DESIGN AND THEORETICAL ANALYSIS OF A HIGHLY COMPACT TRIPLE-FOLDED 20 MHZ QUARTER-WAVE CAVITY FOR SSMB

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Abstract

To meet the energy compensation requirements of Steady-State Micro-Bunching (SSMB), a 20 MHz RF cavity with extreme axial compactness is essential. This paper presents an innovative triple-folded quarter-wave cavity design that overcomes the size limitations of conventional structures. Based on cascaded transmission line theory, we established an analytical model to optimize the cavity's shunt impedance and tuning range. A triple-folded 20 MHz prototype was designed and validated via CST simulations, showing that the axial length is compressed to 1.25 m (approximately 1/3 the length of a standard QWC) while achieving a high shunt impedance of 558 kΩ with excellent thermal stability. This compact design offers a high-efficiency solution for low-frequency RF systems.

INTRODUCTION

The Steady-State Micro-Bunching (SSMB) mechanism [1] offers a promising path toward high-power, high-repetition-rate coherent radiation, particularly for extreme ultraviolet (EUV) lithography. Unlike traditional storage rings, SSMB requires a specialized RF system to compensate for the energy loss of long electron bunches without introducing unwanted longitudinal modulation. For the General Longitudinal Strong Fousing (GLSF) SSMB design, an auxiliary RF system operating at a fundamental frequency of 20 MHz is required.

Designing an efficient RF cavity at such a low frequency presents significant challenges. A standard $\beta = 1$ quarter-wave cavity (QWC) at 20 MHz would exceed 3.7 meters in length, which is impractical for compact lattice integration. While magnetic-alloy loaded cavities or high-capacitive loading cavity can reduce the physical footprint, they often suffer from substantial power dissipation or high risk of multipacting [2–4]. Previous developments, such as the folded QWCs at RHIC, have demonstrated improved compactness [5], but further reduction in axial length is necessary for GLSF SSMB light sources.

In this paper, an innovative multi-folded cavity scheme is proposed, with a specific focus on the triple-folded configuration. A theoretical model based on cascaded transmission line theory is developed to analytical characterize the electromagnetic properties of the structure. Utilizing this model, a comprehensive parametric sweep is performed to evaluate the shunt impedance and frequency sensitivity, leading to the determination of optimized geometric parameters. Finally, the analytical results are validated against CST eigenmode simulations, demonstrating excellent agreement and confirming the effectiveness of the proposed design methodology.

EVOLUTION OF THE CAVITY CONFIGURATION

The multi-folded quarter-wave cavity is evolved from a standard coaxial quarter-wave structure. As shown in Fig. 1(a), a conventional QWC consists of a simple coaxial line that is shorted at one end and open at the accelerating gap, requiring a physical length of $\lambda/4$. By folding the outer conductor back towards the gap, a single-folded QWC (Fig. 1(b)) reduces the axial length by approximately half.

The proposed multi-folded design (Fig. 1(c)) further extends this concept by using several nested coaxial cylinders. This creates multiple "folds" where the electromagnetic wave propagates back and forth within the nested layers (indicated by the red arrows in Fig. 1). This technique effectively compresses the axial footprint to approximately $1/n$ of the original QWC length, where n is the fold number which represents the number of nested coaxial sections between the accelerating gap and the shorted end.

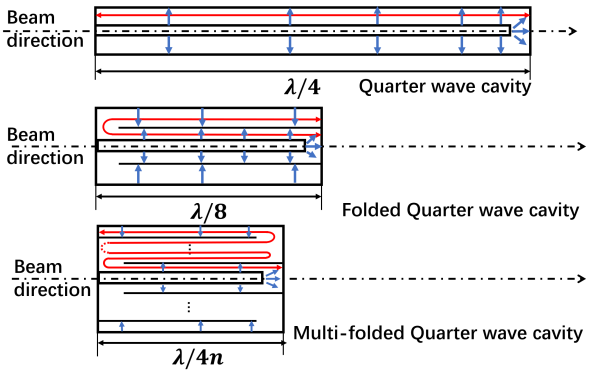

Figure 1: Schematics of a conventional coaxial cavity(a), a folded quarter-wave cavity(b), and the proposed multi-folded quarter-wave cavity(c). Blue arrows indicate the electric field distribution, while red arrows represent the propagation paths of the electromagnetic fields within the cavities.

THEORETICAL SCALING AND FOLD-NUMBER ANALYSIS

To evaluate the fundamental advantages and trade-offs of the multi-folded geometry, an analytical study was conducted focusing on the impact of the fold number n.

To establish a baseline for spatial compression, we first consider an idealized case where the characteristic impedances of all n sections are identical ($Z_1 = Z_2 = \cdots =$

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$Z_n = Z_0$). In this reflectionless propagation model, the axial length and shunt impdance of cavity is :

$$L_{axial}(n) \approx \frac{1}{n} \cdot \frac{\lambda}{4} \tag{1}$$

$$R_{sh}(n) = \frac{1}{n^2} \frac{8}{\pi^2} \frac{\mu_0}{\sqrt{\epsilon_0}} \frac{(\ln(b/a))^2}{1 + (a/b)^{1/n}} \cdot a \left(\frac{b}{a}\right)^{\frac{n-1}{2n}} \sqrt{\pi \sigma f} \tag{2}$$

where μ_0 and ϵ_0 are the permeability and permittivity of vacuum, respectively; σ denotes the electrical conductivity of the cavity wall material (e.g., copper); f represents the resonant frequency; and a, b are the innermost and outermost radii of the multi-folded cavity.

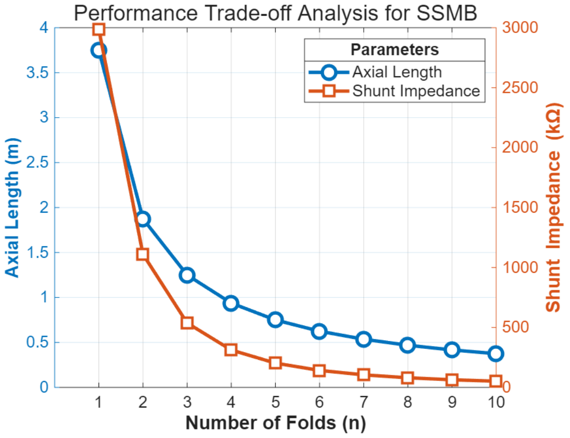

Figure 2: Performance trade-off analysis: Cavity length and shunt impedance as functions of the number of folds n (frequency: 20 MHz; innermost radius: 30 mm; outermost radius: 300 mm; electrical conductivity $\sigma = 5.8 \times 10^7$ S/m).

As indicated by the scaling laws in Eq. (1) and Eq. (2), while the axial length decreases linearly with $1/n$, the shunt impedance R scales approximately with $1/n^2$ which implies that excessive folding leads to a prohibitive surge in RF power requirements.

Figure 2 illustrates the variation of cavity length and shunt impedance as a function of the fold number n for the 20 MHz SSMB cavity. Considering both parameters collectively, $n = 3$ is identified as the optimal design point: it compresses the axial length to the required 1.25 m threshold while preserving a sufficiently high shunt impedance (approx. ~ 500 kΩ) to be driven by standard solid-state power amplifiers. This configuration successfully balances extreme axial compactness with manageable thermal and RF power requirements.Therefore, in the following content, we will conduct detailed theoretical analysis on the triple-folded cavity.

THEORETICAL ANALYSIS AND DESIGN OF THE TRIPLE-FOLDED CAVITY

Theoretical Analysis of Cascaded Transmission Lines

Similar to the traditional QWC can be modeled as coaxial transmission lines [6], the triple-folded QWC can be equivalently modeled as three cascaded coaxial transmission lines. Near the resonant frequency, this cascaded model can be further simplified into a parallel RLC circuit, allowing for the analytical calculation of the resonant frequency, shunt impedance, and quality factor of the triple-folded cavity.

For a parallel RLC circuit, the expression for its input impedance Z_{in} as a function of frequency is:

$$Z_{in} = \frac{R}{1 + 2jQ\frac{\Delta\omega}{\omega_0}} \tag{3}$$

Based on the admittance $Y_{in} = 1/Z_{in}$, the key high-frequency parameters of the parallel RLC circuit are defined as follows:

- **Resonant angular frequency** ω_0: Defined as the angular frequency when the imaginary part of the input admittance is exactly zero, i.e., $\mathrm{Im}(Y_{in}) = 0$.
- **Shunt impedance** R_s: The real part of the input impedance at the resonant frequency ω_0, i.e., $R_s = 2\mathrm{Re}(Z_{in}(\omega_0))$. The factor of 2 originates from the different definitions of shunt impedance used in equivalent circuits and RF cavities.
- **Quality factor** Q: Introducing a small angular frequency perturbation, the quality factor can be expressed as:

$$Q = \frac{\omega_0}{2R_s} \frac{\partial \mathrm{Im}(Y_{in})}{\partial \omega} \tag{4}$$

The triple-folded cavity is modeled as three cascaded coaxial sections. The input impedance $Z_{in,i}$ is derived using the standard transformation:

$$Z_{in,i} = Z_i \frac{Z_{in,i-1} + Z_i \tanh(\gamma_i l)}{Z_i + Z_{in,i-1} \tanh(\gamma_i l)} \tag{5}$$

where $Z_{in,3}$ represents the cavity's total impedance. Near resonance, the structure is equivalent to a parallel RLC circuit. The quality factor Q and shunt impedance R_s are then extracted using the complex input admittance at ω_0. The propagation constant γ_i accounts for power dissipation based on the surface resistance of copper.

The propagation constant of the transmission line is $\gamma_i = \alpha_i + j\beta$. The phase constant is $\beta = \omega/c$ (consistent across all sections), while the attenuation constant α depends on the coaxial structure dimensions and the surface resistance R_{surf}:

$$\alpha_i = \frac{R_{surf}}{2\eta \ln(r_{out,i}/r_{in,i})} \left(\frac{1}{r_{in,i}} + \frac{1}{r_{out,i}} \right) \tag{6}$$

where $R_{surf} = \sqrt{\omega\mu_0/2\sigma}$ is the surface resistance, $\eta = \sqrt{\mu_0/\epsilon_0}$ is wave impedance of the medium inside the cavity.

Under the assumption of a vacuum medium , the characteristic impedance Z_i of each section is approximately:

$$\begin{aligned} Z_i =& \frac{\eta}{2\pi} \ln\left(\frac{r_{out,i}}{r_{in,i}}\right) \times \\ & \left(1 - \frac{jR_{surf}}{2\omega\mu_0 \ln(r_{out,i}/r_{in,i})} \left[\frac{1}{r_{in,i}} + \frac{1}{r_{out,i}}\right]\right) \end{aligned} \tag{7}$$

In summary, by defining geometric dimensions such as the inner and outer radii of the cavity, the resonant frequency, shunt impedance, and quality factor of the triple-folded cavity can be theoretically calculated using the cascaded transmission line formulas presented above.

Parameter Sweep Optimization and CST Simulation Verification

Based on the theoretical model, a 2D parameter sweep and optimization design were performed on the cavity. The innermost and outermost radii were fixed at 30 mm and 300 mm, respectively, with a cavity wall thickness of 10 mm. The target resonant frequency was 20 MHz, and Oxygen-Free Copper (OFC) was selected as the cavity material.

The two radii of the middle folded layers, r_2 and r_3, were selected as variables, and the aforementioned theoretical formulas were used for the parameter sweep. The results are shown in Fig. 3.

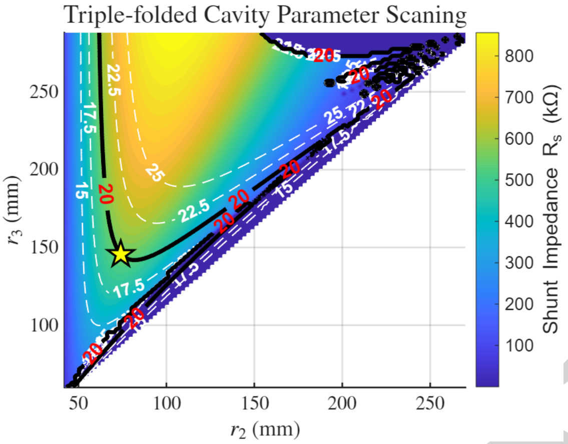

Figure 3: Global parameter sweep of the triple-folded cavity. The background color represents the shunt impedance R_{sh}, while the contour lines indicate the resonant frequency. The bold black line highlights the 20 MHz resonant ridge. The yellow star denotes the design point.

The contour lines in the figure indicate the distribution of the resonant frequency. The analysis demonstrates that in regions with a higher resonant frequency, the shunt impedance would be correspondingly higher. This suggests that appropriately increasing the longitudinal length of the cavity can further improve the shunt impedance while maintaining the target frequency range.

Furthermore, the density of the frequency contour lines reflects the sensitivity of the resonant frequency to the radial dimensions. In practical engineering design, it is highly desirable for the cavity to exhibit lower sensitivity to dimensional variations. This significantly suppresses frequency drifts caused by mechanical machining errors, stress deformations, and thermal expansion effects.

Considering both the maximization of shunt impedance and the minimization of frequency sensitivity, a series of parameters near the high shunt impedance region on the 20 MHz contour line were selected for comparative analysis, as illustrated in Fig. 4. The optimal radius parameters were finally determined to be $r_2 = 74.24$ mm and $r_3 = 145.55$ mm. This operating point perfectly balances high shunt impedance with low frequency sensitivity.

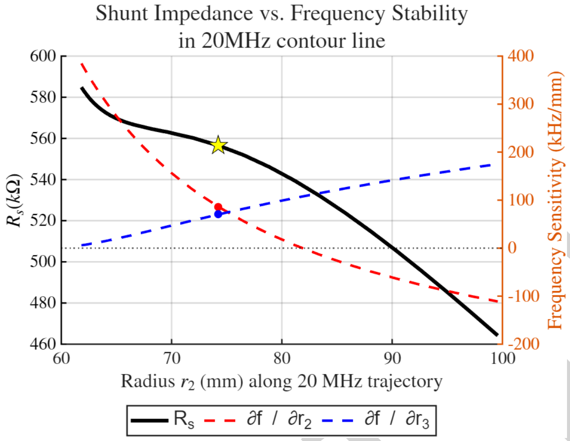

Figure 4: Performance decomposition along the 20 MHz resonant ridge. The left axis shows the shunt impedance (solid black), while the right axis displays the frequency sensitivity components $\partial f/\partial r_2$ (dashed red) and $\partial f/\partial r_3$ (dashed blue). The yellow star denotes the design point.

To verify the accuracy of the theoretical calculations, the optimized parameters were imported into CST Studio Suite, and 3D full-wave electromagnetic simulations were performed using the Eigenmode Solver. As Table 1 shows: the deviation between the high-frequency parameters obtained from the CST simulation and the theoretical derivation of the cascaded transmission lines remains within 1%. This not only verifies the validity and reliability of the theoretical model but also marks the successful completion of the basic electromagnetic design for the triple-folded cavity oriented towards the SSMB light source scheme.

Table 1: Comparison of theoretical calculations and CST results

Parameter	**Theory**	**CST**	**Error [%]**
f_0 [MHz]	20.00	19.81	0.95
R_{sh} [kΩ]	556.5	558.6	0.38
Q	5852	5802	0.86
$\partial f/\partial r_2$ [kHz/mm]	86.26	76.40	11.4
$\partial f/\partial r_3$ [kHz/mm]	70.22	67.05	4.5

CONCLUSION

A compact 20 MHz triple-folded QWC was designed for SSMB energy compensation. An analytical model based on cascaded lossy transmission lines was developed, enabling rapid optimization of the cavity geometry. The optimized design achieves a shunt impedance of 558.6 kΩ and a length of 1.25 m. CST simulations show excellent agreement with the analytical results (error < 1%), confirming the efficiency of the multi-folded scheme for low-frequency RF systems. The corresponding optimization code has been made available as an open-source repository [7].

REFERENCES

[1] X. Deng *et al.*, “Experimental demonstration of the mechanism of steady-state microbunching”, *Nature*, vol. 590, no. 7847, pp. 576-581, 2021. `doi:10.1038/s41586-021-03203-0`

[2] K. W. Shepard *et al.*, “A Superconducting Accelerating Structure for Particle Velocities from 0.12- to 0.23-C”, *IEEE Trans. Nucl. Sci.*, vol. 30, no. 4, pp. 3339-3341, 1983. `doi:10.1109/TNS.1983.4336650`

[3] C. Ohmori *et al.*, “High field-gradient cavities loaded with magnetic alloys for synchrotrons”, in *Proc. PAC’99*, New York, NY, USA, Mar. 1999, pp. 413-417. `doi:10.1109/PAC.1999.795720`

[4] W. Ebeling *et al.*, “The Radiofrequency-Systems of PIA”, *IEEE Trans. Nucl. Sci.*, vol. 28, no. 3, pp. 2898-2900, 1981. `doi:10.1109/TNS.1981.4331952`

[5] C. H. Boulware *et al.*, “Tunable 28 MHz Superconducting Cavity for RHIC”, in *Proc. IPAC’12*, New Orleans, LA, USA, May 2012, paper WEPPC083, pp. 2394-2396.

[6] D. M. Pozar, *Microwave Engineering*, 4th ed. Hoboken, NJ, USA: Wiley, 2011.

[7] A. Fang, ”multi-folded-cavity-analysis-code”. `https://github.com/AlanFFang/multi-folded-cavity-analysis-code`