

# MONOCHROMATIZATION OPTICS FOR FCC-ee OPTIMIZED PERFORMANCE STUDY AND FIRST ATTEMPT FOR IR OPTICS DESIGN FOR FCC-ee LCC LATTICE

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## Abstract

Monochromatization is one of the most intriguing proposed operation modes of the FCC-ee, enabling a significant reduction of the centre-of-mass (CM) energy spread to a level comparable to the Higgs boson's natural width produced through the s-mode direct channel at 125 GeV. Previous studies demonstrated its feasibility using earlier versions of the FCC-ee Global Hybrid Correction (GHC) optics. A first draft implementation of the scheme on the Local Chromaticity Correction (LCC) lattice is explored in this paper. The performances of the new optics types are presented in terms of luminosity and energy spread, supported by the simulation results. This provides an early outlook on the potential operational flexibility of future FCC-ee configurations operating in monochromatization mode.

## INTRODUCTION

While a significant body of previous work successfully demonstrated the feasibility of the monochromatization [1–10] scheme using various versions of the GHC lattice [11–16], this paper presents an updated implementation built upon the LCC lattice configuration with 4 IPs as the primary baseline [17]. Parametric optimization is conducted using a multi-objective optimization algorithm adapted from [18], integrating self-consistent beamstrahlung-induced emittance growth via the formulations for the beamstrahlung (BS) effect in the condition of monochromatization [19–24]. This paper is focused on implementing non-zero vertical dispersion in the interaction point (IP)  $D_y^*$ . Future studies will include performance analysis of horizontal dispersion in the IP  $D_x^*$  implementation and the hybrid mode.

## ANALYTICAL OPTIMIZATION FRAMEWORK

To identify the optimal IP parameters for the monochromatization mode of the LCC lattice at 62.5 GeV, a self-consistent multi-objective optimization framework was developed. This framework relies on a numerical solver structured around the Non-dominated Sorting Genetic Algorithm II (NSGA-II) provided by the pymoo Python library, expanding upon the analytical methodologies for high-energy lepton colliders design [18].

The primary objective of this framework is to navigate a highly constrained, non-linear parameter space to minimize the CM energy spread ( $\sigma_w$ ) and maximize the machine's instantaneous luminosity ( $L$ ).

## Decision Variables and Physical Boundaries

The optimization framework explores a four-dimensional parameter space defined by the key adjustable variables of the IP configuration. These parameters include  $D_y^*$ , the number of bunches per beam ( $N_b$ ), and the horizontal and vertical betatron functions at the IP ( $\beta_x^*$  and  $\beta_y^*$ ). To ensure baseline design compatibility, these variables are scanned within the following predefined physical boundaries:

$$\vec{x} = \begin{bmatrix} D_y^* \\ N_b \\ \beta_x^* \\ \beta_y^* \end{bmatrix} = \begin{cases} 0.0 \leq D_y^* \leq 2.0 \text{ mm} \\ 2000 \leq N_b \leq 12000 \\ 0.09 \text{ m} \leq \beta_x^* \leq 10.0 \text{ m} \\ 0.7 \text{ mm} \leq \beta_y^* \leq 5.0 \text{ mm} \end{cases} \quad (1)$$

To maintain baseline compatibility with the overall LCC lattice design, certain parameters remain fixed during the optimization loops. The horizontal equilibrium emittance ( $\epsilon_x$ ) is kept constant at its purely synchrotron radiation (SR)-dominated baseline value. The vertical emittance ( $\epsilon_y$ ) is not parameterized independently; instead, it is calculated dynamically using a defined betatron coupling factor coupled with the localized BS blow-up. Similarly, the synchrotron tune ( $Q_s$ ) is held fixed.

The design enforces a maximum allowable beam-beam parameter ceiling of  $\xi_{x,y}^{\max} = 0.15$  for both planes to prevent premature beam disruption. The physical constraints are further regularized by defining a lower-bound luminosity threshold  $L_{\text{target}} = 1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ .

## Self-Consistent BS Modeling and Equilibrium Parameters

A key feature of the optimization loop is the calculation of self-consistent equilibrium parameters at each genetic iteration. At 62.5 GeV, intense electromagnetic fields at the collision point trigger strong BS emissions, modifying the global bunch distributions. The underlying physics model calculates the final total energy spread  $\sigma_\delta$  and the subsequent emittance values by iterating the self-consistent analytical formulations [19].

Because the vertical beam size in this setup is heavily dominated by the dispersive component ( $\sigma_y^* \approx D_y^* \sigma_\delta$ ), any BS-induced increase in the energy spread creates a coupled feedback loop that directly impacts the vertical beam dimension and total luminosity. The framework evaluates these loops analytically for every generated parameter set, verifying that the resultant Pareto-optimal entries reflect true steady-state conditions.

In this model, the quantum excitation from BS during a single collision  $\{n_\gamma \langle u^2 \rangle\}_{\text{BS}}$  increases both the nominal

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equilibrium emittance ( $\epsilon_{x,y,\text{tot}}$ ) and the total relative energy spread ( $\sigma_{\delta,\text{tot}}$ ). These equilibrium parameters are expressed as:

$$\epsilon_{x,y,\text{tot}} = \epsilon_{x,y,\text{SR}} + \frac{n_{\text{IP}} \tau_{x,y,\text{SR}}}{4T_{\text{rev}}} \{n_{\gamma} \langle u^2 \rangle\}_{\text{BS}} \mathcal{H}_{x,y}^* \quad (2)$$

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{n_{\text{IP}} \tau_{E,\text{SR}}}{4T_{\text{rev}}} \{n_{\gamma} \langle u^2 \rangle\}_{\text{BS}} \quad (3)$$

where the subscript "SR" denotes the standard synchrotron radiation baseline values without BS,  $n_{\text{IP}}$  is the number of IPs,  $T_{\text{rev}}$  is the revolution period, and  $\tau_{x,y,E}$  are the respective damping times.

The translation of this energy excitation into the transverse emittance depends on the IP dispersion invariant  $\mathcal{H}_{x,y}^*$ :

$$\mathcal{H}_{x,y}^* = \frac{(\beta_{x,y}^* D'_{x,y} + \alpha_{x,y}^* D_{x,y}^*)^2 + (D_{x,y}^*)^2}{\beta_{x,y}^*} \quad (4)$$

For a scheme with purely vertical monochromatization,  $D_x^* = 0$ , so  $\mathcal{H}_x^* \approx 0$ . Assuming a parallel beam at the IP ( $\alpha_y^* = 0$  and  $D_y^* = 0$ ), the vertical invariant simplifies to:

$$\mathcal{H}_y^* \approx \frac{(D_y^*)^2}{\beta_y^*} \quad (5)$$

Because  $\{n_{\gamma} \langle u^2 \rangle\}_{\text{BS}}$  is inversely proportional to the beam size and bunch length ( $\sigma_{z,\text{tot}}^3 \sigma_{x,\text{tot}}^2$ ), the system is cross-coupled. Under the assumption of a planar designed orbit ( $\tau_{x,y} = 2\tau_E$ ), the equations can be rewritten in a self-consistent form:

$$\epsilon_{x,y,\text{tot}} = \epsilon_{x,y,\text{SR}} + \frac{2V \mathcal{H}_{x,y}^*}{\sigma_{\delta,\text{tot}}^2 \sigma_{x,\text{tot}}^3} \quad (6)$$

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{V}{\sigma_{\delta,\text{tot}}^2 \sigma_{x,\text{tot}}^3} \quad (7)$$

where the total horizontal IP beam size is:

$$\sigma_{x,y,\text{tot}}^* = \sqrt{\beta_{x,y}^* \epsilon_{x,y,\text{tot}} + D_{x,y}^{*2} \sigma_{\delta,\text{tot}}^2} \quad (8)$$

The introduction of the coefficient  $V$  decouples the dynamic beam-size and bunch-length terms, making  $V$  a constant determined solely by the optics design and beam parameters. For collisions with a crossing angle characterized by the Piwinski angle  $\phi$ , this coefficient is:

$$V \approx \frac{55\pi^2}{3\sqrt{3}} \left( \sqrt{\frac{2}{\pi}} \right)^3 \frac{n_{\text{IP}} \tau_{E,\text{SR}} Q_s^2 r_e^5 \gamma^2 N_b^3}{T_{\text{rev}} \alpha_C^2 C^2 \alpha} \times \frac{0.77562}{\phi} \quad (9)$$

where  $r_e$  is the classical electron radius,  $\gamma$  is the Lorentz factor,  $\alpha$  is the fine-structure constant,  $Q_s$  is the synchrotron tune,  $C$  is the circumference and  $\alpha_C$  is the momentum compaction factor. For head-on collisions, the final factor  $0.77562/\phi$  is replaced by  $0.7183$ .

Because of the non-zero vertical dispersion, the vertical beam size is dominated by the energy spread ( $\sigma_{y,\text{tot}}^* \approx D_y^* \sigma_{\delta,\text{tot}}$ ). An increase in BS widens  $\sigma_{\delta,\text{tot}}$ , which directly expands the vertical beam size, altering the luminosity and beam-beam parameters. The optimization script evaluates these equations iteratively to ensure that every calculated parameter set represents a true, stable equilibrium.

## Optimization Results and Discussion

The analytical optimization script was executed to identify viable IP parameters for the monochromatization mode. While the primary target is a CM energy spread matching the Higgs boson width of 4.1 MeV [25–28], the optimization is heavily constrained by the trade-off between energy resolution and sustainable luminosity at 62.5 GeV.

Table 1 summarizes a selection of Pareto-optimal solutions identified through the multi-objective scan, while the broader dependence between energy spread, luminosity, and  $\beta_x$  with a fixed dispersion value  $D_y = 0.5$  mm is visualized in Fig. 1. The results clearly demonstrate that decreasing the  $\sigma_W$ , achieved primarily by increasing  $D_y$ , comes at the severe cost of reduced instantaneous luminosity. While standard LCC Z-mode operation provides high luminosity, its natural energy spread of over 63 MeV (the baseline configuration, highlighted in bold at the bottom of the table) makes  $s$ -channel Higgs observation statistically prohibitive.

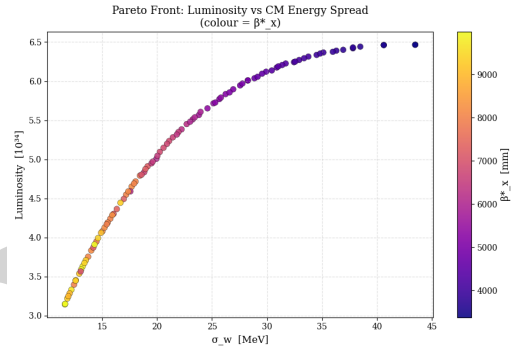


Figure 1: Dependence between  $\sigma_W$  and  $L$ , with the color scale indicating the value of  $\beta_x^*$ . Maintaining  $D_y^* = 0.5$  mm.

Through our analysis, a specific configuration (marked with a † marker in Table 1) was selected as the optimal compromise. This layout achieves a significant reduction in CM energy spread (down to 11.711 MeV) while preserving a luminosity above  $3 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ , providing a meaningful signal-to-background ratio. It is crucial to observe that for all viable monochromatized solutions, including our optimal selection,  $\beta_x$  was increased by nearly two orders of magnitude compared to the baseline (from 90 mm to nearly 10 000 mm). This drastic change is required to suppress the BS-induced emittance blow-up, and it may potentially avoid the X-Z instability zone, resulting in  $\xi_x \gg \xi_y$ , rotating the beam-beam tune footprint, which will be investigated in future work.

## LATTICE IMPLEMENTATION

To translate the analytical optimization parameters into a realistic accelerator design, the monochromatization scheme was implemented in the interaction region (IR) of the LCC lattice. The  $D_y^*$  is generated by a set of dedicated skew quadrupole families positioned in the Final Focus System (FFS). Using the MAD-X optics code [29], the strengths of these skew quadrupoles were matched to provide the target

Table 1: Optimized IP Parameters at 62.5 GeV with 4 IPs (**Baseline; Optimal Monochromatization†**)

$\sigma_w$ [MeV]	L [ $10^{34}\text{cm}^{-2}\text{s}^{-1}$ ]	$D_y^*$ [mm]	$N_b$	$\beta_x^*$ [mm]	$\beta_y^*$ [mm]	$\xi_x$	$\xi_y$	$\epsilon_y$ [pm]
<b>63.03</b>	<b>25.28</b>	<b>0</b>	<b>12 000</b>	<b>90</b>	<b>0.7</b>	<b>0.001</b>	<b>0.03</b>	<b>2.7</b>
9.097	1.071	1.5	11 997	9996.3	0.7	0.1051	0.0014	34.778
9.582	1.543	1.0	11 901	9974.2	0.7	0.106	0.002	18.898
10.131	2.101	0.75	11 976	9998.7	0.7	0.1056	0.0027	11.25
<b>11.711†</b>	<b>3.191</b>	<b>0.5</b>	<b>11 974</b>	<b>9996.2</b>	<b>0.7</b>	<b>0.1057</b>	<b>0.0041</b>	<b>6.519</b>
13.02	3.942	0.4	11 850	9990.6	0.7	0.1068	0.0051	5.386
15.014	5.016	0.35	11 023	8646.8	0.7	0.1092	0.0065	5.55
18.811	7.476	0.25	10 668	6859.3	0.7	0.103	0.0096	4.798

$D_y^* = 0.5$  mm for our selected optimal configuration (†). Simultaneously, the matching routine ensures that the vertical dispersion is closed outside of the IR to avoid emittance growth in the main arcs. The resulting optical functions of the modified IR, as calculated by the code MAD-X, are shown in Fig. 2.

## SUMMARY AND OUTLOOK

In this paper, we presented a first-draft monochromatization scheme on the FCC-ee LCC lattice at 62.5 GeV. Combining multi-objective optimization with analytical BS modeling, we mapped the trade-offs between CM energy spread and luminosity. The required  $D_y^*$  was successfully matched in MAD-X using FFS skew quadrupoles, ensuring local dispersion closure.

Future work will expand on these findings by evaluating horizontal and hybrid monochromatization via  $D_x^*$  to compare its performance with the vertical scheme. Dedicated beam-beam simulations will also be performed to investigate the beam-beam limit, potential X-Z coherent instabilities, and the asymmetric tune footprint dynamics ( $\xi_x \gg \xi_y$ ) inherent to these monochromatized collision parameters.

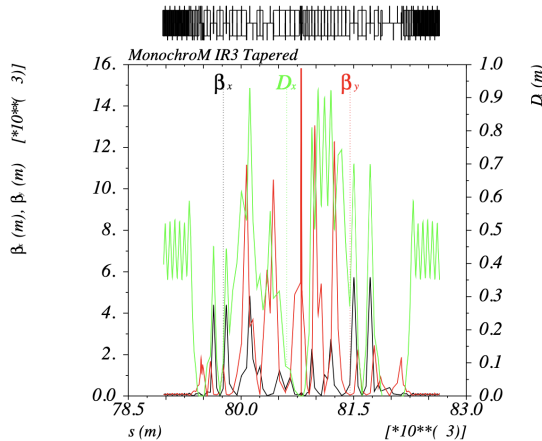


Figure 2: The matched vertical dispersion and beta functions in the LCC v105 IR modeled in MAD-X.

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