

# SIMULATION OF LIOUVILLIAN MULTI-TURN INJECTION FOR DIRECT PROTON ACCUMULATION

E. Salehi\*, N. Milas, M. Eshraqi, European Spallation Source (ESS), Lund, Sweden  
M. Ovegård, Uppsala university, Uppsala, Sweden

## Abstract

In the ESS neutrino super beam (ESSnuSB) project, an efficient injection into the accumulator ring is required for minimizing beam loss during injection, reducing capital cost, and simplifying the overall project. To meet these requirements, the Liouvillian Injection Optimization (LIO) method has been developed for direct proton injection into the accumulator using a 4D multi-turn accumulation process without stripping. This method enables to paint both the horizontal and vertical phase-space simultaneously using a tilted septum. In this study, the LIO method has been investigated using the PyOrbit simulation code. The results demonstrate the method's dependency to the lattice tune, the injected beam beta function, the close-orbit route, and space-charge effects, which play a dominant role on the efficiency of the injection process. This work presents the optimization of the method considering space charge effects, lattice nonlinearities, and considerations on its capabilities.

## INTRODUCTION

The European Spallation Source neutrino Super Beam (ESSνSB) project has been designed to use the high-power proton beam at the European Spallation Source facility near Lund, Sweden, in order to produce a high-intense neutrino beam for precision measurements of charge-parity (CP) violation in the leptonic sector [1]. The current ESS linac delivers a 5 MW proton beam at 2 GeV. In a future upgrade of ESS, the facility would simultaneously operate for both neutron and neutrino production without interfering with the main neutron program of ESS. Achieving this goal would require a doubling of the linac power, along with increasing the pulse repetition rate from 14 Hz to 28 Hz, corresponding to 10% duty cycle. Due to the operational constraint of the magnetic horn in the neutrino target station, the proton bunch length must be compressed from 2.86 ms to 1.2 μs. This can be achieved using a proton accumulator ring.

Conventional accumulator ring operates on charge-exchange injection, which requires the acceleration of H<sup>-</sup> in a linac and stripping its electron at injection. In the ESS case, this approach involves integrating both proton and H<sup>-</sup> sources in the front end. However, using H<sup>-</sup> ions introduce several challenges, including limited H<sup>-</sup> source reliability and lifetime, losses due to H<sup>-</sup> stripping in the linac and transfer line to the ring, and a substantial increase in the complexity of the control and safety system. Recently, a promising approach for achieving a cleaner H<sup>-</sup> injection and reducing beam losses, called laser stripping injection, has been under study [2].

In parallel, in order to avoid the added complexity of the dual source front-end and the challenges associated with H<sup>-</sup> stripping in the linac, a Liouvillian Injection Optimization method has been studied in this paper [3]. This method enables direct proton injection into the accumulator ring through a 4D multiturn accumulation process without H<sup>-</sup> stripping. This method eases an efficient phase-space painting in both horizontal and vertical planes using a tilted septum, which simplifies the injector design and reduces operational complexity.

## LIO METHOD

The developed Liouvillian Injection Optimization (LIO) code is based on a formalism similar to that of MISHIF [4] to minimize the ring acceptance to achieve 100% accumulation efficiency (no beam loss) within the smallest possible emittance. To achieve this goal, a tilted septum is used at an angle  $\theta$  with respect to the horizontal plane at the injection point. This configuration allows both vertical and horizontal planes to be explored simultaneously, enabling more efficient phase space painting and increasing the number of injected particles at a given turn. At the beginning of the injection process, the local closed-orbit is shifted close to the septum using two groups of bump magnets (horizontal and vertical), enabling the beam to be injected near the center of phase space. As the injection continues, the local closed-orbit gradually shifts back to the center position of orbit, thereby distributing the injected particles toward the outer region of the ring acceptance in both planes. By controlling the closed-orbit shift, circulating particles are effectively prevented from hitting the septum.

In order to achieve optimal phase space painting, some parameters must be properly chosen, including the machine tunes ( $\mu_x$  and  $\mu_y$ ), the initial injected beam parameters, the initial closed-orbit shift, and the ring optics at the injection point. The painting process can be described by the following equations, which link the injected beam parameters to those of the accumulated beam [3,4]:

$$\frac{\beta_{r,x}}{\alpha_{r,x}} = \frac{\beta_{i,x}}{\alpha_{i,x}} = \frac{x_i - x_{co}(n)}{x'_i - x'_{co}(n)} \quad (1)$$

$$\frac{\beta_r}{\beta_i} = \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^{1/3} \quad (2)$$

where the subscripts i and r refer to the injected and accumulated beams, respectively. Similar expressions apply to the vertical plane. The parameters  $\alpha$  and  $\beta$  are the Courant-Snyder (C-S) parameters,  $\varepsilon$  denotes the emittance,  $(x_i, x'_i)$  represent the position and angle of the injected beam in the

\* elham.salehi@ess.eu

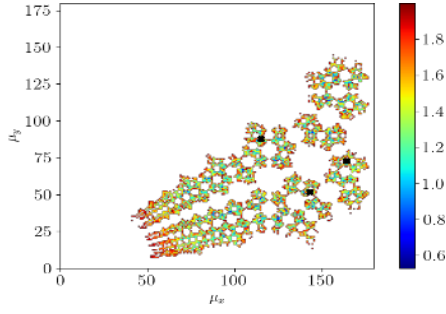


Figure 1: 3D tune diagram of  $\mu_x$  and  $\mu_y$  as a function of KSE parameter for K between 2 and 42 in steps of 0.1, including the effect of the  $1^o$  tune spread.

closed orbit coordinate system, and  $(x_{co}, x'_{co})$  denote the closed-orbit position and angle at turn n.

To accumulate the  $\varepsilon_{100}$  emittance of the injected beam in the ESS  $\nu$ SB accumulator ring over 465 turns without losses, a few assumptions are made. The ring beta functions at the injection point are kept fixed for practical operation, while the beta functions of the injected beam are allowed to vary in order to satisfy Eq. 2. For simplicity during the injection process, C-S  $\alpha$  parameters and beam angles are set to zero at the injection point for both the ring and the injected beam. In addition, the transfer line is optimized to produce equal unnormalized emittance of  $\varepsilon_{100} = 2.8$  mm mrad in both transverse planes (a round beam), which leads to a septum tilt angle of  $45^\circ$  [3]. In addition the combination of injected beam and ring parameters must be optimized to minimize beam loss. The minimum condition for loss-free injection depends on the distance between the beam-center and the septum ( $DS_{bc}$ ) which must be smaller than injected beam size ( $\sqrt{2}dx_i$ ), this can be written as [5]:

$$\cos(n\mu_x) + \cos(n\mu_y) < \frac{2K(n) - 1}{K(n) + 1} \quad (3)$$

which links the horizontal and vertical betatron tunes  $(\mu_x, \mu_y)$  to  $K(n) = dx_{CO}/dx_i$ , where  $dx_{CO} = 1 - x_{CO}$ , and  $dx_i = x_i - 1$ . Eq. 3 allows the calculation of the Number of Turns Before Loss (NTBL) with no closed-orbit shift,  $K(n)=K(0)$ .

Maximizing the NTBL reduces the amplitude of closed-orbit shift, which minimize the stored emittance. Therefore the NTBL is a key parameter to optimize injection parameters. A tune diagram based on Eq. 3 is shown in Fig. 1. It evaluates the figure of merit

$$KSE = \frac{1}{K(N)} \sum_{i=K(0)}^{K(N)} \frac{DS_{sb}(i)}{NTBL(i)}$$

at each working point. This criterion is used to find the optimal working points that provide a reasonable value of  $K(0)$ , maximize NTBL, and avoid resonance regions where injection efficiency is reduced.

After identifying the optimal working points and initial parameter  $K(0)$ , the close-orbit route during injection process must be optimized to minimize the stored emittance

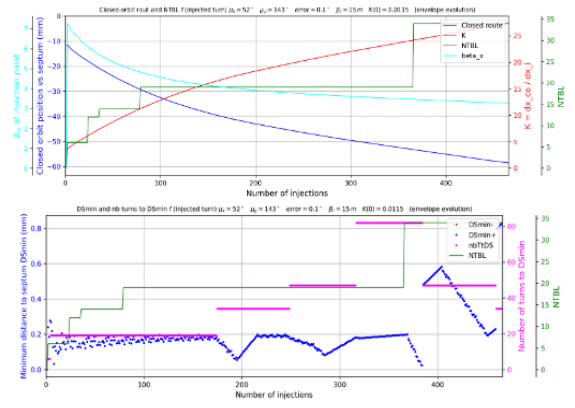


Figure 2: (Top) Optimized closed-orbit route(blue), evolution of the injected beta function (cyan), and NTBL (green) over 465 injections. (Bottom) Minimum distance to the septum(blue) and Number of lost particles(pink).

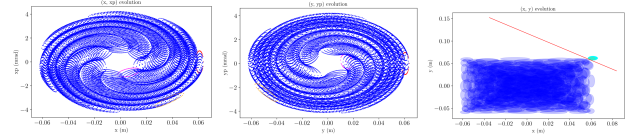


Figure 3: Beam distribution at the horizontal phase space(left), vertical phase space (middle), and transverse beam cross section (right) at the end of injection

while avoiding particle losses. This is achieved through a step-by-step correction of the closed-orbit shift for groups of injected turns [3]. The effect of a  $1^o$  tune spread is also included in the calculations.

The optimal parameter set for ESS  $\nu$ SB is found to be  $K(0) = 3.6$ ,  $\mu_x = 8.144$ , and  $\mu_y = 8.397$ . The results of a well optimized closed-orbit route up to turn 465 are presented in Figs. 2 and 3. This optimization yields a final ring emittance ratio of  $\varepsilon_r/\varepsilon_i = 96.6$ . Figure 2 shows the evolution of optimized closed-orbit route, injected beta function, NTBL, minimum distance to the septum, and number of lost particles during injection process. The final particle distributions at the septum position at the end of injection is shown in Fig. 2 for (i) horizontal phase space, (ii) vertical phase space, and (iii) transverse real space, respectively from left to right. The transverse phase space distributions indicate a hollow structure, which arises because the initial closed-orbit shifted from the center of orbit and is adjusted to maximize NTBL.

## SPACE CHARGE EFFECTS

To evaluate space charge effects on the LIO method, a multi-particle tracking simulation including collective effects is performed using PyOrbit. Space charge effects introduce nonlinear effects that can significantly impact beam dynamic during multi-turn injection and accumulation, which reducing beam stability and increasing particle losses.

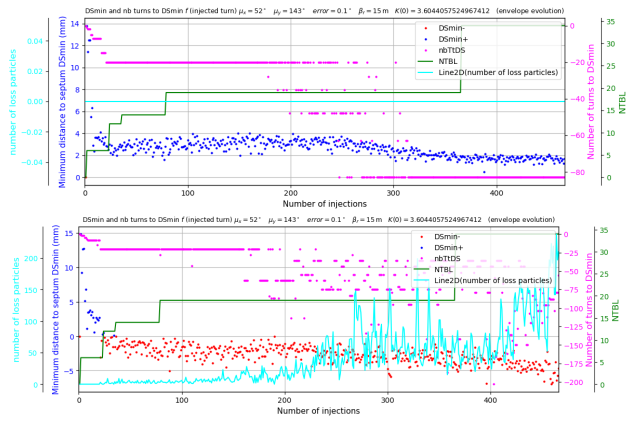


Figure 4: Result of the injection procedure with a tracking simulation code, PyOrbit (top) no space charge effect and (bottom) with space charge effect. Number of particles at each injection is 2000.

In PyOrbit tracking code, space charge effects for a Gaussian distribution are computed using a Particle-In-Cell (PIC) method with 3D solver. The numerical setup includes  $1 \times 10^9$  macro-particles, a transverse grid of  $32 \times 32$  cells, and 32-64 longitudinal slices with open boundary conditions. These parameters ensure sufficient accuracy with reasonable computational cost, as confirmed by convergence tests.

The initial beam parameters, injection painting scheme, working point, and closed-orbit route follow the optimized conditions described in the previous section. The simulations are performed for a 2.5 GeV proton beam with an intensity of approximately  $4.4 \times 10^{11}$  protons per pulse and 0.15 MeV energy spread.

In the absence of space charge, the LIO method provides an almost ideal filling of the phase space with negligible losses as shown in Fig. 4(top). When space charge effects are included, an incoherent tune shift and spread grow during the injection process, causing the tune footprint approaches resonance line and leading to reduced beam stability. Consequently, particle losses increase with the number of injected turns, as presented in Fig. 4(bottom). For the optimal working point of ESSnuSB, the total loss after injection procedure is approximately 3.7% based on simulation using  $2000 \times 465$  macro particles. The final geometrical ring emittance is  $\varepsilon_{r,x,y} = 270$  mm mrad.

In addition, nonlinear space charge forces distort the phase-space distribution, resulting in deviations from the ideal linear behavior. This effect is shown in Fig. 5, which shows the particle distributions in horizontal and vertical phase space without space charge (a-c) and with space charge (d-f). As illustrated in Fig. 5 (c, d), the phase space painting process becomes less uniform in the presence of space charge and fills out all region of transverse phase space, which increases the probability of particle losses at the septum.

These results indicate that, with appropriate choice of working point and injection parameters, the LIO method can operate effectively even in the presence of strong space charge, achieving an accumulation efficiency of approxi-

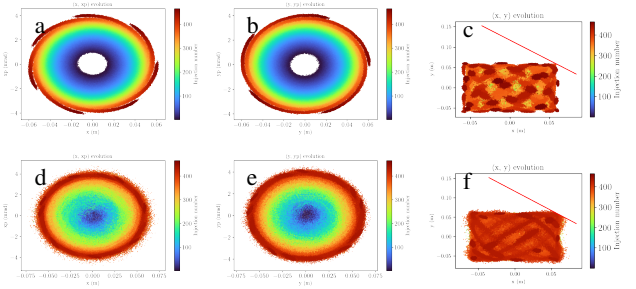


Figure 5: Phase space distribution of surviving particles at the septum position at the end of injection. The top row (a-c) shows the horizontal, vertical phase spaces, and transverse cross section of the surviving particle at the septum position at the end of injection without space charge, while the bottom row (d-f) shows the corresponding distribution with space charge.

mately 95.6% for the ESS $\nu$ SB design parameters. By reducing the number of injection turns to 233, losses can be decreased and the accumulation efficiency improve to 98.7%. However, further improvement in performance of LIO under space charge effects requires re-optimized of the closed-orbit route to enable 100% emittance injection over 465 injection turns with no loss.

## CONCLUSION

Numerical simulations of the Liouvilian Injection Optimization (LIO) method have been performed for ESS $\nu$ SB, with the aim of enabling direct proton injection into the accumulator ring. This method could simplify the overall operation of the ESS linac. First, a linear study was carried out to determine the optimal injection parameters, including the initial closed orbit ( $K(0)$ ), the working point ( $\mu_x, \mu_y$ ), and the closed-orbit route to inject and store the 100% emittance over 465 injection turns without beam loss. Next, the impact of space charge effects during the injection process was studied using the PyOrbit tracking code. Nonlinear space charge induces a shift in the tune parameters and also changes in the particle distributions, which increase beam losses significantly.

The results show that beam losses can reach up to 3.7% when space charge effects are included. By reducing the number of injection turns to 233, losses can be decreased to approximately 1.3% with ring emittance of 172 mm mrad. Future effort will focus on improving the closed-orbit route to achieve full injection with no losses when considering space charge effects.

## ACKNOWLEDGEMENTS

This work has been funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union. Neither the European Union nor the granting authority can be held responsible for them.

## REFERENCES

- [1] A. Alekou et al., “The European Spallation Source neutrino Super Beam conceptual design report”, *Eur. Phys. J. Spec. Top.*, vol. 231, p. 3779, Nov. 2022.  
[doi:10.1140/epjs/s11734-022-00664-w](https://doi.org/10.1140/epjs/s11734-022-00664-w)
- [2] A. Opanasenko, N. Milas, and M. Olvegaard, “Evaluation of laser-assisted charge-exchange injection into the ESSnuSB+ proton accumulator”, presented at the 17th Int. Conf. Part. Accel. (IPAC'26), Deauville, France, paper THP4084, this conference.
- [3] J. M. Lagniel, “ESS $\nu$ SB Liouvillian Injection Optimization”, ESS, Lund, Sweden, Internal Report, Aug. 2023.
- [4] C. R. Prior, “MISHIF: A code to optimize multiturn injection, A guide for users”, STFC-RAL, Chilton, England, Internal Report, Aug. 2020.
- [5] J.-M. Lagniel, M.E. Eshraqi, and N. Milas, “On Liouvillian high power beam accumulation”, in *Proc. 68th Adv. Beam Dyn. Workshop High-Intensity High-Brightness Hadron Beams*, Geneva, Switzerland, Oct. 2023, pp. 511–514.  
[doi:10.18429/JACoW-HB2023-THBP22](https://doi.org/10.18429/JACoW-HB2023-THBP22)