THEORY OF THE WAVEGUIDE VARIABLE POWER DIVIDER AND COMBINER

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Abstract

Theoretically, a 180 ° or 90 ° hybrid bridge can serve as a variable power divider and combiner. This study focuses on related theoretical research, selecting the Magic Tee and 3 dB bridge as the required 180 ° and 90 ° hybrid bridges. Based on the scattering matrix of the four-port microwave network, the relationships between input and output amplitude and phase, the phase difference of the two output signals, and the influence of input amplitude or phase error on the output signals are theoretically deduced. Simulations are conducted, and the results match the theory, validating its correctness. The variable power divider and combiner have wide applications and are worthy of study.

INTRODUCTION

Waveguides are the primary choice for transmitting high-power microwaves due to their high power capacity and low transmission loss. The Waveguide Distribution System (WDS) is widely used in areas like accelerators [1] for the transmission, allocation, monitoring, and absorption of high-power microwaves. The Variable Power Divider (VPD) and Combiner (VPC) are convenient for matching different power allocation schemes without altering the WDS layout [2], thus being widely adopted in high-power WDS and worth researching.

The Magic Tee and 3dB bridge are commonly selected as the 180 ° and 90 ° hybrid bridges to achieve VPD and VPC functions. This paper details the theory regarding the relationships between input and output amplitude and phase, the phase difference of the two output signals, and the influence of input amplitude or phase error on the output signals.

THEORY OF THE MAGIC-TEE-BASED VPD AND VPC

The Magic Tee is shown in Fig. 1, it is a four-port 180 ° hybrid bridge with the following scattering matrix

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad . \tag{1}$$

Theory of the VPD

Define port 3 and port 4 as input ports with signals $E_3 \times \sin\varphi_3$ and $E_4 \times \sin\varphi_4$. Let E and φ be the amplitude and phase of the input signal respectively. Assume $E_3 = \alpha \times E_4$, $E_4 = 1$, $\alpha \in [0, +\infty)$ and $\varphi_3 = \varphi_4 + \theta$, where α and θ are the magnitude ratio and phase difference of the two input signals at port 3 and port 4. Substituting these conditions into Eq. (1), the output signals of port 1 and port 2 are

$$P_{1out} = \frac{\alpha\cos\theta + 1}{\sqrt{2}}\sin\varphi_4 + \frac{\alpha\sin\theta}{\sqrt{2}}\cos\varphi_4 \quad . \tag{2}$$

$$P_{2out} = \frac{\alpha\cos\theta - 1}{\sqrt{2}}\sin\varphi_4 + \frac{\alpha\sin\theta}{\sqrt{2}}\cos\varphi_4 \quad . \tag{3}$$

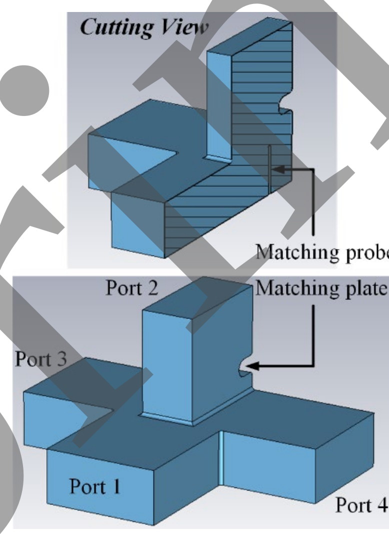

Figure 1: Structure and port definition of the Magic Tee.

So, the output magnitude ratio is

$$\frac{|P_{1out}|}{|P_{2out}|} = \sqrt{\frac{\alpha^2 + 2\alpha\cos\theta + 1}{\alpha^2 - 2\alpha\cos\theta + 1}} \quad . \tag{4}$$

If $\alpha = 1$, then

$$\frac{|P_{1out}|}{|P_{total-out}|} = \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}} \quad . \tag{5}$$

If $\theta \in [0, \pi]$, the value of Eq. (5) varies between [1,0]. This means that if one input signal is equally divided and the two output signals are fed into port 3 and port 4 of the Magic Tee after adjusting their phase difference from 0 to π, output signals of any magnitude ratio can be obtained at ports 1 and port 2, realizing the VPD function.

Influence of the Input Amplitude Error In practice, it is difficult to achieve an ideal 1:1 magnitude ratio of the two input signals at port 3 and port 4. Define the error of deviation from a 1:1 ratio as δ, which can be positive or negative, corresponding to a ratio larger or smaller than 1:1. Replacing the input magnitude ratio with 1 + δ in Eq. (4) gives

$$\frac{|P_{1out}|}{|P_{2out}|} = \sqrt{\frac{\delta^2 + 2\times(1+\cos\theta)\times\delta + 2\times(1+\cos\theta)}{\delta^2 + 2\times(1-\cos\theta)\times\delta + 2\times(1-\cos\theta)}} \quad . \tag{6}$$

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Equation (6) shows the relationship between the output magnitude ratio $|P_{1out}|/|P_{2out}|$ and the input phase difference θ when the input magnitude ratio of the Magic Tee deviates from 1:1 with an error of δ, and can be used for theoretical analysis and correction of measurement curves.

Phase Difference of the Two Output Signals For the two output signals of port 1 and port 2 of the Magic Tee, besides the output magnitude ratio, the phase difference is also crucial. Assuming $\alpha = 1$ and $\theta \in (0, \pi)$, then $\sin\theta > 0$. Substituting into Eq. (2) and (3) and calculating the phase difference between port 1 and port 2 gives

$$\Delta\varphi_{(p_1-p_2)} = -\arctan\left(\frac{\cos\theta+1}{\sin\theta}\right) + \arctan\left(\frac{\cos\theta-1}{\sin\theta}\right) = \frac{\pi}{2} \quad (7)$$

Equation (7) indicates that the phase difference of the two output signals remains constant as the output magnitude ratio changes.

Theory of the VPC

From Eq. (4), if $\theta \equiv 90^\circ$ or $\theta \equiv 270^\circ$, then $\cos\theta \equiv 0$ and $|P_{1out}|/|P_{2out}| \equiv 1:1$. By setting the phase difference of the two input signals to 90 ° or 270 ° and feeding them to port 3 and port 4 of the Magic Tee, two output signals with the same amplitude but different phases can be obtained at port 1 and port 2. Adjusting them to the same phase and using an equal power combiner can achieve the VPC function, regardless of the input magnitude ratio.

Influence of the Input Phase Error Define the error of phase deviation from 90 ° as σ. Replacing the input phase difference with $90^\circ + \sigma$ in Eq. (4) gives

$$\frac{|P_{1out}|}{|P_{2out}|} = \sqrt{\frac{\alpha^2-2\alpha\times\sin\sigma+1}{\alpha^2+2\alpha\times\sin\sigma+1}} \quad . \quad (8)$$

Equation (8) shows the relationship between the output magnitude ratio $|P_{1out}|/|P_{2out}|$ and the input magnitude ratio α when the input phase difference of the Magic Tee deviates from 90 ° with an error of σ, and can be used for theoretical analysis and correction of measurement curves.

Phase Difference of the Two Output Signals The relationship between the phases of the two output signals at port 1 and port 2 of the Magic Tee is important for the final combination of the two input signals. Assuming $E_3 = \alpha \times E_4$, $E_4 = 1$, $\alpha \in (0, +\infty)$, $\varphi_3 = \varphi_4 + \theta = \varphi_4 + 90^\circ$. Substituting into Eq. (2) and (3) and calculating the phase difference between port 1 and port 2 gives

$$\Delta\varphi_{(p_1-p_2)} = -2 \times \arctan\left(\frac{1}{\alpha}\right) . \quad (9)$$

Equation (9) indicates that the phase difference of the two output signals at port 1 and port 2 varies from $-\pi$ to 0 as the input magnitude ratio α varies from 0 to $+\infty$.

THEORY OF THE 3DB-BRIDGE-BASED VPD AND VPC

The 3dB bridge is shown in Fig. 2, it is a four-port 90 ° hybrid bridge with a scattering matrix as follows

$$S = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & j & 1 \\ 0 & 0 & 1 & j \\ j & 1 & 0 & 0 \\ 1 & j & 0 & 0 \end{bmatrix} . \quad (10)$$

Theory of the VPD

Define port 1 and port 2 as input ports with signals $E_1 \times \sin\varphi_1$ and $E_2 \times \sin\varphi_2$. Assuming $E_1 = \alpha \times E_2$, $E_2 = 1$, $\alpha \in [0, +\infty)$ and $\varphi_1 = \varphi_2 + \theta$, where α and θ are the magnitude ratio and phase difference of the two input signals. Substituting into Eq. (10) gives

$$\frac{|P_{3out}|}{|P_{4out}|} = \sqrt{\frac{\alpha^2-2\alpha\sin\theta+1}{\alpha^2+2\alpha\sin\theta+1}} \quad . \quad (11)$$

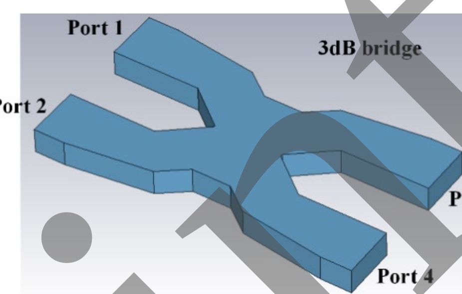

Figure 2: Structure and port definition of the 3dB bridge.

If $\alpha = 1$, then

$$\frac{|P_{3out}|}{|P_{total-out}|} = \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}+\sqrt{1-\sin\theta}} \quad . \quad (12)$$

If $\theta \in [-\pi/2, \pi/2]$, the value of Eq. (12) varies between [1,0]. This is the theoretical basis for realizing the VPD based on the 3dB bridge, with a similar plan to that of the Magic Tee.

Influence of the Input Amplitude Error Define the error of deviation from an ideal 1:1 input magnitude ratio as δ. Replacing the input magnitude ratio with 1+δ in Eq. (11) gives

$$\frac{|P_{3out}|}{|P_{4out}|} = \sqrt{\frac{\delta^2+2\times(1-\sin\theta)\times\delta+2\times(1-\sin\theta)}{\delta^2+2\times(1+\sin\theta)\times\delta+2\times(1+\sin\theta)}} \quad . \quad (13)$$

Equation (13) can be used for theoretical analysis and correction of measurement curves considering measurement errors.

Phase Difference of the Two Output Signals Assuming $\alpha = 1$ and $\theta \in (-\pi/2, \pi/2)$, then $\cos\theta > 0$, calculating the phase difference between port 3 and port 4 gives

$$\Delta\varphi_{(p_3-p_4)} = \arctan\left(\frac{\cos\theta}{1-\sin\theta}\right) - \arctan\left(\frac{1+\sin\theta}{\cos\theta}\right) = 0 . \quad (14)$$

Equation (14) indicates that the phase difference of the two output signals remains constant as the output magnitude ratio changes.

Theory of the VPC

From Eq. (11), if $\theta \equiv 0^\circ$ or $\theta \equiv 180^\circ$, then $\sin\theta \equiv 0$ and $|P_{3out}|/|P_{4out}| \equiv 1:1$. This is the theoretical basis for realizing the VPC based on the 3dB bridge, with a similar plan to that of the Magic Tee.

Influence of the Input Phase Error Define the error of the phase deviation from 0 ° as σ. Replacing the input phase difference with $\boldsymbol{\sigma}$ in Eq. (11) gives

$$\frac{|P_{3out}|}{|P_{4out}|} = \sqrt{\frac{\alpha^2-2\alpha\times\sin\sigma+1}{\alpha^2+2\alpha\times\sin\sigma+1}} \quad . \quad (15)$$

Equation (15) is the same as Eq. (8), indicating that an input phase error affects the output magnitude ratio in the same way for both the Magic-Tee-based and 3dB-bridge-based VPCs.

Phase Difference of the Two Output Signals The relationship between the phases of the two output signals at port 3 and port 4 of the 3dB bridge is important for the final combination of the two input signals. Assuming $E_1 = \alpha \times E_2$, $E_2 = 1$, $\alpha \in (0, +\infty)$, $\varphi_1 = \varphi_2 + \theta = \varphi_2$, calculating the phase difference between port 3 and port 4 gives

$$\Delta\varphi_{(p_3-p_4)} = \arctan\left(\frac{\alpha^2-1}{2\alpha}\right) \quad . \tag{16}$$

Equation (16) indicates that the phase difference of the two output signals varies from $-\pi/2$ to $\pi/2$ as the input magnitude ratio α varies from 0 to $+\infty$.

Relationship of the Output Phase Difference Between the Magic-Tee-Based and 3dB-Bridge-Based VPCs

Comparing Eq. (9) and (16) gives

$$\begin{aligned}\Delta\varphi_{(p_1-p_2)} &= -2 \times \arctan\left(\frac{1}{\alpha}\right) \\ &= -\left[\pi + \arctan\left(\frac{2\times\frac{1}{\alpha}}{1-\left(\frac{1}{\alpha}\right)^2}\right)\right] \\ &= -\left[\pi - \frac{\pi}{2} - \arctan\left(\frac{\alpha^2-1}{2\alpha}\right)\right] \\ &= -\frac{\pi}{2} + \arctan\left(\frac{\alpha^2-1}{2\alpha}\right) \\ &= -\frac{\pi}{2} + \Delta\varphi_{(p_3-p_4)}, \qquad 0 < \alpha < 1\end{aligned} \tag{17}$$

and

$$\begin{aligned}\Delta\varphi_{(p_1-p_2)} &= -2 \times \arctan\left(\frac{1}{\alpha}\right) = -\arctan\left(\frac{2\times\frac{1}{\alpha}}{1-\left(\frac{1}{\alpha}\right)^2}\right) \\ &= -\left[\frac{\pi}{2} - \arctan\left(\frac{\alpha^2-1}{2\alpha}\right)\right] \\ &= -\frac{\pi}{2} + \arctan\left(\frac{\alpha^2-1}{2\alpha}\right) \\ &= -\frac{\pi}{2} + \Delta\varphi_{(p_3-p_4)}, \qquad \alpha > 1\end{aligned} \tag{18}$$

Combining them gives

$$\Delta\varphi_{(p_1-p_2)} = -\frac{\pi}{2} + \Delta\varphi_{(p_3-p_4)}, \ \alpha > 0 \quad . \tag{19}$$

In summary, for the 3dB-bridge-based VPC, the phase difference curve of its output ports shifts upward by $\pi/2$ compared to that of the Magic-Tee-based VPC, while the curve shape remains unchanged, as shown in Fig. 3.

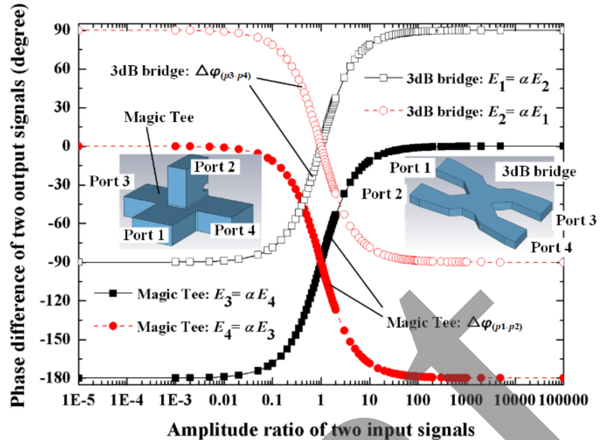

Figure 3: Output phase difference curves of the 3dB-bridge-based and Magic-Tee-based VPCs.

All theoretical results derived from the equations in this paper have been verified by software simulation, and the results are in excellent agreement. Due to space limitations, the simulation curves are not presented here.

CONCLUSION

The theoretical equations of the VPD and VPC based on the Magic Tee or 3dB bridge are derived, including the relationships between input and output amplitude and phase, the phase difference of the two output signals, and the influence of input amplitude or phase error on the output signals. The simulation results match the theory, validating its correctness. These theoretical equations provide effective guidance for actual fabrication and measurement.

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