NONLINEAR DYNAMICS STUDY FROM COUPLED BEAM–BEAM AND SPACE-CHARGE EFFECTS IN A REALISTIC COLLIDER LATTICE *

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Abstract

Beam–beam interactions are well known to limit the performance of high-energy hadron colliders, as they can degrade beam quality and reduce particle lifetime. When the strengths of the beam–beam and space-charge forces become comparable, as in high-intensity, mid-energy colliders, their interplay results in complex nonlinear dynamics and the excitation of resonances.

In this project, we perform GPU-enabled simulations, motivated by the forthcoming Electron–Ion Collider (EIC), to study the combined effects of beam–beam and space-charge interactions within a realistic accelerator lattice. In support of this endeavor, we have developed a preliminary 2.5D space-charge symplectic model incorporating self-consistent space-charge forces and the complete magnetic lattice. This model enables the study of long-term nonlinear dynamics, which is essential for achieving high average luminosity.

INTRODUCTION

In the context of next-generation hadron collider projects, understanding collective effects along the beam line is of fundamental importance, as they directly influence beam stability, intensity limits, and overall machine performance. Deeper insight into these mechanisms enables optimization of beam quality, leading to improved luminosity, operational efficiency, and long-term reliability of the accelerator complex.

However, accurately modeling collective effects—particularly space-charge forces—across the full beamline, and capturing their interplay with individual accelerator components, is computationally demanding. This challenge is especially pronounced for high-intensity beams, where self-consistent simulations must be performed over many turns or extended transport sections. As a result, developing efficient computational approaches for collective-effects simulations is essential.

To address these limitations, tracking frameworks can be adapted to exploit graphics processing units (GPUs), leveraging their massively parallel architecture to significantly reduce execution time and improve scalability for simulations with large macroparticle distributions.

This approach is supported by recent advances in high-performance computing, which have established GPUs as powerful accelerators for large-scale scientific simulations, particularly in computationally intensive areas such as beam dynamics and collective-effects modeling [1–3].

This paper is organized as follows. Section 1 presents the GPU-accelerated computational framework developed for element-wise simulation of a representative lattice similar to that of the Electron–Ion Collider (EIC) at Brookhaven National Laboratory [4, 5].

Section 2 details the formulation and implementation of the 2.5D symplectic space-charge model and examines the resulting beam dynamics under collective effects. Section 3 reports simulation results incorporating the coupled influence of space-charge and beam–beam interactions.

GPU FRAMEWORK FOR ELEMENT-WISE LATTICE TRACKING

In this work, the `CUDA.jl` package is employed as the programming interface for NVIDIA GPU acceleration within the Julia ecosystem [6]. The implementation exploits fine-grained parallelism to advance particles in the full six-dimensional phase space using symplectic transfer maps. This approach enables efficient element-by-element lattice tracking while rigorously preserving the underlying Hamiltonian structure and nonlinear dynamics. Consequently, it facilitates high-fidelity simulation of proton beam evolution over several thousand turns.

The lattice optical parameters, extracted using `TrackPad.jl`, are shown in Figs. 1 and 2 [7, 8].

A central component of the implementation is the evaluation of the transverse electric field using the Bassetti–Erskine formalism. This formulation is consistently employed in both the space-charge and synchro-beam maps to model weak–strong beam–beam interactions. This approach was previously implemented in Ref. [9], where the Weidemann algorithm is utilized to precompute the polynomial contribution to the Bassetti–Erskine function. This strategy reduces the computational cost of field evaluation, yielding an approximate threefold performance improvement [10]. The nominal design specifies a synchrotron tune of $Q_s = 0.01$. This tune is achieved using an RF cavity operating at 591 MHz to maintain longitudinal beam bunching [4].

SELF-CONSISTENT SIMULATION OF SPACE-CHARGE EFFECTS IN HADRON RING LATTICE

The space charge model applied to this study has been described in Refs. [11, 12]. The potential, denoted by $\varphi_0(x, y, z)$, is evaluated assuming a 2.5D Gaussian distribution, i.e., $\varphi_0(x, y, z) = (\varphi(x, y) + \varphi_{00})\lambda_z(z)$, which is valid under the condition

$$\frac{\partial^2 \phi}{\partial x, y^2} \gg \frac{1}{\gamma^2}\frac{\partial^2 \phi}{\partial z^2} . \tag{1}$$

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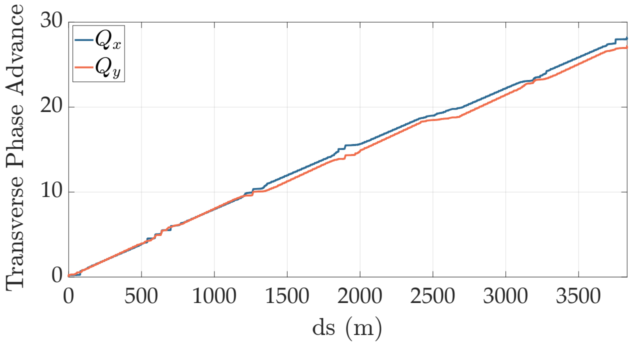

Figure 1: Cumulative transverse phase advance along the ring circumference.

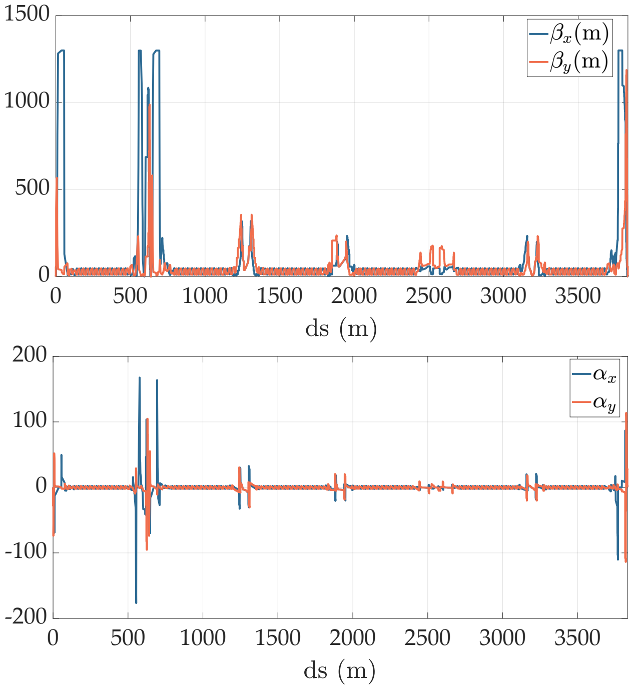

Figure 2: Optical Twiss parameters of the simulated lattice along the ring circumference. The top figure illustrates the horizontal (β_x, blue) and vertical (β_y, orange) betatron functions. The bottom figure shows the horizontal and vertical α functions.

To account for the distributed nature of space-charge forces around the ring, a discretized approximation is employed. In this approach, space-charge kicks are uniformly distributed in phase advance, with each kick scaled proportionally to the corresponding lattice segment length. For the following simulations, approximately 245 space-charge kicks are used, which is sufficient to achieve numerical convergence. This choice represents a practical compromise between accuracy and computational cost, as the execution time scales linearly with the number of space-charge kicks.

Space-Charge–Real Lattice Interactions

These initial simulations are designed to investigate tune shifts in the regime where the space-charge force is comparable in magnitude to the beam–beam interaction. To this end, the proton beam is tracked over several thousand turns to enable detailed frequency map analysis, with tunes extracted using the NAFF algorithm implemented in Python [13].

In comparison with the bare lattice tunes shown in Fig. 3, Fig. 4 reveals pronounced defocusing tune shifts of up to about $\Delta\nu_x = 0.01$ and $\Delta\nu_y = 0.03$. At high beam intensity, strong space-charge forces drive a systematic downward shift of the tune footprint, causing a fraction of particles to cross synchro-betatron resonances (SBRs). This phenomenon, is consistent with previous observations reported in Ref. [12].

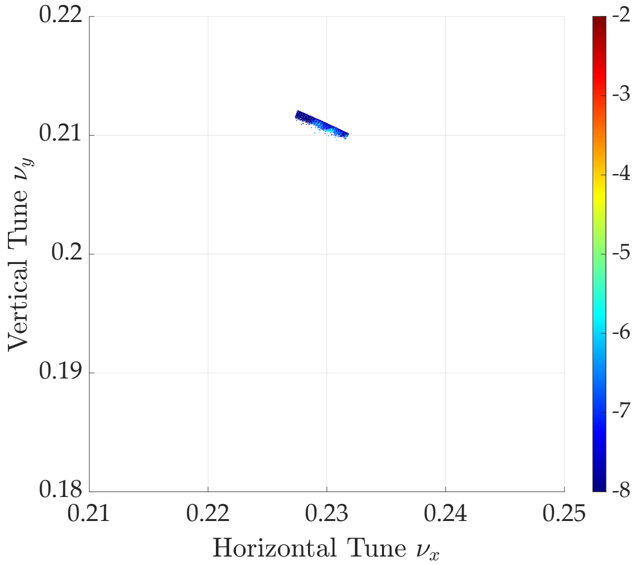

Figure 3: Frequency map analysis obtained from tracking simulations of a bare EIC-like lattice. The color scale represents the diffusion index highlighting regions of enhanced tune variation and resonance crossing.

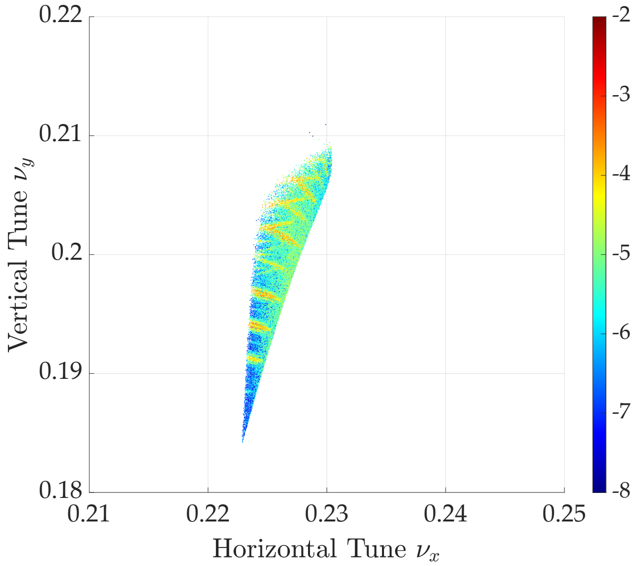

Figure 4: Frequency map analysis illustrating space-charge effects in the EIC lattice with a strength equivalent to the beam–beam interaction.

The computational efficiency of GPUs enables the simulation of long-term emittance evolution throughout the hadron ring lattice. Specifically, Fig. 5 compares the transverse emittance evolution of the bare lattice against a model incorporating 2.5D symplectic space-charge effects. The results demonstrate stable evolution in both cases, with no significant divergence observed between the two models over the simulated timescale.

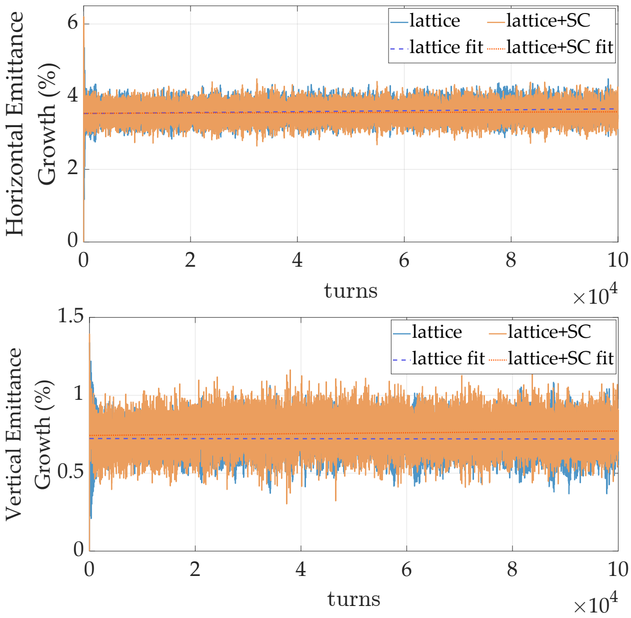

Figure 5: Horizontal (top) and vertical (bottom) emittance growth (%) tracked over 10^5 turns. The solid blue line represents the baseline lattice without space charge, while the orange line illustrates the lattice with space-charge effects integrated along the entire ring. Dashed lines indicate linear fits to the simulation data.

Combined Space-Charge and Beam–Beam Effects in Real Lattice

In this preliminary study, the weak–strong beam–beam interaction is modeled for head-on collisions without the inclusion of crab cavities. Their inclusion is left for future work, where their impact on the synchrotron tune will be systematically investigated.

The proton beam–beam parameters were calculated using the proton and electron beam parameters reported in Ref. [5], yielding $\xi_x = 0.012$ and $\xi_y = 0.011$. Figure 6 shows the tune footprint obtained from beam–beam simulations, in which particles are tracked for 10^5 turns through the full lattice. A comparison with Fig. 7 reveals a pronounced downward shift of the footprint when space-charge effects are included. This shift arises from the cumulative impact of space-charge kicks distributed along the ring. Furthermore, a fraction of particles is observed to cross the resonance at $\nu_y = 0.2$, as well as higher-order resonances, including fourth- and sixth-order lines.

CONCLUSION

This work presents a preliminary study of a GPU-based framework for simulating the complex collective effects critical to the early design stages of hadron rings. The model

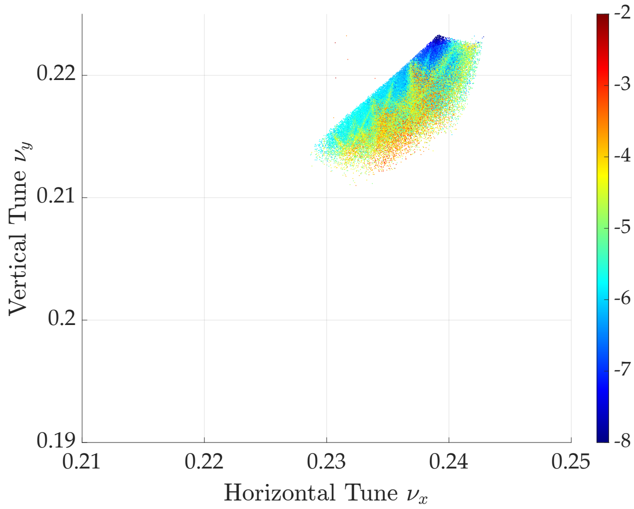

Figure 6: Frequency map analysis of the EIC lattice incorporating the weak-strong beam–beam kick.

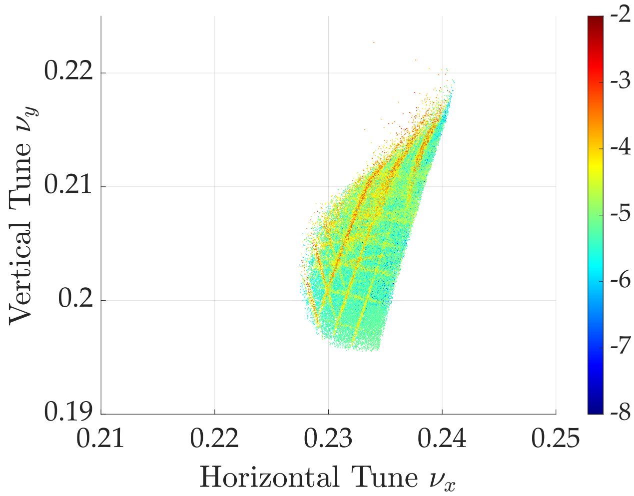

Figure 7: Frequency map analysis of the EIC lattice incorporating both space-charge and beam–beam effects of comparable magnitude.

captures the impact of strong nonlinear interactions on particle dynamics within a realistic lattice without relying on linear approximations, all while maintaining high computational efficiency.

By accounting for beam–beam and space-charge interactions, the simulations identify resonance crossings that could compromise beam stability. This capability provides deep insight into long-term beam performance, supporting the design and optimization of future high-intensity accelerator configurations.

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