TOMOGRAPHIC RECONSTRUCTION OF LONGITUDINAL PHASE SPACE FROM FCT MEASUREMENTS

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Abstract

For the analysis of longitudinal dynamics in synchrotrons and storage rings, the reconstruction of the beam distribution in longitudinal phase space is of high interest. This study applies a tomographic method to longitudinal beam profiles measured in the ESR to reconstruct longitudinal phase space. We show the limitations of these methods for the nonlinear beam dynamics in an RF bucket.

INTRODUCTION

Tomography provides the reconstruction of an n-dimensional distribution from projections in $(n-1)$-dimensional image-space with a range of projection angles. In case of a beam in longitudinal phase space, the distribution of beam density is measured along the direction of motion s while the beam itself rotates in the RF bucket. Other works on longitudinal phase space tomography typically use an iterative approach [1–3] to ensure small errors of reconstruction despite the typically small numbers of projections and the non-linear character of synchrotron oscillations. A direct reconstruction method, however, is computationally beneficial and therefore desirable for real time diagnostics during machine operation. In this study we apply the direct filtered back projection (FBP) method for reconstruction and analyze the resulting uncertainties of reconstruction for different types of beam distribution. The measurement of these longitudinal beam profiles is performed by a Fast Current Transformer (FCT).

Beam Rotation

Under the influence of the RF cavity, the beam follows trajectories in the bucket, each identified by its maximum phase $\psi_{\max}$. For small $\psi_{\max}$ the beam particles describe an elliptic rotation with constant synchrotron frequency. For large $\psi_{\max}$, both the trajectories are non-elliptic and the rotation frequency is not constant. The synchrotron frequency has the value

$$\omega_s = \omega_{\text{rev}} \sqrt{\frac{qVh}{2\pi\beta^2 E} \left| \left(\alpha_p - \frac{1}{\gamma^2} \right) \right|} , \tag{1}$$

with ω_{rev} the revolution angular frequency, V the cavity gap voltage, E the beam energy, h the harmonic number, α_p the momentum compaction factor, relativistic β, γ.

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Reconstruction

To reconstruct the 2D phase space from longitudinal beam profiles, it is assumed that the rotation frequency is constant for all particles of the beam. Under this assumption the measured FCT beam profile shows the 2D phase space distribution projected onto the coordinate s under continuosly changing angles. Since the FCT performs triggered measurements of current over time, the position s is measured as the time τ of arrival of a beam particle in the FCT. The beam and machine parameters, yielding the synchrotron frequency, and the time steps Δt between consecutive profile measurements determine the steps of projection angle

$$\Delta\vartheta = \omega_s \Delta t \ . \tag{2}$$

For the beam phase space reconstruction the FBP method is applied. All measured profiles are first filtered by a high-pass Ram-Lak filter. Then these pre-processed profiles are back projected onto the 2D space. The background that results from the back projection is obtained from the density in the region outside the bucket and is used as a lower cut-off for the resulting distribution. To retrieve the distribution in phase space, the time coordinate τ is renormalized to phase

$$\psi = 2\pi \frac{h\tau}{T_{\text{rev}}} , \tag{3}$$

with h the harmonic number and T_{rev} the revolution time of the reference particle. While the rotation used in the tomographic method yields coordinates of ψ in both directions, the energy deviation dE/E can be estimated by the height of the bucket $(dE/E)_{\max}$

$$\frac{dE}{E} = \frac{\psi}{\psi_{\max}} \left(\frac{dE}{E} \right)_{\max} . \tag{4}$$

Measurements

The measurements of a stored Ar^{18+} beam were made during machine development (MD) time at the Experimental Storage Ring (ESR) [4] at GSI. The beam was injected, RF bunched and cooled by the electron cooler. During storage at injection energy of 400 MeV/u sets of profile measurements were made with the FCT after varying storage duration. The number of beam profiles in one set was limited to 49 for technical reasons. The time interval between consecutive profiles was chosen so that roughly two periods of oscillation were measured in one set and therefore the oscillation

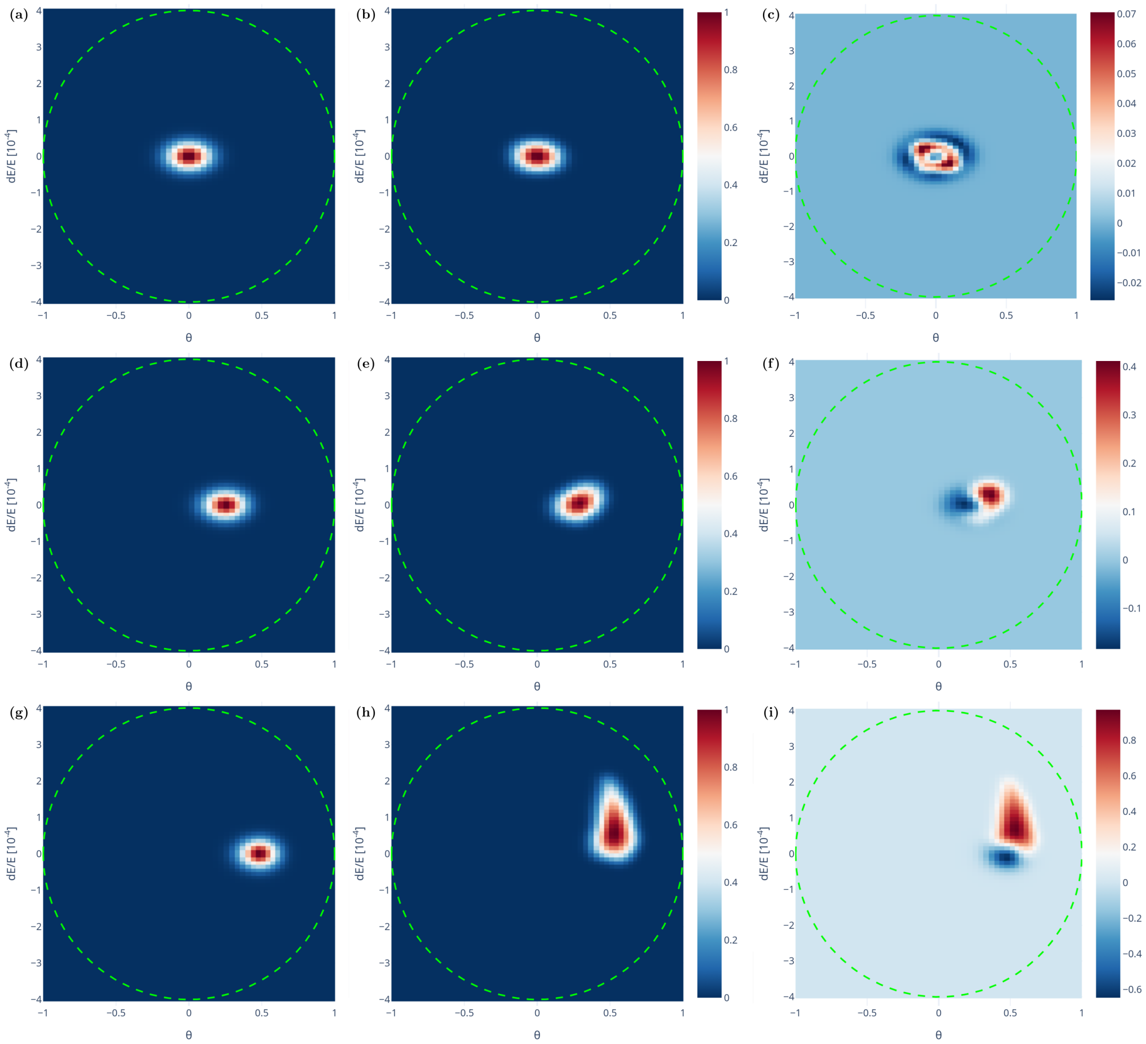

Figure 1: Normalized initial distribution (a), normalized reconstructed distribution after tracking (b) and the absolute difference of both (c) for a small centered distribution. The same for a distribution with small offset from the center (d), (e), (f) and larger offset (g), (h), (i). The dashed green line shows the separatrix, which in the transformed coordinates is circular.

frequency could be determined reliably. A detailed analysis of this measured synchrotron oscillation frequency and the dependency on the amplitude is shown in [5].

SIMULATED DISTRIBUTIONS

For the evaluation of the reconstruction method, simulated distributions are used. The distributions are created by tracking a set of Gaussian distributions in the phase space over a number of turns using the same optics and RF parameters as in the measurements. The resolution of beam profiles as well as the number of profiles along roughly two periods of synchrotron oscillation are chosen to match the measurements. The projections onto τ are created and the FBP reconstruction method is applied. To compare the original and reconstructed distributions, the cavity phase ψ is transformed to a renormalized coordinate θ given by

$$\theta = \mathrm{sgn}(\psi)\sqrt{1-\cos^2(\psi/2)}\,, \tag{5}$$

so that the trajectories in phase space are circular. After normalizing the distributions, the uncertainty of the reconstruction is found by

$$\sigma_{\mathrm{rel}} = \frac{\sqrt{\sum_i \Delta n_i^2}}{\sum_i n_i}\,, \tag{6}$$

with Δn_i the difference between reconstructed and initial normalized distribution and with n_i the normalized initial distribution on the ith point on the $(\theta,\ dE/E)$ grid.

Rigid Rotation

The initial distribution we show in Fig. 1(a) is a centered Gaussian distribution with the parameters given in Table 1. It is tracked for 9×10^{-3} s. At time steps $\Delta t = 1.875\times10^{-4}$ s the longitudinal profiles are stored. Then the FBP is applied to reconstruct the distribution. After renormalization, the two distributions are subtracted and the relative uncertainty is calculated from Eq. 6. Figure 1(b) shows the reconstructed

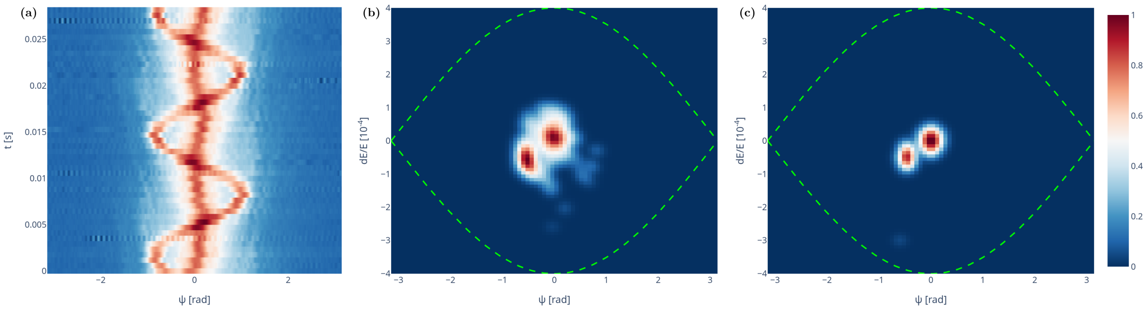

Figure 2: Time development of beam profiles with time t of an example measurement at ESR (a). Normalized reconstructed distribution of this measurement (b). Normalized reconstructed distribution of three overlapping Gaussian distributions (see Table 1) after tracking (c). The dashed green line is the separatrix.

Table 1: Parameters σ_ψ, $\sigma_{dE/E}$, μ_ψ, $\mu_{dE/E}$ of the Gaussian Distributions and the Number of Macroparticles N in each Distribution Used for the Simulated Reconstructions of Figs. 1 and 2 and the Resulting Uncertainties σ_{rel} of the Reconstruction

	σ_ψ	$\sigma_{dE/E}$	μ_ψ	$\mu_{dE/E}$	N	σ_{rel}
Fig. 1(a)	$0.05\,\pi$	2×10^{-5}	0	0	100 000	9.06×10^{-3}
Fig. 1(d)	$0.05\,\pi$	2×10^{-5}	0.5	0	100 000	4.78×10^{-2}
Fig. 1(g)	$0.05\,\pi$	2×10^{-5}	1	0	100 000	1.83×10^{-1}
Fig. 2(c)	0.2	6×10^{-5}	0	0	50 000	
	0.1	2×10^{-5}	0	0	25 000	
	0.06	2×10^{-5}	-0.4	-4×10^{-5}	25 000	2.53×10^{-2}

distribution, Fig. 1(c) the absolute difference of reconstruction and initial distribution.

Shear Rotation

To investigate the error for larger oscillations, where the rotation of the beam in the bucket is a shear rotation, distributions with larger offsets from the bucket center are used. The same methods as before are applied. For the reconstruction the steps of angle are still calculated by Eq. 2, while clearly this introduces an error with larger amplitudes and therefore varying rotation frequency. The initial distributions, reconstructions and the reconstruction differences are shown in the middle and bottom row of Fig. 1 with the Gaussian parameters and the obtained relative uncertainties given in Table 1. As expected, the uncertainty of the reconstruction grows at larger amplitudes. However also the distribution itself is deformed during the motion in the outer region of the bucket and therefore the deviation from the initial distribution is an effect of both the assumption of fixed rotation frequency as well as the assumption that the distribution is not altered during the time of capturing the profiles.

MEASURED DISTRIBUTION

One example of a large number of measurement sets taken during the MD 2025 and analyzed in detail in [5] was chosen to apply the tomographic reconstruction shown above to the measured beam profiles. We show in Fig. 2 the oscillations of the measured beam profiles (a) and the reconstructed beam distribution for this measurement (b). As an estimate for the reconstruction uncertainty, a simulated reconstruction of a distribution of three overlapping Gaussians was performed and is shown in Fig. 2(c). The distribution used and the relative uncertainty of the simulated reconstruction are given in Table 1.

DISCUSSION

While it was shown that for large ψ_{max} the direct reconstruction assuming rigid rotation has expectedly high uncertainties, it was also observed, that the algorithm still can be useful for quick computation of estimated beam distribution. A reconstruction with resolution (2048x2048) using a non-optimized simple image rotation algorithm is performed on 4 cores in roughly 10 s. For smaller maximum phase, which is common especially for cooled stored beams, the uncertainty is small. The example distribution from a measurement performed at ESR shows that the beam distribution in longitudinal phase space could be reconstructed, with the comparable simulated distribution yielding a relative uncertainty of 2.53%.

OUTLOOK

Since an error is introduced when the rigid rotation is assumed for larger amplitudes, a direct method for tomographic reconstruction including shear would be beneficial. Such a method will be developed in a future study.

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