GPU-ACCELERATED SIMULATION FRAMEWORK FOR PARTIALLY COHERENT EUV LIGHT TRANSPORT IN TSINGHUA SSMB BEAMLINE OPTICS*

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Abstract

Accurate simulation of partially coherent beam transport and coherence evolution is critical for next-generation accelerator-based light sources. In this work, we develop a dedicated numerical approach for partially coherent EUV beamline propagation to support the optical design and optimization of Steady-State Microbunching (SSMB) beamlines under development at Tsinghua University.* A GPU-accelerated mutual optical intensity framework is developed for two-dimensional propagation of partially coherent radiation. Using spatially partitioned diffraction-integral kernels with locally reconstructed paraxial approximation, the method achieves substantial speedup over conventional mutual-intensity and wavefront-based propagation methods while maintaining high accuracy. Complex optical components, including arbitrary curved and rough reflective surfaces, are fully supported for efficient simulation of realistic EUV systems. In summary, the proposed framework provides a practical tool for optical design and end-to-end simulation of EUV beamlines, enabling coherence analysis and accurate light propagation modeling of partially coherent radiation.

INTRODUCTION

With the development of advanced lithography technologies and high-brightness accelerator-based light sources, accurate simulation of EUV radiation propagation and coherence evolution has become increasingly important [1,2]. Tsinghua University is currently developing a Steady-State Microbunching (SSMB) accelerator light source aiming to generate high-brightness narrow-bandwidth EUV radiation in a storage ring [3].

The Mutual Optical Intensity (MOI) formalism provides a rigorous theoretical framework for describing the propagation of partially coherent radiation [4,5]. However, its numerical implementation involves four-dimensional propagation calculations, resulting in high computational complexity and limiting its practical application in complex optical systems.

To address this challenge, we develop a GPU-accelerated numerical framework for partially coherent radiation propagation optimized for SSMB EUV beamline optics. The proposed method preserves full two-dimensional propagation coupling and supports efficient simulation of complex curved optical components and rough surfaces.

MUTUAL OPTICAL INTENSITY FORMALISM

In the analysis of partially coherent radiation, the MOI formalism provides a rigorous theoretical framework for describing statistical coherence properties [4,5]. Unlike scalar field descriptions that characterize instantaneous field distributions, MOI describes the second-order spatial correlation of the optical field through statistical averaging, making it suitable for modeling partially coherent radiation.

For monochromatic partially coherent radiation in free space, the optical field can be described by the mutual optical intensity function [4]

$$J(\mathbf{r}_1, \mathbf{r}_2) = \langle E(\mathbf{r}_1) E^*(\mathbf{r}_2) \rangle$$

where $\langle \cdot \rangle$ denotes statistical averaging.

Assuming that the radiation propagates from a source surface Σ_0 to an observation surface Σ_1, the propagation of the mutual optical intensity can be written as

$$J_1(\mathbf{P}_1, \mathbf{P}_2) = \iint_{\Sigma_0} \iint_{\Sigma_0} J_0(\mathbf{Q}_1, \mathbf{Q}_2)\, h(\mathbf{P}_1, \mathbf{Q}_1)\, h^*(\mathbf{P}_2, \mathbf{Q}_2)\, dS_{\mathbf{Q}_1} dS_{\mathbf{Q}_2}.$$

Here $h(\mathbf{P}, \mathbf{Q})$ denotes the scalar diffraction propagation kernel. To support propagation between arbitrary curved surfaces, a general formulation based on the free-space Green's function is adopted:

$$h(\mathbf{P}, \mathbf{Q}) = \frac{1}{i\lambda} \frac{e^{ik\rho}}{\rho} \cos(\hat{\mathbf{n}}(\mathbf{Q}), \mathbf{P} - \mathbf{Q}), \qquad \rho = |\mathbf{P} - \mathbf{Q}|.$$

This expression is applicable to propagation between arbitrary surfaces and provides a unified theoretical basis for partially coherent propagation simulations involving complex optical components such as curved mirrors and rough surfaces.

For two-dimensional transverse coordinates, the mutual optical intensity depends on four spatial variables (x_1, y_1, x_2, y_2). If each transverse direction is discretized using N sampling points, the direct numerical evaluation leads to a computational complexity of $\mathcal{O}(N^4)$, resulting in substantial computational and memory requirements in practical beamline simulations. Therefore, efficient numerical methods are required to enable fast simulation of partially coherent radiation propagation in complex EUV optical systems.

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GPU-ACCELERATED PROPAGATION METHOD

To reduce the computational complexity of MOI propagation, an efficient numerical method is developed based on the mutual optical intensity propagation formulation introduced above. In this approach, the source surface Σ_0 and the observation surface Σ_1 are discretized into surface elements, and the propagation operator is evaluated through the contributions between surface elements. For two observation points $\mathbf{P}_i$ and $\mathbf{P}_j$, the discrete form of the mutual optical intensity propagation can be written as

$$J_1(\mathbf{P}_i, \mathbf{P}_j) \approx \sum_{m,n} H_{im} H^*_{jn} J_0(\mathbf{Q}_m, \mathbf{Q}_n),$$

where $\mathbf{Q}_m$ and $\mathbf{Q}_n$ denote discrete surface elements on the source surface, and H_{im} represents the propagation contribution from source element m to observation point i. The propagation kernel can be written as

$$H_{im} = \iint_{\Delta S_m} \exp\left[i\frac{2\pi}{\lambda}\left(r_{im} - \alpha_i \cdot \mathbf{u}\right)\right] \frac{\chi(\gamma_{im})}{\sqrt{\lambda r_{im}}}\, dS,$$

where $r_{im} = |\mathbf{P}_i - \mathbf{Q}_m|$ is the geometric distance between the source element and the observation point, $\mathbf{u}$ denotes the local coordinate within the source element, and α_i represents the phase gradient introduced by local surface tilt. The function $\chi(\gamma_{im})$ represents a geometric weighting factor including both the obliquity factor and polarization correction, where γ_{im} is determined by the local propagation geometry. For rectangular elements, the above integral can be evaluated analytically, avoiding additional discretization inside each element.

To reduce the computational cost, a local paraxial approximation is introduced for each pair of surface elements. In this approximation, the two surfaces are locally treated as tilted planes, allowing analytical evaluation of the propagation kernel.

In conventional scalar diffraction theory, the scalar approximation typically requires a small-angle condition. However, for radiation generated by the SSMB EUV source, the polarization state is usually close to linear polarization. By incorporating a polarization correction factor in the propagation kernel, the dependence of the scalar approximation on strict small-angle conditions can be relaxed while maintaining propagation accuracy.

Widely used beamline simulation tools such as Synchrotron Radiation Workshop (SRW) propagate coherent wavefronts directly [6]. In contrast, the MOI formalism describes the statistical properties of partially coherent radiation through second-order field correlations.

Unlike methods that rely on complex analytical approximations of the propagation distance, the proposed approach directly evaluates the geometric distance between surface elements and computes the propagation kernel under the local tilted-plane paraxial approximation. Since the propagation between different surface-element pairs is independent, the computation can be efficiently parallelized on GPU architectures.

Since the propagation calculation is based on the superposition of contributions between surface elements, the method naturally supports arbitrary curved optical components, which is important for realistic EUV beamline optics. In addition, the full two-dimensional coupling between transverse coordinates is preserved during propagation. Unlike commonly used separable one-dimensional propagation methods [7], the present framework can be directly applied to optical systems with intrinsic two-dimensional coupling and can reproduce the resulting two-dimensional interference effects.

It should be noted that, in fully coupled two-dimensional propagation, the mutual optical intensity depends on four spatial variables, leading to data structures of size $O(N^4)$. For large spatial sampling grids, the primary limitation of the method often arises from memory requirements rather than computational cost. Therefore, high-resolution simulations of large EUV optical systems may still require GPUs with large memory capacity or high-performance computing resources.

SIMULATION OF SSMB EUV BEAMLINE OPTICS

To verify the numerical correctness and physical consistency of the proposed partially coherent propagation framework, several representative test cases with known analytical results or well-understood physical characteristics are studied [4]. These include partially coherent double-slit interference, two-dimensional speckle pattern formation, and the point spread function (PSF) near the focus of a curved optical system. These examples are used to validate the capability of the method in modeling partially coherent interference, two-dimensional propagation coupling, and focusing behavior in curved optical systems.

Partially Coherent Double-Slit Interference

We first consider the classical double-slit interference system to verify the capability of the proposed method in modeling partially coherent interference phenomena [4].

In the simulation, partially coherent EUV radiation passes through the double-slit structure and propagates to the observation plane. The intensity distribution is obtained from the diagonal elements of the propagated mutual optical intensity matrix. Figure 1 shows the simulated interference patterns under two different spatial coherence lengths and compares them with theoretical expectations.

When the coherence length is set to 1 mm, the radiation behaves nearly as fully coherent light, producing high-contrast interference fringes. When the coherence length is reduced to 10 µm, the fringe visibility decreases significantly, exhibiting typical characteristics of partially coherent interference.

It should be noted that in the present numerical model, each discretized surface element is treated as a fully coherent radiation unit. Therefore, the completely incoherent limit can only be approached by reducing the element size. As the discretization scale becomes finer, the model gradually approaches the limit of fully incoherent propagation.

The simulated intensity distributions agree well with the expected interference structures, demonstrating that the proposed propagation method can correctly reproduce partially coherent interference and the variation of fringe visibility determined by spatial coherence.

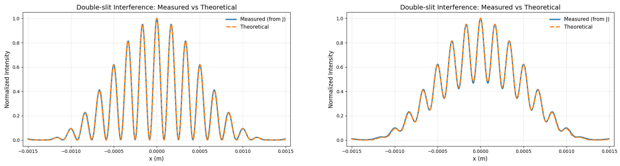

Figure 1: Double-slit interference under different spatial coherence lengths. Left: coherence length = 1 mm (quasi-coherent case). Right: coherence length = 10 µm (partially coherent case).

Two-Dimensional Speckle Simulation

The second example demonstrates the capability of the proposed method to simulate two-dimensional propagation coupling. In this case, a coherent EUV beam is reflected from a surface with random roughness and then propagates to the observation plane. The random phase distribution introduced by the rough surface leads to the formation of a characteristic speckle pattern [8].

Figure 2 shows the simulated two-dimensional intensity distribution obtained from the diagonal elements of the propagated mutual optical intensity matrix.

Since the propagation calculation preserves the full two-dimensional coupling between transverse coordinates, the proposed method can naturally reproduce such two-dimensional interference patterns. In addition, the surface-element-based propagation formulation allows direct simulation of arbitrary curved and rough optical surfaces.

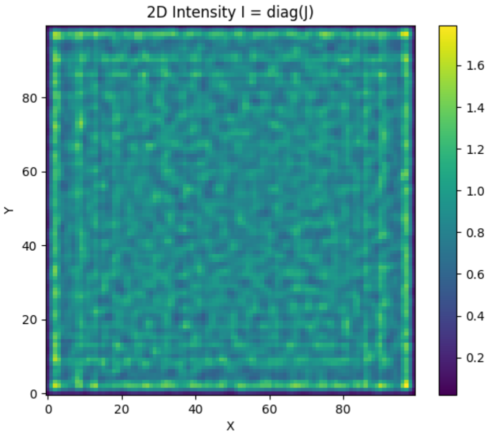

Figure 2: Simulated speckle pattern generated by coherent EUV radiation reflected from a rough surface.

PSF Verification of Curved Surface Propagation

To further verify the numerical accuracy of the proposed propagation algorithm for curved optical systems, the point spread function (PSF) near the focal region of a curved mirror is calculated.

Partially coherent EUV radiation is reflected by a curved mirror with a focal length of 0.1 m.

Figure 3 shows the simulated intensity distributions at propagation distances of 0.08 m, 0.10 m, and 0.12 m. The position 0.10 m corresponds to the theoretical focal point, while 0.08 m and 0.12 m correspond to defocused distributions before and after the focus.

The results show a clear focusing structure at the focal position, while the distributions away from the focus exhibit the expected defocused spreading patterns [9]. The results verify the numerical correctness of the curved-surface propagation algorithm.

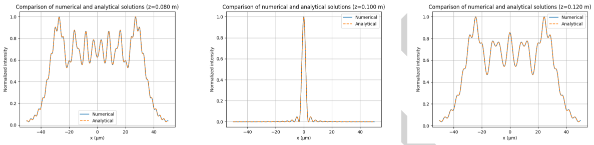

Figure 3: Intensity distributions near the focus of a curved mirror with focal length 0.1 m. From left to right: $z = 0.08$ m, $z = 0.10$ m (focus), and $z = 0.12$ m.

CONCLUSION

In this work, a GPU-accelerated numerical framework based on the mutual optical intensity formalism has been developed for simulating the propagation of partially coherent EUV radiation. The method discretizes optical surfaces into surface elements and efficiently evaluates the MOI propagation operator under a local propagation approximation, enabling fast simulation of partially coherent radiation in complex optical systems. Since the propagation contributions between different surface-element pairs are mutually independent, the algorithm can fully exploit the massive parallelism of GPU architectures, significantly improving computational efficiency.

The method is validated using three representative examples including partially coherent double-slit interference, speckle formation from rough surfaces, and PSF evaluation near the focus of a curved mirror.

The framework supports arbitrary curved optical surfaces and provides a practical numerical tool for the optical design of the SSMB EUV beamline under development at Tsinghua University.

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