ORBIT CORRECTION STUDIES ON THE ELECTRON TRANSPORT LINE FROM RCS TO ESR *

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Abstract

A dedicated electron transfer line from Rapid Cycling Synchrotron (RCS) to Electron Storage Ring (ESR), referred to as the RTE line has been designed for the Electron-Ion Collider (EIC). The beamline follows a straight-line geometry, with a length of 133 m, and is consists with two matching sections and a FODO section for beam diagnostics. Imperfections with magnet alignments introduce orbit distortions, making orbit correction scheme a critical component in the design. To facilitate orbit correction, each quadrupole magnet is equipped with a pair of beam position monitors (BPMs) and kickers. The Singular Value Decomposition (SVD) algorithm is used for orbit correction and tolerance studies. This paper presents the ongoing progress in the optics design and error correction scheme of the RTE line.

INTRODUCTION

The U.S. Electron-Ion Collider (EIC) is being designed to be build at Brookhaven National Lab as a collaborative effort between Jefferson Lab and and Brookhaven National Lab. The new electron injector system for the EIC consists of electron linac, Beam Accumulator Ring (BAR), Rapid Cycling Synchrotron (RCS), and Electron Storage Ring (ESR). In the RCS, electrons get accelerated to three different energies; 5 GeV, 10 GeV and 18 GeV [1]. Accelerated electrons will then be transported to ESR using a dedicated transport line referred to as RTE line.

The RTE line extracts electrons using three pulsed kicker magnets and two septum magnets, making 3° extraction angle. A FODO section consists with 9 compact quadrupole magnets reusing 30 cm MIT-Bates quadrupole magnets [2]. Two matching sections are at the ends of FODO section to match Twiss functions from RCS extraction and to ESR injection points. The ESR injection is designed with a 6° or 9° injection angle, using two or three 3° DC septum magnets along with 14 fast-pulsed kickers. More details on the ERS injection is found in [3].

TRAJECTORY CORRECTION WITH SVD

The design of the RTE line evolved to accommodate the changes and requirements of the RCS and ESR ring designs. For this study an RTE line design with 9° ESR injection angle is used. The lattice layout of this line is illustrated in Fig. 1.

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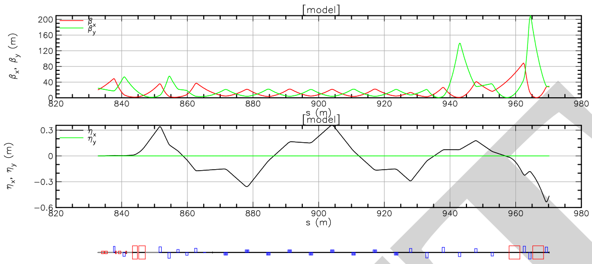

Figure 1: Beta functions, dispersion and the magnet layout of the RTE line with 9° ESR injection

Singular Value Decomposition

The transfer matrix R between two positions in a beamline is,

$$R_{1,2} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \tag{1}$$

For a perturbed orbit due to an applied kick ($\Delta x'_1$) at point 1 is measured with a BPM at the point 2. The beam position at point 2 can be described as;

$$\Delta x_2 = R_{12} \Delta x'_1 \tag{2}$$

Similarly, for a series of "M" BPMs and "N" kicker magnets, deviated orbit at j^{th} BPM is,

$$\Delta x^j = \Sigma_{i=1}^{N} (R_{12})_{ji} \Delta x'_i \tag{3}$$

Two set of vectors are then created using the beam obit position (X) and the kicks (X'), which the used in the Singular Value Decomposition (SVD) algorithm.

$$X = \begin{pmatrix} \Delta x^1 \\ . \\ . \\ . \\ \Delta x^M \end{pmatrix} \quad , \quad X' = \begin{pmatrix} \Delta x'^1 \\ . \\ . \\ . \\ \Delta x'^N \end{pmatrix}$$

These two vectors are then written as;

$$X = AX' \tag{4}$$

Where, A is a N x M matrix know as the response matrix.

In SVD, this matrix A is decomposed into 3 matrices, U, V and Σ, where U and V are orthogonal and Σ is a rectangular diagonal matrix containing singular values $\sigma_{ii} = \Sigma_{ii}$. One can easily remove the singular vectors with small singular values to increase the robustness of the trajectory correction. After singular vector removal the "pseudo inverse" of A is obtained as $A^{-1} = V\Sigma^{-1}U^T$. The required corrector kicks are then determined by, [4, 5]

$$X' = A^{-1}X \tag{5}$$

QUADRUPOLE ERRORS AND CORRECTION IN RTE LINE

The error correction study is carried out to estimate the tolerances associated with quadrupole magnet alignment and power supply errors in the RTE line design. Correction of the perturbed beam orbits is performed using the SVD based trajectory correction method described in the previous section. We then evaluated the allowable magnet error tolerance by determining the rms orbit offset after trajectory correction. We used, a BPM and a horizontal and vertical corrector magnet placed near the downstream of each quadrupole in the RTE line for trajectory correction. The correctors are assumed to be similar to RCS correctors with a length 25.7 cm. To evaluate the required corrector kicks, first the orbit response matrices (A) for the X and Y planes are calculated using the unperturbed lattice. The correction procedure is then performed using a BMAD program, integrated in a python script. Since this is a transport line the matrices A_x and A_y are triangular matrices, as only the BPMs downstream of a given kicker can see the orbit offset due to the kicker.

The error tolerance study started with only quadrupole misalignment errors and later added different quadrupole errors and used the cumulative distribution functions (CDF) change to evaluate the different error levels. A cutoff value of 0.01 is used to remove smaller singular values.

Cumulative Distribution Function

The cumulative distribution function (CDF) is a well-known statistical function of a random variable X, used to determine the probability that X takes a value less than or equal to a given value x, and is expressed in Eq. 6, where r.h.s represents the probability [6].

$$F_X(x) = \mathrm{P(X \leq x)} \tag{6}$$

To evaluate the probability of orbit correction, Eq. 7 is used and calculate the CDF of rms orbit deviation and rms $\beta(s)$ deviation as given below.

$$\sigma_x = \sqrt{\frac{\Sigma_{i=1}^{N}(x_i - x)^2}{N}} \tag{7}$$

For misalignment errors, x_i in Eq. 7 represents the orbit offset measured at the i^{th} BPM, while σ_x denotes the RMS orbit deviation. In contrast, for other quadrupole errors, such as tilt and mispowering errors, x_i represents the deviation of $\beta(s)$ from its design value at the i^{th} BPM. Then the σ_x is the RMS $\beta(s)$ deviation from the design. CDF is then calculated using the calculated σ_x corresponding to each error set.

Quadrupole Misalignment Errors

Misalignment of a quadrupole magnet introduces additional transverse kicks to the beam because the magnetic field is non-zero, away from its magnetic center. These kicks perturb the beam orbit and must be corrected. Therefore,

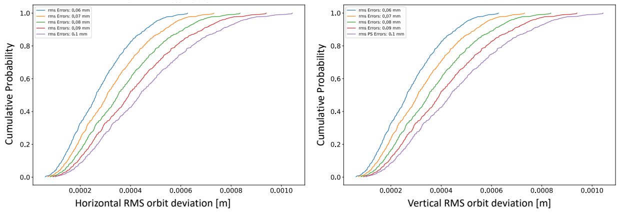

Figure 2: Cumulative Distribution Function (CDF) of the corrected orbit

estimating acceptable quadrupole alignment tolerances is essential for efficient accelerator operation.

A set of 500 random error configurations for each RMS misalignment value are generated. The required corrector strengths at each corrector magnet are calculated using SVD based orbit correction algorithm as described above.

The calculated corrector kicks are then applied to compensate for the orbit deviations. For each corrected orbit, the standard deviation from the design orbit is evaluated and used to construct the CDF for the corresponding error set. This analysis helps estimate the quadrupole alignment precision required to achieve a desired level of orbit accuracy by using 2.5σ cutoff, corresponding to a 98.78% CDF criterion. From Fig. 2, ≈ 99% of the orbits generated by RMS misalignment errors less than 0.1 mm are corrected with an rms orbit deviation less than 1 mm.

Quadrupole Tilt error

Quadrupole tilt errors arise when a quadrupole magnet is rotated around its longitudinal axis. This rotation introduces 'skew' quadrupole field components, which couple the beam motion in horizontal and vertical planes. A pure quadrupole tilt, in the absence of any orbit offset, does not directly distort the beam orbit because an on-axis beam does not receive transverse kicks from an ideal quadrupole, but affects the beam focusing changing the Twiss functions. Consequently, SVD-based orbit correction is generally ineffective for correcting pure tilt errors [7].

To study this effect, a set of random tilt errors was applied to all quadrupoles along with an RMS misalignment error of 0.1 mm. Figure 3 shows the cumulative distribution function (CDF) of the rms $\beta(s)$ deviations after orbit correction using the SVD algorithm in the presence of these tilt errors. Form the CDF curves, it is observed that with 0.4 mrad error, 99% cases are corrected with RMS $\beta(s)$ deviation of less than 3 mm. The results can also be used to estimate acceptable tolerances for quadrupole tilt angles.

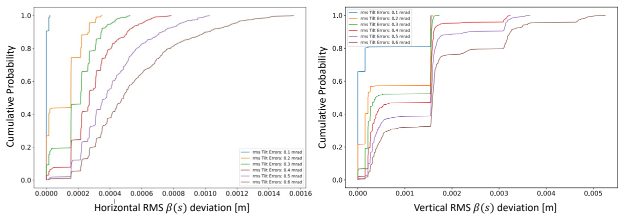

Figure 3: CDF of $\beta(s)$, after orbit correction due to quadrupole misalignment and different tilt errors

Quadrupole Pitch Errors

Quadrupole pitch error is known as the rotation around its transverse axis (either horizontal or vertical), resulting in beam offsets due to field asymmetry on the beam center. To study the error tolerances with quadrupole pitch errors, along with an RMS misalignment error of 0.1 mm and RMS tilt error of 0.4 mrad are used. These values are consistent with the APSU magnet tolerance parameters [8]. As shown in Fig. 4, no significant difference is RMS $\beta(s)$ deviation is observed within the error sets used. For RMS pitch angle error of 0.4 mrad along with an RMS misalignment error of 0.1 mm, 99% of cases are corrected with less than RMS $\beta(s)$ deviation of 2.1 mm.

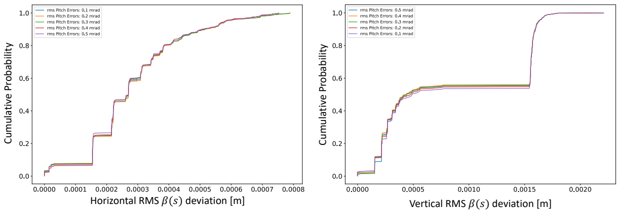

Figure 4: CDF of $\beta(s)$, after orbit correction due to quadrupole misalignment, tilt and different pitch errors

Quadrupole Power Supply Errors

Errors in a power supply connected to a quadrupole magnet results deviations in the quadrupole focusing strengths (k1) as the applied current and k1 are directly proportional as given in Eq. 8.

$$\frac{\Delta I}{I} \approx \frac{\Delta k1}{k1} \tag{8}$$

Quadrupole power supply errors are introduced to the lattice along with the errors described above. From survey, it was measured that the tolerances of power supply for MIT-Bates quadrupole magnets is $\approx$ 2 mA. Hence only changed the power supply errors of the matching quadrupoles by keeping the RMS error form Bates quadrupoles fixed at the measured value. Figure 5 illustrates the CDF of the RMS $\beta(s)$ deviations for different errors. These curves suggest that for RMS power supply error of 4 mA, along with RMS misalignment error of 0.1 mm and tilt error of 0.1 mrad, 99% cases with are corrected with RMS $\beta(s)$ deviation up to less than 12 m.

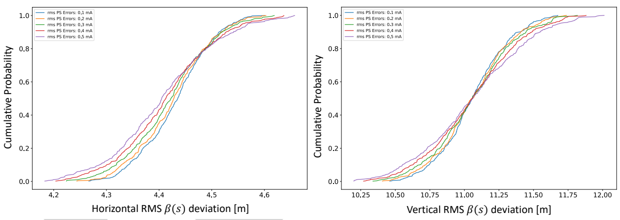

Figure 5: CDF of the rms $\beta(s)$ deviation from the design due to misalignment, tilt and pitch errors and power supply errors in quadrupole magnets

Figure 6 illustrates the perturbed and corrected orbits with all the errors corrected using SVD based lattice correction method. Top two plots corresponds to horizontal(X) orbits, while the bottom two plots corresponds to vertical(Y) orbit. The results demonstrate that the SVD-based correction algorithm effectively restores the perturbed beam orbits close to the design trajectory.

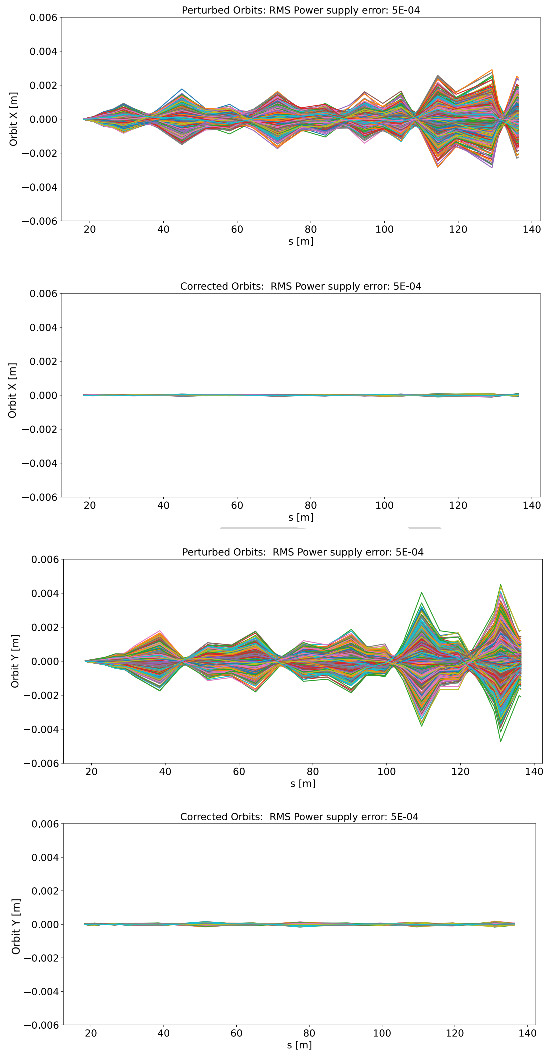

Figure 6: Perturbed and corrected orbits in X & Y, with all the errors.

CONCLUSION

Error tolerances for quadrupole magnets in the RTE line is studied using a error correction method based on SVD. A misalignment error of 0.1 mm is used to study the affects of the other quadrupole errors and Table 1 summarizes the tolerance values of the rms errors.

Table 1: Tolerance Balues of Quadrupole Errors

Error	Tolerance value
Misalignment	0.1 mm
Tilt angle	0.4 mrad
Pitch angle	0.4 mrad
Power supply error	4 mA

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