BETA FUNCTION MEASUREMENTS USING QUADRUPOLE VARIATION IN SLS 2.0

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Abstract

The Swiss Light Source upgrade, SLS 2.0, is a fourth generation storage ring based on a seven-bend achromat design and is currently under commissioning. Precise knowledge and control of the linear optics are essential for optimal machine performance. This contribution presents measurements of the beta function using the quadrupole variation method at 264 locations around the ring. The corresponding tune shifts were determined with high resolution via the mixed BPM technique combined with Numerical Analysis of Fundamental Frequencies (NAFF).

INTRODUCTION

The Swiss Light Source upgrade, SLS 2.0, features a seven-bend achromat lattice incorporating novel technologies such as longitudinal gradient bends and reverse bending magnets [1]. It has a circumference of 288 meters, operates with a stored beam current of 400 mA, and an electron beam energy of 2.7 GeV. The upgrade results in a more than 40-fold reduction in horizontal emittance compared to the previous storage ring.

With the commissioning phase of SLS 2.0 now in an advanced stage, it is essential to verify that the machine performs as designed [2]. This requires accurate measurement of the linear optics, and comparison with the modeled values.

SLS 2.0 is equipped with 264 octupoles, each featuring auxiliary windings that can be activated to generate normal- and skew-quadrupolar fields. In the following, these windings are referred to as quadrupoles for simplicity. By varying their integrated strength (ΔK) and measuring the resulting tune shift ($\Delta Q_{x,y}$), the average local beta function at the location of a quadrupole can be determined using the expression given in [3]:

$$\beta_{x,y} = \pm\frac{2}{\Delta K}\left\{\cot(2\pi Q_{x,y})\left[1-\cos(2\pi\Delta Q_{x,y})\right] + \sin(2\pi\Delta Q_{x,y})\right\}. \quad (1)$$

The positive and negative signs correspond to the horizontal (x) and vertical (y) planes, respectively. This technique is commonly referred to as *quadrupole variation* (QV) or *K-modulation*.

Several alternative techniques exist for linear optics measurements; a comprehensive review can be found in [4]. Many of these methods rely on beam position monitor (BPM) data. Since SLS 2.0 is equipped with 115 BPMs, the QV method provides a significantly denser sampling of the lattice functions and therefore serves as a valuable complement to BPM-based measurements.

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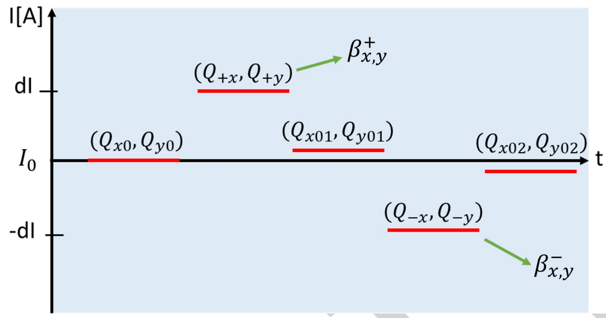

Figure 1: Measurement procedure. First, the initial tune is measured. For each quadrupole, the current is varied by $\pm dI$, and the tune is measured after each change. Using Eq. (1), the beta functions $\beta^{\pm}_{x,y}$ are obtained. After each variation, the current is restored to its initial value and hysteresis is corrected. The procedure is repeated for all quadrupoles.

The main challenges associated with this measurement are tune measurement resolution, hysteresis effects following quadrupole excitation, orbit perturbations induced by quadrupole feed-down, and accurate knowledge of the magnet transfer functions. In this contribution, the first three aspects are addressed.

BETA FUNCTION MEASUREMENT

Measurement Procedure

The basic layout of the measurement technique is sketched in Fig. 1.

Several comments are in order. First, to mitigate the effect of tune jitter, each tune measurement was repeated five times. Second, the excitation current was set to 1 A (corresponding to $\Delta K \approx 0.0072$ m^{-1}, depending on the specific magnet). This value was chosen to be sufficiently large to exceed the tune jitter level while remaining small enough to avoid significant optics perturbations. This choice was based on previous experimental runs in which both higher and lower excitation currents were tested. The selected current produced tune shifts ranging from 6.6×10^{-4} to 6.72×10^{-3} in the horizontal plane and from 6.5×10^{-4} to 3.70×10^{-3} in the vertical plane.

The quadrupole current is changed in both positive and negative directions to compensate for the optics perturbations induced by the quadrupole strength variation.

After returning the quadrupole current to its initial value, the original tune is not guaranteed to be recovered due to hysteresis effects. An additional current must be added as in [5]. The current is varied iteratively according to

$$I = I_0 - \frac{\Delta Q_0}{r}, \qquad r = \frac{\Delta Q}{dI},$$

where I_0 is the current at the previous step, ΔQ_0 is the difference between the initial tune and the current tune in the horizontal or vertical plane, ΔQ is the tune shift induced by the quadrupole excitation, and dI is the excitation current. The procedure is stopped when ΔQ_0 becomes smaller than a threshold fixed at 1×10^{-5}, or when a maximum of ten iterations is reached.

A waiting time of two seconds is introduced between successive current changes. This reduces the number of iterations required for hysteresis correction and, in some cases, makes the correction unnecessary. For faster measurements, the threshold on ΔQ_0 could be relaxed.

Hysteresis was compensated in only one plane at a time. Fig. 2 compares the tune change after each quadrupole measurement for the case in which hysteresis is corrected only in the horizontal plane (Run 1) with the case in which the correction plane is alternated between the vertical and horizontal planes (Run 2). In both cases, the vertical tune change increases along the measurement sequence; however, the increment is smaller in the latter case. Several factors may explain this tune shift. On the one hand, there is the cumulative effect of small tune variations below the hysteresis-correction threshold, as well as a few cases in which the correction scheme did not converge. On the other hand, a tune drift was observed even without applying any changes to the machine, as a consequence of machine warm-up. Over approximately ten minutes, a tune shift of about 1×10^{-4} was observed in both planes. Since measuring the beta functions in the entire SLS 2.0 ring takes about two hours, a tune drift is expected. Nevertheless, its impact on the measurement is negligible because the initial tune is updated before exciting the current in each quadrupole.

The fast orbit was active during the measurements to counteract orbit changes induced by the quadrupole current variations.

Tune Measurement

Since the resolution of the beta-function measurement is limited by the tune resolution, it is essential to improve it as much as possible. To this end, the Numerical Analysis of Fundamental Frequencies (NAFF) method [6,7] is employed. NAFF first uses a Fast Fourier Transform (FFT) to obtain an initial estimate of the dominant frequency components of the signal, and subsequently applies an interpolation algorithm to determine their values with high precision. Compared to a standard FFT, NAFF achieves higher frequency resolution and requires a smaller number of turns to extract the tune value, which is beneficial in presence of decoherence effects from chromaticity or amplitude detuning. These advantages are further enhanced when signals from all BPMs are combined, a technique referred to as the mixed BPM method [8].

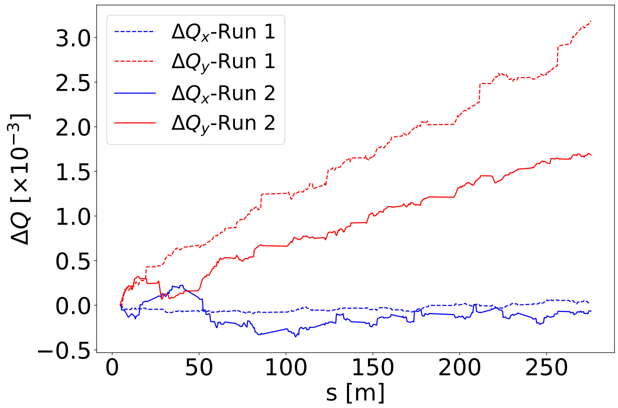

Figure 2: Tune change after each quadrupole measurement. Run 1 corresponds to hysteresis correction applied only in the horizontal plane. Run 2 corresponds to the case in which the correction plane is alternated between successive magnets.

Figure 3 shows the tune jitter. It can be seen that its contribution to the measurement error is small. In addition, the figure highlights the high tune resolution achieved with this method.

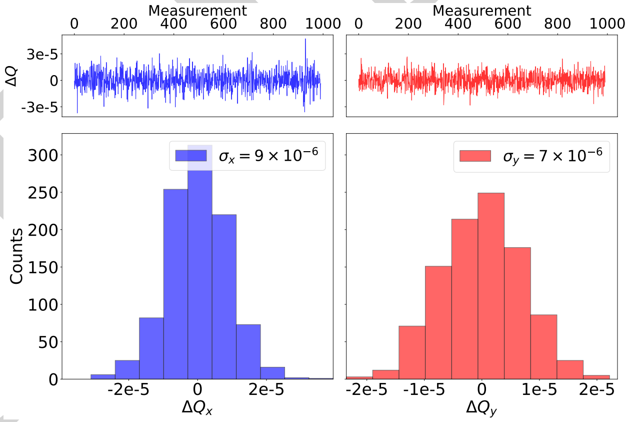

Figure 3: Tune jitter estimation. Top: 1000 consecutive tune measurements in both planes after subtraction of a slowly varying component originating from machine warm-up. Bottom: Corresponding histograms. The standard deviation is indicated in the legend.

Beta Function and Beta-Beat

Since the current of each quadrupole was varied in both positive and negative directions, two sets of tune shifts were obtained, and consequently two sets of beta functions: β_z^+ and β_z^-, with $z = x, y$, see Fig. 1. We define $\bar{\beta}_z$ as the average of the two values.

The averaged beta functions $\bar{\beta}_z$ are shown in Fig. 4 for the first two arcs of SLS 2.0. The error bars, corresponding to the statistical uncertainty due to tune noise, are too small to be visible. Systematic uncertainties are not taken into account.

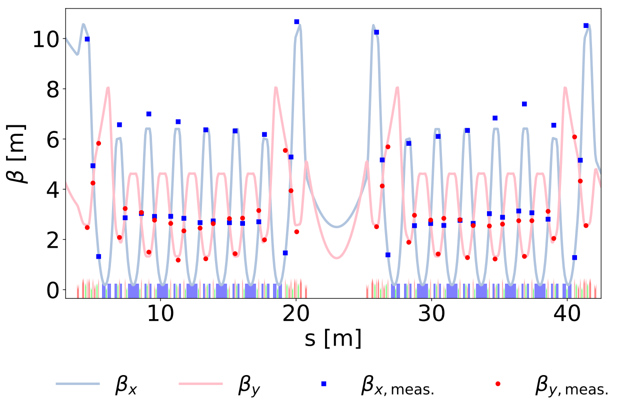

Figure 4: Comparison between the measured beta functions and the modeled values in two arcs of SLS 2.0.

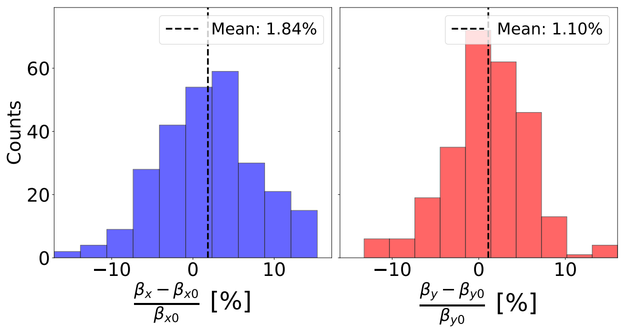

Figure 5: Beta-beat distribution at the quadrupoles of SLS 2.0.

The beta-beat is defined as

$$\frac{\Delta \beta_{z,i}}{\beta_{z_0,i}} = \frac{\beta_{z,i} - \beta_{z_0,i}}{\beta_{z_0,i}}$$

where $\beta_{z_0,i}$ denotes the model value at the position of the i-th quadrupole. It is customary to summarize the results of beta function measurements by providing the rms (root mean square) beta-beat. For the averaged beta function $\bar{\beta}_z$, the rms beta-beat is 6.32 % and 4.86 % in the horizontal and vertical plane, respectively.

Figure 5 shows the distribution of the measured beta-beat at the quadrupoles in the two first arcs of SLS 2.0. The mean beta-beat values are 1.84% and 1.10% in the horizontal and vertical planes, respectively. A similar offset was observed when the QV method was applied to the SLS [9]. This offset could indicate the presence of systematic errors. Nevertheless, as discussed in [10], a positive mean beta-beat is also expected from random errors, a result that was corroborated by simulations performed for SLS 2.0.

The QV method is robust, as consecutive measurements yield consistent results. This is illustrated in Fig. 6, which shows the beta-beat in the first arc for three consecutive measurements.

CONCLUSION

The beta functions in SLS 2.0 were determined using the quadrupole variation method, while simultaneously compensating for hysteresis effects and orbit distortions. High-resolution tune measurements, achieved with the mixed BPM method combined with NAFF, were essential for this task. The QV method proved to be reliable for beta-function measurements; however, it is time-consuming due to the large number of quadrupoles available, and systematic errors arising from uncertainties in the magnet transfer functions still need to be quantified. The next steps consist of comparing this method with the results provided by LOCO (Linear Optics from Closed Orbits) and subsequently using the measured beta functions to perform an optics correction.

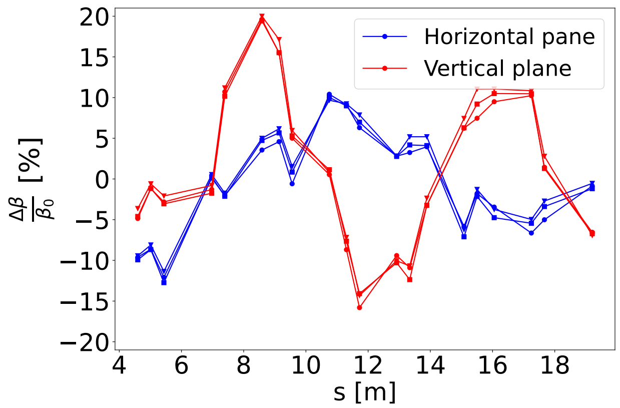

Figure 6: Beta-beat in the first arc of SLS 2.0 for three consecutive measurements.

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