboratory served as the injector for the Relativistic Heavy Ion Collider (RHIC) for the entirety of the RHIC physics program, which recently concluded in February 2026 [1]. It will continue to be the injector for the planned Electron Ion Collider for all hadron species including polarized protons and helium-3.

Polarization of proton beam is preserved in the AGS mainly by use of a pair of helical dipole partial snakes, separated by 1/3 of the ring circumference, that rotate the proton spin about the longitudinal direction by 18°and 10.8°, respectively. These magnets allow avoidance of the strongest depolarizing resonances associated with the vertical beam motion. In the presence of horizontal betatron motion, however, partial snakes can themselves drive numerous weak horizontal intrinsic resonances which collectively can lead to substantial polarization loss. Polarized protons are accelerated in the AGS from $G\gamma$= 4.5 to 45.5 which requires passing through 82 horizontal intrinsic resonances where $G\gamma = N \pm \nu_x$ (G is the anomalous gyromagnetic ratio, γ the relativistic factor and N is an integer). Left uncorrected, the horizontal resonances in the AGS result in a 15-20% relative loss in proton beam polarization [2].

Since 2011, a fast tune jump has been used to increase the resonance crossing rate, but this only prevents about half the polarization loss, leaving an 8–10% drop [3–5]. Instead of jumping through the resonances and incurring residual polarization loss, these resonance can instead be compensated and reduced to zero strength via additional betatron coupling. A set of 15 fast ramping thin skew quadrupoles has been installed in the AGS lattice and deployed operationally to individually compensate each of the 82 resonances. The report the correction mechanism and our operational experience.

Horizontal Resonances

The primary purpose of the AGS partial snakes is to manipulate the spin tune in such a way as to allow avoidance of strong vertical intrinsic resonances and to produce dominating imperfection resonance terms to allow for total spin flips instead of polarization loss at imperfection resonance [6]. A particle executing horizontal betatron oscillations will experience an additional resonance term with frequency at the horizontal betatron tune, ν_x. In the spinor formalism, we can model the spin motion as

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} F & -\xi \\ -\xi^* & -F \end{pmatrix}\Psi \tag{1}$$

with

$$\begin{aligned} \xi &= iX_s\delta(\theta-\theta_s) + \epsilon_x e^{i\nu_x\theta} \\ F &= G\gamma - (1+G\gamma)x''(\theta)\rho \end{aligned}$$

where $x(\theta)$ is the particle position relative to the reference orbit, X_s is the angle of rotation produced by the snake, δ is the periodic Dirac delta, θ_s the azimuthal position of the snake and ρ is the bend radius of the local reference orbit. Prime denotes differentiation with respect to the azimuthal angle. A particle on the design orbit has $x = 0$, $\epsilon_x = 0$, so only the intended precessional and snake drive term remain. The term proportional to ϵ_x is an additional resonant term excited at the ν_x by, for example, betatron coupling.

It can be shown that this motion results in a net resonance term given by [7]

$$\begin{aligned} \epsilon = &\frac{1}{2\pi}\int \delta_n(\theta-\theta_s)e^{-iG\gamma\theta+iK\theta} \\ &\times \left[1-\left(\tilde{x}'\frac{(1+G\gamma)}{2}+\epsilon_x\right)e^{\pm i\nu_x\theta}\right]d\theta\,. \end{aligned} \tag{2}$$

The unity term in square brackets is just the snake imperfection. The term proportional to $\tilde{x}'$ is a driving term that appears due to the modulation of the spin phase that occurs, turn by turn, due to betatron motion at the entrance to the snake. One can see that by exciting a particular coupling resonance it is possible to reduce this intrinsic term to zero.

RESONANCE COMPENSATION

For a typical horizontal tune near 8.72 and the standard ramp rate ($dG\gamma/d\theta = 4.7 \times 10^{-5}$), the horizontal resonance crossings are separated in time by 4-5 ms. Between injection and extraction energy ($G\gamma = 4.5 - 45.5$) eighty-two of these resonances are crossed. The main difficulty in designing a compensation system is that the relative phasing of a resonance term driven by a skew quad and that of the

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[*] schoefer@bnl.gov

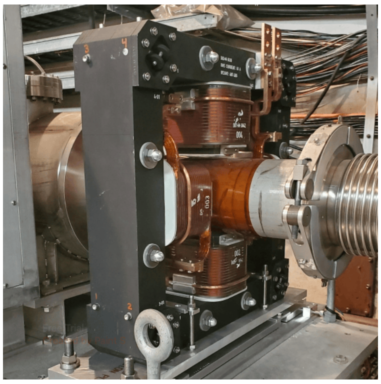


Figure 1: Thin skew quadrupole for snake resonance compensation.

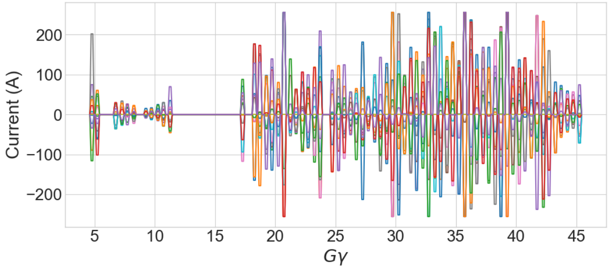


Figure 2: Typical excitation currents for the skew quadrupole correctors. Each trace is a different magnet, independently powered. The gap around $G\gamma$=15 is near the transition crossing. These were omitted during commissioning.

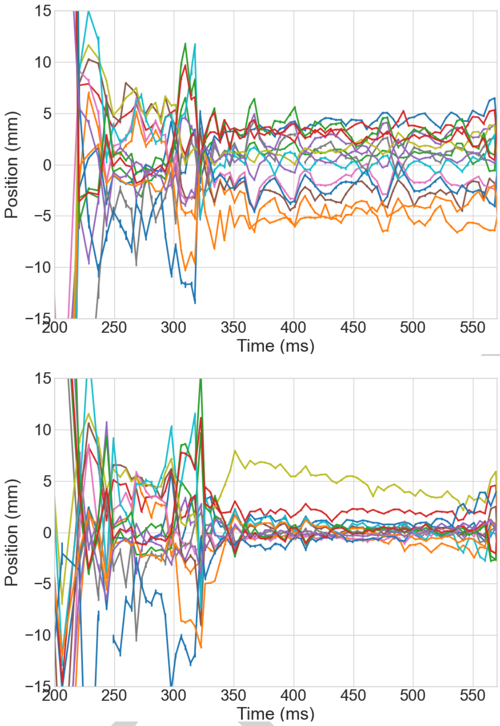


Figure 3: Horizontal orbit position in each skew quadrupole during acceleration. Before (top) and after (bottom) correction. Position is inferred from vertical difference orbits resulting from skew quadrupole excitation.

snakes is highly energy dependent. This means that even though there are only four beam parameters to control (two complex terms each for the spin resonance and to control the overall coupling), more than four skew quadrupole knobs are required to compensate for all of the resonances.

A set of 15 skew quadrupoles is chosen for the AGS. The quadrupoles (Fig. 1) are only 17 cm in physical length to maximize the number of possible installation locations and thereby optimize the correction phasing. The magnets pulse with ramp up and down times of 1 ms and hold a flattop for 1.3 ms centered on each resonance crossing time. Pulses go to a maximum of 275 A (for an integrated skew quad gradient of 0.3 T) [8]. The voltage for the 82 pulses is supplied by drawing down from a 450 V capacitor bank during the 400 ms acceleration ramp. The bank recharges during the rest of the 4 second injector cycle.

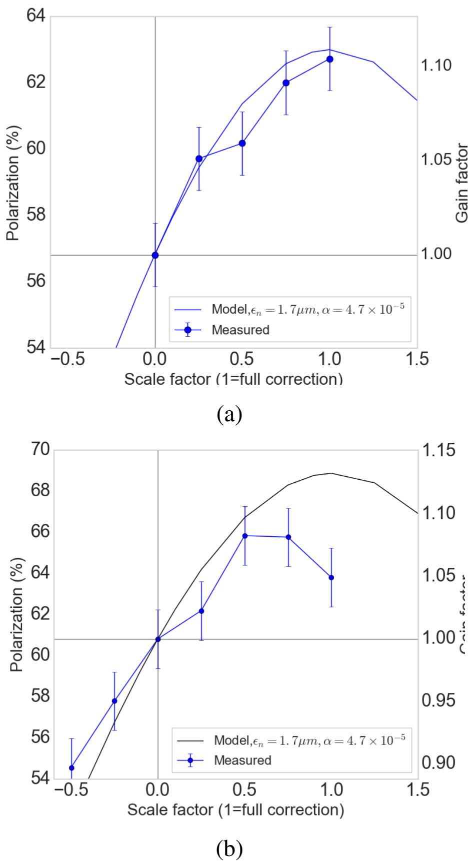
The 15 skew quadrupole strengths at each resonance are determined by three constraints: reduction of the model-calculated net spin resonance strength to zero, maintaining the betatron tune shifts from the resultant coupling to <0.005 and minimization of orbit distortion effects. The resonance strengths from the snakes and the skew quads are calculated using the Bmad code [9]. A typical set of corrector currents is shown in Fig. 2.

OPERATIONAL EXPERIENCE

A practical difficulty with deployment of skew quadrupoles in the AGS is the closed orbit effects. The horizontal orbit in the AGS has large excursions (e.g. ±2 cm near injection energy, ±1 cm at higher energies). Since the vertical betatron tune is relatively close to an integer (8.985-8.99), powering a skew quadrupole in the presence of such large horizontal orbit excursions can lead to beam loss. Fully global correction of these offsets is not currently possible due to the weak corrector magnets and relatively large errors.

A beam-based approach is taken to skew-quad centering, since beam position monitors are not available adjacent to the skew quad locations. Each skew quadrupole is pulsed individually at each resonance time at the highest current that beam loss from orbit excursions will allow. Figure 3 shows the results of beam-based measurements of the horizontal beam positions at the skew quads before and after correction. At low energy (prior to ≈320 ms), the orbit is constrained by the needs of injection and accommodation of the orbit effects of the helical dipoles. Correction at those energies will require improvements to the AGS orbit control system.

With orbit effects minimized, the effects of the correction on the polarization transmission through acceleration can be measured. Figure 4 shows the increase in polarization while the correction currents are gradually introduced from zero (no correction) to full correction. The model prediction of the polarization gain is calculated using SPRINT [10] to calculate the residual resonance strengths and the Froissart-Stora equation to calculate the resulting polarization loss. Figure 4a shows the measurement to model comparison



(a)

(b)

Figure 4: Polarization improvement from increasing correction strength using (a) pulses above $\gamma = 8$ and (b) pulses over the whole ramp. Future work will focus on improving the performance of the low energy pulses.

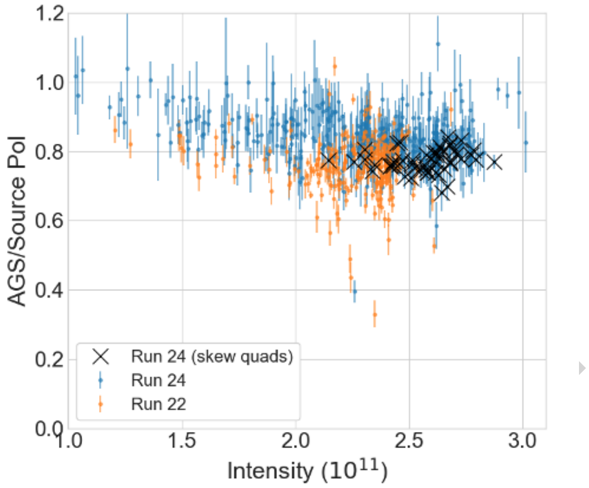


Figure 5: Polarization transmission efficiency from the end of the Linac (200 MeV) to AGS extraction energy (23.8 GeV). Blue and orange show performance with tune jump, black crosses with the skew quads. Performance demonstrated so far is comparable to the tune jump but does not exceed it.

when only high energy (γ>8) resonances are included. The results show good agreement with the model, which assumes full correction of any resonance for which the skew quadrupoles pulse. The agreement with the model is worse when the skew quads are pulsed during the lower energy resonances (Fig. 4b). This suggests that the calculation of the resonance correction is largely accurate for the high energy cases and less so for the low energy resonances. Possible error sources for correction of the low energy resonances are optical errors due to large orbit offsets or model deficiencies (which would change the required skew quad strengths) and betatron coupling, which would add vectorially with the resonance driven by the snakes.

The skew quadrupole compensation was deployed operationally as a replacement for the last weeks of RHIC Run 24. Comparison of fill by fill polarizations are shown in Fig. 5. One can see that the typical polarization transmission with the skew quadrupole was comparable to that when operating with the tune jump. Improvement of the skew quadrupole compensation efficiency is expected to improve as the model and orbit control for the AGS at low energy is improved. These are likely dominated by closed orbit effects due to the large residual orbit excursions, especially due to orbit feed-down from the sextupoles. Offsets in sextupoles will drive both beta-beating (horizontal) and unknown additional betatron coupling (vertical), both of which tend to degrade the skew quad compensation. Some discussion of the residual error sources can be found in a companion paper [11].

CONCLUSION

A set of 15 fast pulsing skew quadrupoles has been installed in the AGS in order to fully compensate horizontal resonances driven by the partial snakes. The early commissioning and operational results show that the initial performance is comparable to the previously employed tune jump method. Since the skew quadrupole compensation relies on accurate model predictions for full compensation, the main impediment to increasing performance of the system is accurate knowledge of the AGS model, including closed orbit offset effects.

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