SIMULATION-BASED OPTIMIZATION OF RF CAVITY PI CONTROLLERS FOR BEAM STABILITY IN HIGH-CURRENT STORAGE RINGS*

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Abstract

In high-current electron storage rings, tuning the radio-frequency (RF) cavity's proportional-integral (PI) controller is crucial for beam stability. Traditional online trial-and-error methods risk beam loss due to improper parameter settings. This paper proposes a pre-commissioning method based on particle-tracking simulations that self-consistently model the coupled beam-cavity-feedback dynamics. The method delineates a theoretically stable operating region for the PI controller, providing reliable guidance for initial online settings. Applied to the Super Tau-Charm Facility (STCF) parameters, it successfully identifies stable operating ranges for the feedback parameters. This simulation-based approach not only promises to guide design and commissioning but also substantially mitigates the risk of beam loss during operation.

INTRODUCTION

In high-current electron storage rings, longitudinal coupled-bunch instability (CBI) driven by the RF cavity accelerating mode can impose stringent limits on the achievable stored beam current. Digital low-level RF (DLLRF) systems employing proportional–integral (PI) controllers are widely used to regulate cavity fields; however, the PI gains together with the total loop delay reshape the closed-loop cavity response and, consequently, the effective impedance seen by the beam. As a result, stability margins can be highly sensitive to controller settings [1].

In practice, PI loop parameters often require optimization via parameter scans during operation [2]. The strong coupling among the beam, cavity, and feedback systems, combined with variations in operating conditions—such as beam current, cavity detuning, and system latency—can alter the optimal PI control parameters. More critically, inappropriate PI parameter settings can induce beam instability or even lead to beam loss [3].

To address these challenges, this paper presents a simulation-based methodology for pre-commissioning PI parameters. The method utilizes the GPU-accelerated particle-tracking code STABLE [4]. By implementing a PI controller model based on I/Q signal processing, we perform systematic parameter scans to identify the boundaries of beam stability. In parallel, the Nyquist criterion is applied to evaluate the intrinsic stability of the feedback loop and to quantify its gain margin. This combined analysis produces a stability map informed by both beam-tracking results and Nyquist analysis. The map provides reliable a priori guidance for the design and commissioning of DLLRF systems, significantly reducing the risk associated with online tuning and helping to prevent instabilities arising from suboptimal parameter choices.

SIMULATION WORKFLOW AND STABILITY ANALYSIS

Figure 1 illustrates the proposed workflow to identify the stable region of PI loop parameters. Based on the coupled beam–cavity–DLLRF model, we construct a stability map by evaluating the growth rate of the fastest-growing coupled-bunch mode. In parallel, we apply a Nyquist-based screening to eliminate parameter points that render the feedback loop intrinsically unstable. The resulting safe operating region for PI tuning is given by the intersection of the beam-stable region and the loop-stable region.

Beam–Cavity–LLRF Interaction Model

The interaction is simulated using the GPU-accelerated code STABLE [4], which implements a digital PI controller with I/Q baseband processing [5]. Dynamics are advanced at the bucket level: cavity voltage is updated per RF bucket using a pulse-discretized generator current, and beam loading is applied according to the bunch filling pattern. Key factors, including FIR filtering and the total loop delay, are explicitly included.

Scan Strategy and Stability Definition

A two-dimensional parameter scan over the proportional gain K_P and the loop delay τ_d is performed, as these parameters significantly affect the closed-loop cavity impedance distribution and the system's stability margins. Given that tracking simulations indicate the integral gain K_I has a relatively minor influence on beam instability, it is fixed at a specified nominal value. To improve computational efficiency, all scan points are initialized using the steady-state solution from a single reference point. For each parameter set, tracking simulations are run for up to 4×10^4 turns; if the beam oscillation amplitude exceeds a preset threshold, the current simulation is terminated early and the computation proceeds to the next parameter point.

The operational stability boundary is determined jointly by beam-dynamical stability and feedback-loop stability.

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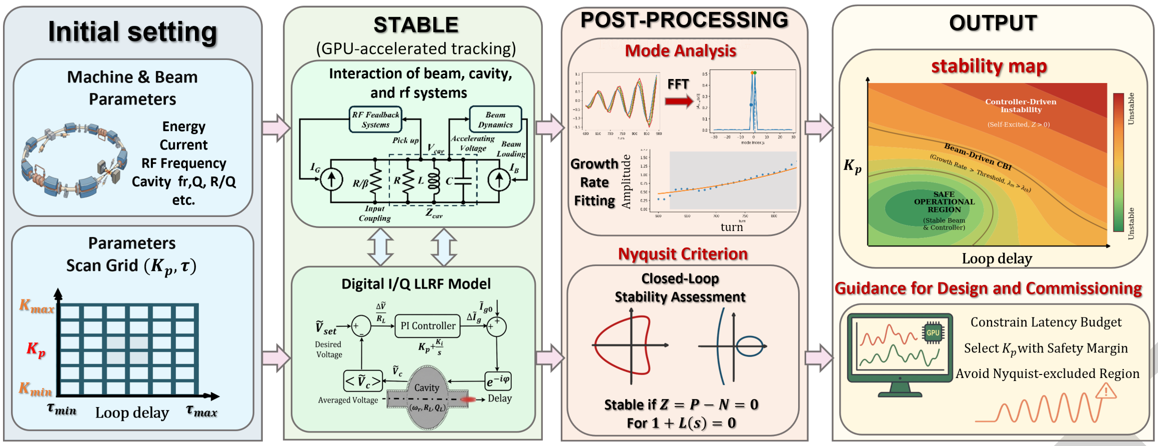

Figure 1: Schematic overview of the simulation-based workflow for tuning PI feedback-loop parameters to ensure beam stability.

Specifically, a stable operating point must satisfy the following two conditions simultaneously:

1. **Beam-dynamical stability:** The growth rate of the fastest-growing mode must be less than the radiation damping rate.

2. **Loop stability:** The feedback loop itself must be stable and possess sufficient stability margin, which is evaluated using the Nyquist criterion.

The final stability map is given by the intersection of the beam-stable region and the Nyquist-stable region.

Mode Identification and Growth-Rate Extraction from Tracking Data

Let $q_{m,n}$ denote the longitudinal centroid of bunch m ($m = 0, \ldots, M-1$) recorded at sample index n ($n = 1, \ldots, N$). The data are down-sampled by a fixed interval of 10 turns. To ensure the instability is well-developed, only the final third of the recorded data is used for the subsequent analysis.

At the last recorded sample, we remove the common-mode component to highlight the coupled bunch mode:

$$q_m = q_{m,N} - \frac{1}{M}\sum_{k=0}^{M-1} q_{k,N}^{\mathrm{raw}}. \tag{1}$$

A candidate coupled-bunch mode is determined by searching the maximum magnitude of the discrete Fourier transform (DFT) for q_m:

$$\mu_{\mathrm{cand}} = \arg\max_{\mu} \left| \frac{1}{M}\sum_{m=0}^{M-1} q_m \exp\!\left(-i\frac{2\pi\mu}{M}m\right) \right|. \tag{2}$$

Because q_m is real-valued, its DFT is conjugate symmetric; therefore both $+\mu$ and $-\mu$ cannot be distinguished. To determine the sign of μ and to obtain the evolution of the mode envelope, we construct, for each bunch, an analytic signal by applying a Hilbert transform $\mathscr{H}_n[\cdot]$ along the sample index n [6, 7]:

$$\tilde{q}_{m,n} = q_{m,n} - i\,\mathscr{H}_n[q_{m,n}]. \tag{3}$$

The complex amplitude of mode μ at sample n is then obtained by projecting $\tilde{q}_{m,n}$ onto the corresponding spatial harmonic:

$$A_n(\mu) = \frac{1}{M}\sum_{m=0}^{M-1} \tilde{q}_{m,n} \exp\!\left(-i\frac{2\pi\mu}{M}m\right). \tag{4}$$

The envelope $|A_n(\mu)|$ is used to compare the two candidates $\pm\mu$. The mode with the larger mean envelope is identified as the most unstable mode μ^*. The growth rate, $1/\tau_{\mu^*}$, is obtained by fitting an exponential to the exponential-growth phase. Finally, the intrinsic growth rate is given by $1/\tau_{\mu^*} + 1/\tau_{\mathrm{rad}}$, with τ_{rad} being the radiation-damping time.

Intrinsic Loop Stability via Nyquist Criterion

The PI feedback parameters, in particular the proportional gain K_P and the loop delay τ_d, affect not only the beam dynamics but also the intrinsic stability of the LLRF feedback loop itself. To separate controller-driven oscillations from beam-driven coupled-bunch motion, we evaluate the closed-loop stability using the Nyquist criterion [8].

For a given parameter set (K_P, τ_d), the open-loop transfer function is

$$L(i\omega) = G(i\omega)\, Z_{\mathrm{cav}}(i\omega)\, e^{-i\omega\tau_d}, \tag{5}$$

where $Z_{\mathrm{cav}}(i\omega)$ is the loaded cavity impedance. In the frequency range relevant for loop stability, the controller response is typically dominated by the proportional action, so we approximate

$$G(i\omega) \simeq \frac{K_P}{R_L}, \tag{6}$$

where R_L is the loaded shunt impedance. Closed-loop stability requires that the Nyquist plot of $L(i\omega)$ does not encircle the critical point $(-1, 0)$. Equivalently, the winding number of $1 + L(i\omega)$ around the origin must be zero:

$$N = 0. \tag{7}$$

Any parameter point that violates Eq. (7) is excluded from the stable operating region. Within the stable region, we further quantify robustness using the gain margin (GM) evaluated at the phase-crossover frequency ω_π (defined by $\angle L(i\omega_\pi) = -\pi$):

$$\mathrm{GM}\,[\mathrm{dB}] = -20\log_{10}|L(i\omega_\pi)|. \qquad (8)$$

APPLICATION TO STCF

We take the parameters of the Super Tau-Charm Facility (STCF) at a beam energy of 3.5 GeV as an example to demonstrate the simulation-based optimization and commissioning method proposed in this paper. As described previously, the simulation is performed using the beam–cavity–DLLRF coupled model implemented in the STABLE code. For each scanned point (K_P, τ_d), it is tracked for 4×10^4 turns, after which the fastest-growing coupled-bunch mode and its growth rate are extracted from the tracking data. Finally, a stability map is generated to guide the design and commissioning of the DLLRF system.

Stability Map

The resulting stability map in the (K_P, τ_d) plane is shown in Fig. 2. The color scale indicates the growth rate of the fastest-growing mode, and markers denote the corresponding mode index μ.

The influence of K_P and τ_d on coupled-bunch mode instability is visually evident. At low K_P, mode -1 dominates; as K_P and τ_d increase, the dominant mode successively transitions to mode -2 and then to mode $+1$. The allowable parameter region is quantified by three key boundaries. First, the solid contour labeled "149" corresponds to the radiation damping rate, $1/\tau_{\mathrm{rad}} \approx 149\mathrm{s}^{-1}$. Parameters within this contour are stable, as the resulting growth rate is lower than this damping rate. Second, the white region in the upper-right corner is excluded because it fails to satisfy the Nyquist loop-stability criterion. Third, the dashed contours indicate gain margins of 0, 3, 6, and 9 dB, quantifying the robustness of the loop.

Guidance for Design and Commissioning

The stability map provides direct guidance for both DLLRF system design and commissioning. For the DLLRF system design, it constrains the allowable latency budget by identifying the range of loop delays that permit stable operation. For commissioning, the map enables a simulation-guided tuning procedure. We recommend first measuring the total loop delay τ_d, then selecting K_P from the stable interval at that specific delay, as indicated in Fig. 2. A safety margin relative to the predicted stability boundary ($\alpha_{\max} = 1/\tau_{\mathrm{rad}}$) should be maintained, and the Nyquist-excluded region must be strictly avoided to prevent loop instability. This method can provide direct guidance for DLLRF system commissioning and minimizes the risk of beam loss.

CONCLUSION

A simulation-based method has been developed for tuning PI feedback parameters in RF cavity voltage regulation,

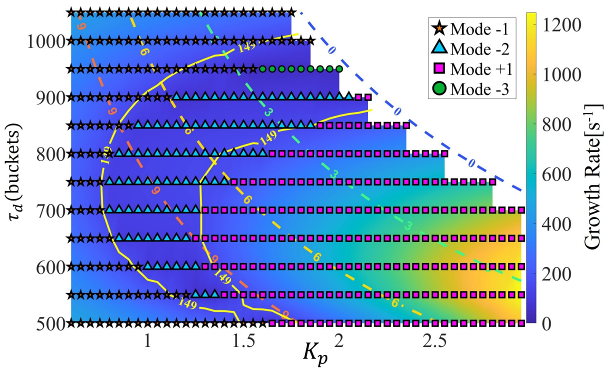

Figure 2: Stability map for STCF at 3.5 GeV in the (K_P, τ_d) plane. The color scale represents the growth rate of the fastest-growing mode, with markers indicating the mode index (stars: mode -1; triangles: mode -2; squares: mode $+1$). The solid contour labeled "149" marks the radiation damping threshold $1/\tau_{\mathrm{rad}}$, while dashed contours denote the gain margin. The white upper-right region is excluded by the Nyquist criterion.

specifically to mitigate accelerating-mode driven coupled-bunch instabilities in high-current storage rings. Using the GPU-accelerated tracking code STABLE—which captures the coupled beam–cavity–DLLRF dynamics—the beam-stable operating region is mapped in the (K_P, τ_d) plane by evaluating the growth rate of the fastest-growing coupled-bunch mode. Concurrently, a Nyquist-based screening excludes intrinsically unstable loop configurations and quantifies the corresponding gain margins.

The method was demonstrated using the 3.5 GeV STCF parameters. The resulting stability map delineates a theoretically grounded safe operating region, providing valuable guidance for online commissioning and constraining key DLLRF design parameters, notably the allowable latency budget.

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