ANALYSIS AND RESEARCH ON MEASUREMENT ERRORS CAUSED BY MAGNET MISALIGNMENT IN ENERGY ANALYSIS SYSTEMS*

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Abstract

Accurate evaluation of the beam energy and momentum spread plays a crucial role in the commissioning and operation of electron accelerators. The energy analysis (EA) system, consisting of a dipole with associated upstream and downstream drift sections, is widely applied in accelerator facilities due to its structural simplicity, large measurement range, and high resolution. However, assembly errors are inevitably introduced during engineering implementation. These errors lead to offsets or tilt angles between the input beam and the designed system axis. Such misalignment necessarily affect the accuracy of the measurement results of the EA system, and the impact may be even more significant for low-energy e-beams with the energy of a few MeV. In this paper, based on the low-energy injector experimental platform currently under construction, the misalignment induced errors are analyzed in depth, and potential compensation methods are explored. The results offer critical theoretical and practical guidance for enhancing measurement precision.

INTRODUCTION

With the widespread application of particle accelerators, diagnostic techniques for high-quality electron beams [1, 2] have advanced significantly. The performance of such accelerators is highly dependent on the quality of the electron beam, necessitating precise characterization of the beam energy spectrum.

Energy analysis systems based on magnetic deflection are widely adopted [3–5] due to their stable operation and low cost. A typical system consists of a dipole magnet, an entrance slit, a downstream fluorescent screen, and the drift sections in between. The working principle relies on combining the dispersive properties of sector magnets with beam transport theory [6–8]. The momentum deviation δ is measured via magnetic dispersion, and the corresponding kinetic energy T is determined by the relativistic relation $T = \sqrt{(BRec)^2 + m_0^2 c^4} - m_0 c^2$, where B is the magnetic field, R the bending radius, c , e and m_0 represents the light speed, the electron charge and mass, respectively. For an electron with initial transverse offset x_0, angular divergence x_0', and momentum deviation δ, after passing through a beamline described by the transfer matrix M, its final horizontal position can be expressed as $x_1 = m_{11}x_0 + m_{12}x_0' + m_{13}\delta$, where m_{11}, m_{12}, and m_{13} are the corresponding elements of the transfer matrix M.

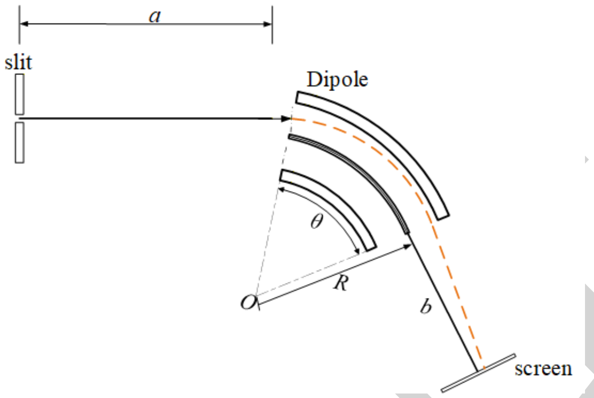

Figure 1: Schematic Diagram of Beam Bunch Deviation from the Center Caused by Magnet Translation and Tilting.

However, in practical engineering implementations, installation and alignment errors [9] of magnets are unavoidable. This paper analyzes the particle trajectories to calculate the momentum spread and energy measurement errors caused by dipole magnet translation and tilt. Furthermore, a correction method for the translation-induced energy measurement error is proposed.

ALIGNMENT ERROR IMPACTS ON ENERGY AND MOMENTUM SPREAD MEASUREMENTS

To facilitate analysis, the design parameters of the HuTeX system are adopted as an example, where the bending radius of the dipole magnet is 300 mm and the bending angle is $\pi/3$ radians. The drift lengths a and b are 607 mm and 450 mm, respectively. Given that the vertical alignment errors within the uniform field gap have a negligible impact on the beam trajectory, only errors in the horizontal plane are considered. Furthermore, to simplify the numerical analysis, a one-dimensional (1D) bunch simulation model is employed, where the influence of the initial transverse beam size is neglected.

Under the current engineering accuracy, maximum deviations of ±1 mm for magnet translations and ±1 mrad for tilts are defined based on the practical mechanical alignment tolerances. A schematic of the off-center beam injection caused by magnet translation and tilting is presented in Fig. 1.

For a given energy analysis system, the multi-particle beam may be treated as a single electron when measuring the central energy. The focus is placed on the trajectory of the central-energy electron and its position on the screen. The electron trajectory can be analyzed using a geometric

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method. By taking point O as the coordinate origin, the circular equation corresponding to the trajectory within the dipole, when no translation or tilt of the magnet occurs, can be expressed as Eq. (1).

$$x^2 + (y - n)^2 = r^2 \tag{1}$$

where, n=r_0-r, r_0 and r denote the bending radii of the reference and off-momentum particles, respectively.

The linear equation representing the exit edge of the dipole magnet can be expressed as Eq. (2).

$$y = kx = \tan(\pi/2 - \theta)x \tag{2}$$

where, k is the slope of the linear exit edge, and θ denotes the bending angle of the dipole in radians.

By combining the circular equation with the linear equation, the particle coordinates at the dipole exit can be obtained as Eq. (3).

$$\begin{cases} x = \dfrac{2nk + \sqrt{4n^2k^2 - 4(1+k^2)(n^2 - r^2)}}{2(1+k^2)} \\ y = kx \end{cases} \tag{3}$$

For the circle represented by Eq. (1), the divergence angle at the dipole exit can be determined by calculating the slope of the tangent at the coordinates (x, y) .

$$x' = \arctan \frac{k_2 - k_1}{1 + k_2 k_1} \tag{4}$$

where, $k_1 = -\frac{x}{y-n}$, $k_2 = \tan(\pi - \theta)$.

The particle position on the screen, x_1, can then be expressed as Eq. (5).

$$x_1 = x\cos(\pi/2 - \theta) + y\sin(\pi/2 - \theta) - b\tan(x') - r_0 \tag{5}$$

When translation or tilt occurs, the particle trajectory circular equation can be adjusted according to the particle incident position and angle. By employing this method, the influence of alignment errors on the measurement results is investigated via MATLAB. The analysis is conducted for a beam with an energy range of 3 - 15 MeV and a momentum spread of 0 - 12‰.

Figures 2(a) and 2(b) show the energy measurement errors under different dipole translation offsets and tilt angles, respectively. Here, particle injections at the high-energy side and the low-energy side are denoted as positive (+) and negative (−) signs, respectively. For a beam within 15 MeV, when the dipole is translated within ±1 mm, the energy measurement error remains within ±30 keV. Specifically, the evaluation results indicate that for beam injection at the high-energy side (+), the reconstructed energy is smaller than the true value, whereas for injection at the low-energy side (−), the measured result is larger. Conversely, the impact of dipole tilt on energy measurement results is negligible. For tilt angle is within ±1 mrad, the energy measurement errors are within 0.02 keV.

Figures 2(c) and 2(d) show the momentum spread measurement errors. When the beam is injected at the high-energy end due to magnet translation or tilting, the beam size is increased; conversely, a decrease in size is observed for low-energy injection. The larger the offsets, the greater the deviation of the beam size.

Based on the beam size and third-order correction [10], the momentum momentum calculation error can be obtained, which follows the same trend as the beam size error. This total measurement deviation is essentially the coupled result of the intrinsic calculation error remaining after the third-order correction and the alignment errors. The results indicate that, within a translation of ±1 mm and a tilt of ±1 mrad, the measurement error of momentum spread remains below 0.5‰, , which is sufficiently small to be safely neglected in practical engineering applications.

ENERGY MEASUREMENT ERROR CORRECTION

As established by the analysis in the preceding chapter, the influence of magnet tilting on energy measurement errors is negligible. Furthermore, the momentum spread measurement errors are all within the system's allowable accuracy limits. Therefore, this chapter focuses on the correction of energy measurement errors induced by magnet translation.

In Fig. 1, when alignment errors are present, the particle trajectory is represented by an orange dashed line; without alignment errors, the trajectory is shown by a black solid line. Using geometric relationships, the particle position at the system exit, x_1, under magnet translation can be expressed as a function of the translation displacement, Δx, and the bending radius, r, as shown in Eq. (6).

$$x_1 = f(r, \Delta x) \tag{6}$$

Based on the energy measurement formula, the radius r can be expressed as a function of the T and B. By substituting this relation into the Eq. (6), the terminal position can be written as Eq. (7).

$$x_1 = g(T, B, \Delta x) \tag{7}$$

Based on Eq. (7), the magnetic field strength can be varied multiple times to record the terminal positions x_{1n}. Multiple overdetermined equations relating T and Δx can then be established. By solving this set of equations, the particle kinetic energy T can be determined.

Taking 7 MeV, 10 MeV, and 15 MeV beams as examples, the energy correction results obtained by the above method are calculated for translations of ±1 mm and ±0.5 mm, respectively. The results are summarized in Table 1.

The energy measurement errors induced by magnet translation are reduced from several tens of keV to the level of 0.1 keV after correction. This result demonstrates the effectiveness of the correction method.

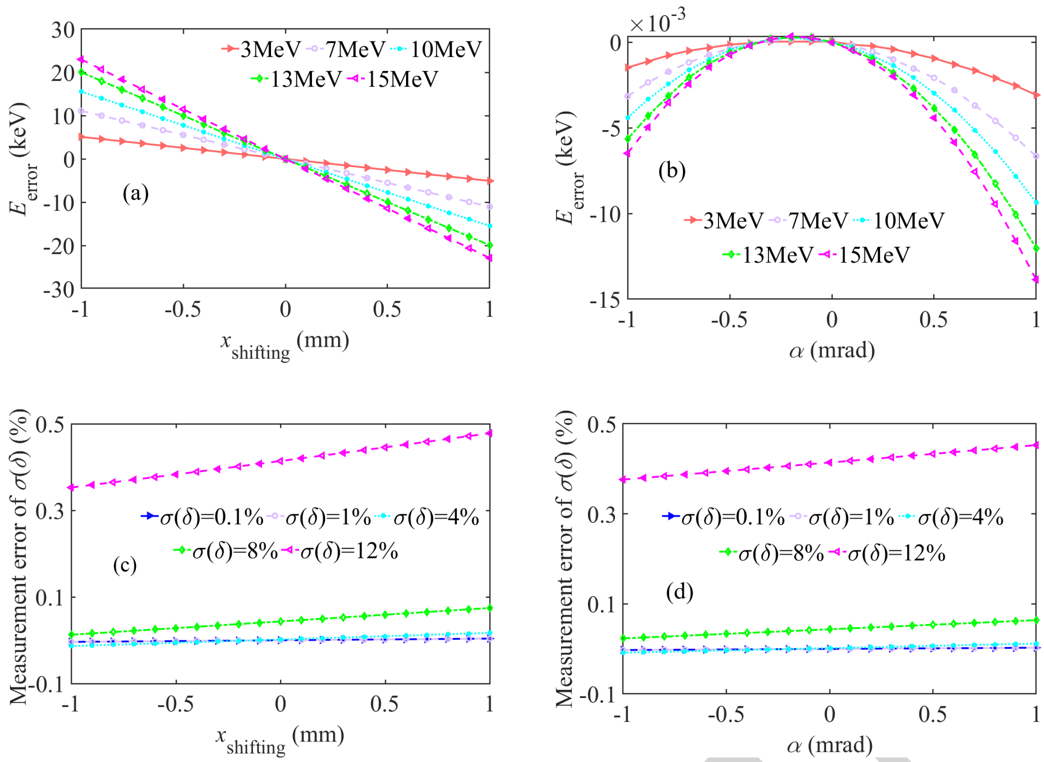

Figure 2: (a) Energy Measurement Errors Under Different Translation Offsets; (b) Energy Measurement Errors Under Different Tilt Angles; (c) Momentum Spread Measurement Errors for a 10 MeV Beam Under Different Translation Offsets; (d) Momentum Spread Measurement Errors for a 10 MeV Beam Under Different Tilt Angles.

Table 1: Energy Correction Results Under Different Offsets

Δx (mm)	**7 MeV**	**10 MeV**	**15 MeV**
+1	6.9999	10.0001	15.0002
+0.5	7.0000	10.0000	15.0001
−0.5	7.0000	9.9999	15.0000
−1	6.9999	10.0001	14.9998

CONCLUSION

In this paper, a comprehensive alignment error analysis for the magnetic deflection-based energy analysis system has been completed. Within the current engineering alignment tolerances (±1 mm translation and ±1 mrad tilt), the measurement errors of the momentum spread and the energy deviations induced by dipole tilts are proved to be negligible. Although dipole translations can introduce a maximum energy measurement error of up to tens of keV, this error is successfully suppressed to approximately 0.1 keV via the proposed correction methodology. These results systematically validate the high-precision capability of the system and provide robust theoretical and technical support for its practical engineering application.

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