IMPACT OF WIGGLER FIELD PROFILES ON KEY BEAM PARAMETERS IN THE SUPER TAU-CHARM FACILITY*

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Abstract

The Super Tau-Charm Facility (STCF) is a next-generation high-luminosity e+e- collider operating in the center-of-mass energy range of 2-7 GeV. To satisfy the requirements of damping time, emittance, and energy spread, damping wigglers are installed to enhance radiation damping. In practical wiggler, the longitudinal magnetic-field distribution often deviates from the ideal sinusoidal profile, especially for devices with large period lengths where the field near the peak becomes approximately flat. To describe such realistic field distributions, a trapezoidal field model with a rectangularity parameter k is proposed. Based on this model, analytical expressions for synchrotron radiation integrals and beam parameters are derived. The effects of the rectangularity parameter on emittance, energy spread, and damping time are analyzed, providing theoretical support for the parameter optimization of damping wigglers in the STCF collider rings.

INTRODUCTION

Damping wigglers are widely used in e+e- colliders to enhance synchrotron radiation damping and control beam parameters such as emittance and damping time [1–3]. In modern colliders, damping wigglers have been successfully applied to improve beam performance and collision luminosity, such as in DAΦNE and SuperKEKB [4, 5].

The Super Tau-Charm Facility (STCF) is a next-generation circular e+e- collider proposed in China. It operates at a center-of-mass energy range of 2–7 GeV and aims to achieve a luminosity exceeding $0.5 \times 10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at center-of-mass energy of 4 GeV [6, 7]. STCF imposes significantly stricter requirements on short damping time, suitable emittance, and large energy spread, which cannot be satisfied by the lattice itself. Therefore, 16 damping wigglers are installed in each ring to enhance radiation damping.

In conventional beam-dynamics analysis, the longitudinal magnetic field of a wiggler is usually approximated by an ideal sinusoidal distribution [8].

$$B_y = B_w \sin\left(\frac{2\pi}{\lambda_w} s\right). \quad (1)$$

This approximation is suitable for wigglers with relatively short period lengths, which are commonly used in synchrotron radiation light sources. However, the damping wigglers designed for STCF employ relatively large period lengths to achieve high peak magnetic fields. In this case, the longitudinal magnetic field near the peak region becomes noticeably flat, leading to a significant deviation from the ideal sinusoidal profile. As synchrotron radiation integrals strongly depend on the magnetic-field distribution, the sinusoidal approximation may introduce deviations in the evaluation of beam parameters.

To better describe realistic magnetic-field distributions, a trapezoidal field model with a rectangularity parameter k is proposed in this work. Based on this model, analytical expressions for synchrotron radiation integrals and equilibrium beam parameters are derived and applied to the parameter optimization of the STCF damping wigglers.

TRAPEZOIDAL FIELD MODEL

The trapezoidal field model is shown in Fig. 1.

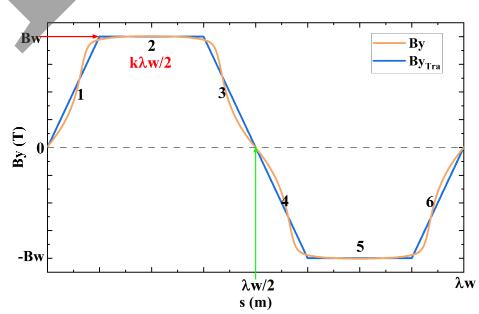

Figure 1: Trapezoid field profile.

The trapezoid height equals to B_w, the lower base corresponds to $\lambda_w/2$, and the upper base is k times the lower base. The model preserves the same peak field B_w and satisfies the same integral $\int_0^{\frac{\lambda_w}{2}} {B_y}^2\, ds$ as the actual magnetic field. The longitudinal magnetic field is parameterized by a rectangularity parameter k. It quantifies the degree of rectangularity of the magnetic field profile.

$$k = \frac{4}{\lambda B_w} \int_0^{\frac{\lambda_w}{2}} {B_y}^2\, ds - 1. \quad (2)$$

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In this model, the magnetic field can be expressed in a simplified form as a piecewise function over one wiggler period:

$$B_y(s) = \begin{cases} \frac{4B_w}{\lambda_w(1-k)}s & \left[0, \frac{\lambda_w(1-k)}{4}\right] \\ B_w, & \left[\frac{\lambda_w(1-k)}{4}, \frac{\lambda_w(1+k)}{4}\right] \\ -\frac{4B_w}{\lambda_w(1-k)}\left(s-\frac{\lambda_w}{2}\right) & \left[\frac{\lambda_w(1+k)}{4}, \frac{\lambda_w}{2}\right] \\ -\frac{4B_w}{\lambda_w(1-k)}\left(s-\frac{\lambda_w}{2}\right) & \left[\frac{\lambda_w}{2}, \frac{\lambda_w(3-k)}{4}\right] \\ -B_w, & \left[\frac{\lambda_w(3-k)}{4}, \frac{\lambda_w(3+k)}{4}\right] \\ \frac{4B_w}{\lambda_w(1-k)}(s-\lambda_w) & \left[\frac{\lambda_w(3+k)}{4}, \lambda_w\right] \end{cases}. \quad (3)$$

In synchrotron storage rings, damping times, energy spread, and horizontal emittance are governed by the integrals I_2, I_3, and I_5 [9]. These synchrotron radiation integrals are separated into contributions from dipoles (subscript 0) and damping wigglers (subscript w). By solving the dispersion equation $\eta_x'' + \frac{{B_y}^2}{(B\rho)^2}\eta_x = \frac{B_y}{B\rho}$, the corresponding dispersion η_x and dispersion derivatives η_x' are obtained and substituted into the H_x.

Given that β_{xw} varies slowly within the damping wiggler section, α_{xw} can be neglected and β_{xw} is approximated by its average value $\langle\beta_{xw}\rangle$. For a typical damping wiggler, λ_w is on the order of centimeters to several hundred centimeters, while $\langle\beta_{xw}\rangle$ is on the order of meters.Under this approximation, the dispersion invariant H_{xw} can be approximated as in (4).

$$H_{xw} \approx \langle\beta_{xw}\rangle({\eta_{xw}}')^2. \quad (4)$$

Integrating the piecewise expression of the trapezoidal magnetic field profile given in Eq. (3), the analytical expressions of the synchrotron radiation integrals I_{2w}, I_{3w}, and I_{5w} are derived.

$$I_{2w} = \frac{(1+2k){B_w}^2 L_w}{3(B\rho)^2}. \quad (5)$$

$$I_{3w} = \frac{(1+3k){B_w}^3 L_w}{4(B\rho)^3}. \quad (6)$$

$$I_{5w} = \frac{\langle\beta_{xw}\rangle{\lambda_w}^2{B_w}^5 L_w}{(B\rho)^5} \cdot \frac{(15k^3+11k^2+5k+1)}{1536}. \quad (7)$$

Then, the analytical expressions for the damping time, energy spread, and horizontal emittance are then derived.

$$\tau_u = \frac{\pi T}{C_r E^3} \cdot \frac{12(B\rho)^2}{3(B\rho)^2 I_{20} + {B_w}^2 L_w(1+2k)}. \quad (8)$$

$${\sigma_E}^2 = \frac{C_q\gamma^2}{J_s} \cdot \frac{12(B\rho)^3 I_{30} + 3{B_w}^3 L_w(1+3k)}{12(B\rho)^3 I_{20} + 4{B_w}^2 L_w(B\rho)(1+2k)}. \quad (9)$$

$$\begin{aligned} \varepsilon_x = \frac{C_q\gamma^2}{J_x} \cdot \Big[&1536(B\rho)^5 I_{50} \\ &+ \langle\beta_{xw}\rangle\lambda_w^2 B_w^5 L_w(15k^3+11k^2+5k+1)\Big] \\ &\Big/\Big[1536(B\rho)^5 I_{20} + 512(B\rho)^3(1+2k)B_w^2 L_w\Big]. \end{aligned} \quad (10)$$

APPLICATION TO STCF

The STCF collider rings employ 16 room-temperature damping wigglers in each ring. The damping wiggler adopts an even number (2N) of poles. The field arrangement follows the sequence +1/4, -3/4, +1, -1, … +1, -1, +3/4, -1/4 [10–12]. The main parameters of the damping wigglers used in this study are listed in Table 1.

Table 1: Main Parameters of the STCF Damping Wigglers

Parameter	**Value**
Peak field	1.6 T
Period length	0.8 m
wiggler length	4.8 m
Number of wigglers	16
Total wiggler length	76.8 m

The lattice of the damping wigglers straight section consists of two matching sections and eight triplets, with the optical functions. Each triplet provides a long drift for installing the damping wiggler, and two matching sections connect the straight section to the main ring [10]. Figure. 2 shows the actual field distribution obtained from Opera simulations, along with the trapezoidal and sinusoidal field approximations. When the actual magnetic field is approximated by either a trapezoidal profile with the same peak field strength, k is calculated as 0.5 from Eq. (2).

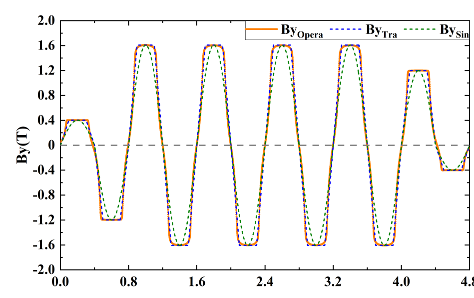
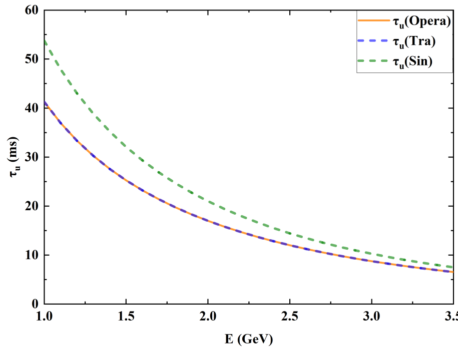
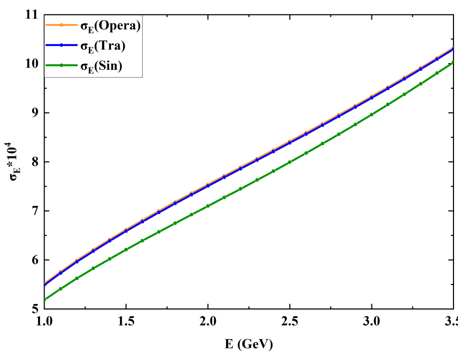


Figure 2: The trapezoidal and sinusoidal field approximations.

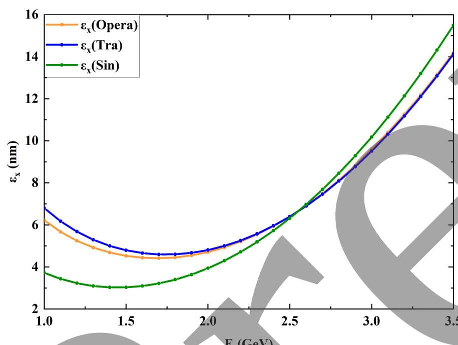
To evaluate the impact of the damping wigglers on the beam parameters of the STCF collider rings at 1-3.5 GeV, the dipole contributions are given as $I_{20} = 7.07 \times 10^{-1}$ m^{-1}, $I_{30} = 7.90 \times 10^{-2}$ m^{-2}, and $I_{50} = 1.06 \times 10^{-3}$ m^{-1}. The actual wiggler magnetic field obtained from Opera is modeled using 1 mm dipole slices in MAD-X, and the corresponding beam parameters are then calculated. The trapezoidal and



(a) Damping time.



(b) Energy spread.



(c) Horizontal emittance.

Figure 3: Beam parameters of the STCF collider rings.

sinusoidal field formulas are used to calculate damping time, energy spread, and horizontal emittance.

In Fig. 3a, 3b and 3c, the trapezoidal approximation shows better agreement with the sliced actual field results for damping time, horizontal emittance, and energy spread, with smaller errors than the sinusoidal approximation.

For the STCF collider rings, the horizontal emittance is designed to 5 nm at 2 GeV, and should remain as close as possible at other beam energies. The damping wigglers are turned off above 2.7 GeV. As shown in Fig. 4, the dependence of horizontal emittance on k is nonuniform and energy dependent. For each k, a curve is obtained showing the variation of horizontal emittance with beam energy. The standard deviation σ_{5nm} is evaluated using Eq. (11).

$$\sigma_{5nm} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_{x_i} - 5\right)^2}. \tag{11}$$

As shown in Fig. 5, σ_{5nm} is minimized at k=0.48. At this value, the horizontal emittance is reduced from 2.2-16.0 nm to 4.3-7.4 nm over the 1–2.7 GeV range. This represents the best overall agreement with 5 nm, as indicated by the red curve in Fig. 5. In practical wiggler design, reducing the pole-head length can optimize k from 0.5 to 0.48.

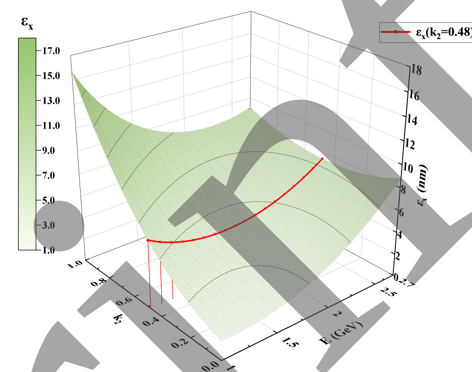


Figure 4: Changes in horizontal emittance with beam energy and k.

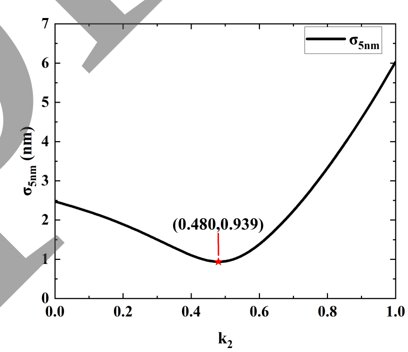


Figure 5: The relationship between σ_{5nm} and k.

SUMMARY AND DISCUSSION

A trapezoidal magnetic-field model with a rectangularity parameter k was developed to describe the longitudinal field distribution of large-period damping wigglers. Based on this model, analytical expressions for synchrotron radiation integrals, damping time, energy spread, and horizontal emittance were derived.

The trapezoidal model was applied to the STCF collider rings and compared with results calculated by sinusoidal and actual field. The results show that the trapezoidal approximation reproduces the synchrotron radiation integrals and beam parameters with significantly better accuracy than the conventional sinusoidal model.

The dependence of beam parameters on the rectangularity parameter was further investigated. An optimal value of $k = 0.48$ was identified, which provides the closest agreement with the target horizontal emittance over the 1–2.7 GeV energy range. This study provides theoretical support for the parameter optimization and engineering design of STCF damping wigglers.

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