IMPACT OF INITIAL ENERGY SPREAD ON THE ACCURACY OF ENERGY SPREAD GROWTH FORMULA IN A FREE ELECTRON LASER

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Abstract

Evaluating the energy spread growth due to radiation is crucial for the design of steady-state microbunching (SSMB) insertion section. This study systematically benchmarks a classical analytical formula against three-dimensional numerical simulations to validate its accuracy under varying initial energy spreads. Our findings reveal that while the formula provides reliable estimations for beams with small initial energy spreads, its predictions deviate significantly as the initial energy spread increases. Such discrepancies arise because larger initial energy spreads amplify three-dimensional effects and nonlinear beam dynamics, which compromise the idealized analytical model. This study shows that comprehensive 3D simulations are indispensable for robust and accurate SSMB insertion section design.

INTRODUCTION

Storage ring-based light sources, such as the proposed SSMB scheme [1–3], have the potential to provide high average power EUV radiation for lithography.

In the design of the insertion section for SSMB storage rings, a crucial requirement is the implementation of reversible laser modulation [4]. Specifically, the laser modulation imparted on the electron beam upstream must be compensated by an inverse modulation downstream, aiming to restore the beam as closely as possible to the steady-state parameters of the storage ring. During this process, the energy spread growth of the electron beam due to radiation is a critical parameter. Excessive energy spread growth will result in incomplete cancellation of the laser modulation, thereby decreasing the radiation power.

In a storage ring FEL, Eq. (1) is commonly used to calculate the energy spread growth of the electron beam in the lasering process [5, 6]:

$$\Delta\sigma_\delta = \sqrt{\frac{2\rho P}{P_{\text{beam}}}}, \tag{1}$$

$$\rho = \left[\frac{1}{8\pi}\frac{I_p}{I_A}\left(\frac{K[JJ]}{1+K^2/2}\right)^2\frac{\gamma\lambda_r^2}{2\pi\sigma_x^2}\right]^{\frac{1}{3}},$$

here P is the radiation power, P_{beam} is the beam power and ρ is the FEL(Pierce) parameter. For an SASE FEL starting up from electron shot noise, the radiation power can be written as $P \approx \frac{1}{9}P_n \exp\left(\frac{z}{L_G}\right)$, where $P_n = \sqrt{2\pi}\frac{\rho^2\gamma mc^3}{\lambda_r}$ is the shot noise power, z is the distance along the undulator, and L_G is the FEL gain length. This equation is valid for the one-dimensional (1D) high-gain FEL regime. To account for 3D FEL processes, the authors in [5, 6] adapted it by setting the power to $P = P_n \exp\left(\frac{z}{L_G^{3D}}\right)$ and substituting the gain length L_G^{3D} with Ming Xie's formula [7].

Nevertheless, the accuracy of this formula in 3D FEL processes has yet to be validated by simulation results, especially in cases with different initial energy spreads. In this study, we focus on comparing the predictions of this formula with simulation results under various initial energy spreads.

METHODOLOGY

To evaluate the applicability of Eq. (1), we perform numerical simulations using the FEL code GENESIS [8].

Simulation Setting

The simulation parameters are mainly chosen from [6], which are listed in Table 1 and the optical functions of one undulator section is shown in Fig. 1.

Table 1: Main Simulation Parameters

Parameters	**Value**
Beam energy E (MeV)	988
Radiation wavelength λ_r (nm)	13.5
Bunch current I_p (A)	150
Normalized emittance $\gamma\epsilon_{x,y}$ (mm mrad)	1.66
Undulator period λ_u (cm)	2.6
Undulator parameter K	2.4
β-functions in undulator $\beta_{x,y}$ (m)	~2.5
FEL parameter ρ	0.001 29

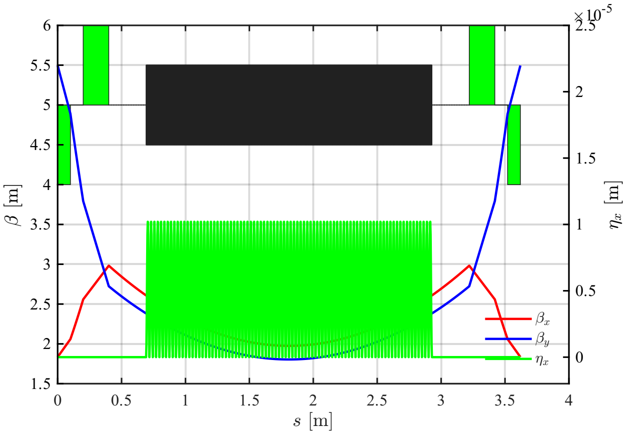

Figure 1: Optical functions of one undulator section.

For simplicity, a longitudinally uniform electron bunch with a length of 100 µm is adopted for the simulation. We conduct 3D FEL simulations with varying initial energy spreads of the electron bunch.

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RESULTS AND DISCUSSION

The results are shown in Fig. 2, where the energy spread growth calculated using Eq. (1) is compared with the numerical results from GENESIS. The energy spread growth data were extracted when the radiation power reached 1 MW, to ensure that the process was well within the exponential growth regime.

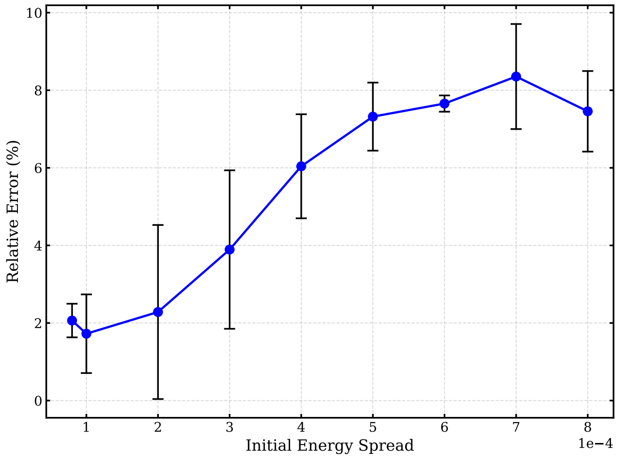

Figure 2: Relative error of the Eq. (1) with respect to the simulation results. The data were extracted when the radiation power reached 1 MW.

As illustrated in the figure, when the initial energy spread is below 3×10^{-4}, the relative error between the analytical formula and the simulation results is small, remaining at approximately 2%, which aligns well with theoretical expectations. However, as the initial energy spread exceeds 4×10^{-4}, the discrepancy grows significantly, reaching roughly 8% within the chosen parameter range and continuing to rise. This pronounced deviation is primarily attributed to the fact that a larger initial energy spread exacerbates 3D effects and nonlinear beam dynamics, causing the actual process to depart from the idealized analytical model's predictions. In Ref. [6], the steady-state energy spread of the storage ring designed using Eq. (1) is 8.64×10^{-4}. Therefore, we believe that the result simulated by them with Genesis has an error of approximately 8%.

Consequently, it is risky to directly use this formula for SSMB insertion section design. The SSMB storage ring has a relatively large steady-state energy spread on the order of 10^{-3} , and applying this formula results in an unacceptable error of about 10% or even higher. Nevertheless, this formula can be used as an upper limit to predict the energy spread growth. This is because the undulator in SSMB insertion section is very short, meaning the longitudinal phase space of the beam will not rotate drastically transversing the undulator, so the energy spread will not grow significantly.

CONCLUSION

In conclusion, we validate the accuracy of the analytical formula for electron beam energy spread growth in 3D high-gain FEL processes through GENESIS simulations. The formula achieves high precision with a relative error of approximately 2% when the initial energy spread is below 3×10^{-4}, making it an efficient tool for preliminary theoretical analysis and quick estimation.

However, the deviation between the formula results and GENESIS simulations rises significantly to around 8% as the initial energy spread exceeds 4×10^{-4}, due to enhanced 3D effects and nonlinear beam dynamics that are not fully accounted for in the formula.

Thus, this formula is not suitable for direct application in SSMB insertion device design, as the large steady-state energy spread of SSMB storage rings would cause unacceptable errors. Nevertheless, it can be used as an upper limit to estimate the energy spread growth, serving as a reference for preliminary design and performance evaluation.

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