THEORY OF CAVITY TUNING WITH PERPENDICULAR BIASED FERRITE

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Abstract

Yttrium-Iron-Garnet (YIG) ferrites have one-to-two orders of magnitude smaller magnetic permeability than Ni-Zn ferrites. In the frequency range above 10 MHz, Ni-Zn ferrites are too lossy to be useful. However in the frequency range roughly 30-120 MHz, YIG has high magnetic (and electrical) quality factor. Further, ferrites perpendicular-biased near saturation, have large incremental permeability due to the ferro-magnetic resonance (FMR). These properties presented an opportunity for effective wide-range RF cavity tuning, with low losses. Although high-Q narrow-range tuning was achieved, the promise of both wide tuning range and high Q was not achieved (despite more than a decade of development).This has never been explained, even prompting Shapiro to suggest the magnetic Q (material property) varies quickly with frequency. Here we present a simple theory of cavity tuning with perpendicular biased ferrite (and FMR) that predicts tuning range and effective quality factor; and explains why the high Q cannot be maintained across a large range, and instead Q falls at low frequency.

INTRODUCTION

The quasi-DC, large amplitude magnetic bias H-field sweeps out a *major loop* hysteresis curve. The radio frequency, small amplitude RF H-field sweeps out a *minor loop* starting from a location on the major loop. The RF incremental permeability $\mu_\Delta = \Delta B/(\mu_0 \Delta H)$ differs greatly from the DC value μ. μ_Δ has many dependencies: location on the B-H curve, frequency of the RF wave, polarization, temperature, etc, and has to be measured for every point on the main B-H curve.

The cavity drive frequency must be perfectly matched to an integer multiple of the charged particle revolution frequency around the synchrotron. If the particles accelerate, there will be a program of values drive-frequency versus time called a *frequency law.* The magnetic bias field must track a corresponding program of values. Thus we wish to find the bias field (law) that makes the cavity resonance frequency $\omega_{\rm res}$ and drive frequency equal. Now $\omega_{\rm res}$ is a function of the incremental permeability, which in turn is a function of the resonance frequency and DC bias. This recursive relationship has to be found/solved self-consistently. We shall illustrate by a crude example. Suppose ω_c is the RF cavity resonance frequency without ferrite. If filled with ferrite, the frequency becomes:

$$\omega_{\rm res} = \omega_c / \sqrt{\mu_\Delta [H, B(H), \omega_{\rm res}]} \tag{1}$$

This is a nonlinear equation that is solved iteratively for bias as a function of frequency, $H(\omega_{\rm res})$.

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Symbols

- Let H_z be the applied bias magnetic field, M_z the induced magnetization.
- Let h_y be the applied RF magnetic field, and m_y the induced RF magnetization.
- Let $\rho \equiv M_z/M$ be the ratio of magnetization in the bias direction to the total magnetization M modulus.
- Let $\delta \equiv \alpha\rho$ where α is the LLG relaxation parameter.
- Let $g \equiv \mu_0 \gamma_e$ where μ_0 is the permeability of free space and γ_e is the gyromagnetic ratio of the electron.
- We drop the subscript "res": $\omega_{\rm res} \to \omega$.

PERPENDICULAR BIAS

In the particular regime of perpendicular bias, we may appeal to the Landau-Lifshitz-Gilbert (LLG) equation to give the frequency dependence of $\mu_\Delta = 1 + \Re[m_y/h_y]$. The LLG equation and the form of the effective field $H_{\rm eff}$ used here assumes that the material is near saturation (M almost independent of H) and that the static values H_z and M_z are mutually self-consistent.

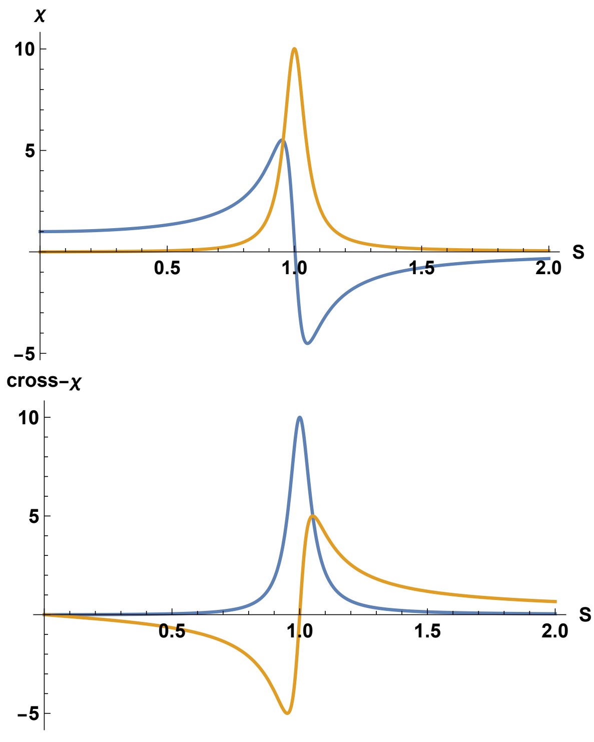

Figure 1: Complex susceptibility components of m_y (left graph) and m_x (right graph). $\delta = 0.05$ The in-phase (real) component χ' shown blue; the quadrature (imaginary, lossy) component χ'' shown gold. The bias field is fixed, as is the scale parameter between angular frequency ω and S.

Polder Tensor

The Polder tensor [2] gives the (linearized) incremental susceptibility. Figure 1 shows the in-phase (real) and quadrature (imaginary) components of the complex susceptibility m_y/h_y and cross-susceptibility m_x/h_y versus (normalised) frequency at fixed bias field. $S = \omega/\omega_0$ is the ratio of applied frequency to the gyromagnetic resonance frequency $\omega_0 = g\tilde{H}_z\sqrt{1+\delta^2}$.

Remarkably, the incremental permeability $\mu_\Delta = 1 + \chi$ due to m_y/h_y may become negative for $S = \omega/\omega_0 > 1$. To be useful, the applied RF must be below the gyromagnetic frequency. The susceptibility formulae for a cylinder or sphere aligned with H_z are:

$$\begin{aligned}\Re[\chi_{yy}] &= g^2 M_z H_z[\omega^2(-1+\delta^2) + g^2H_z^2(1+\delta^2)^2]/D \\ \Im[\chi_{yy}] &= gM_z\omega\delta[\omega^2 + g^2H_z^2(1+\delta^2)]/D \qquad (2)\\ D &= \omega^4 + 2(gH_z\omega)^2(-1+\delta^2) + (gH_z)^4(1+\delta^2)^2\ .\end{aligned}$$

Here H_z is the effective field including the demagnetization factor.

Bias/Frequency Law

Equation (2) is substituted into Eq. (1). In principle, this leads to a quartic equation for the bias $H_z(\omega)$. We know that $\delta \ll 1$. Suppose we set $\delta = 0$. The bias equation simplifies to a quadratic:

$$\frac{\omega}{\omega_c} = \left[\frac{H_z(H_z+M_z) - (\omega/g)^2}{H_z^2 - (\omega/g)^2}\right]^{-1/2}\ . \qquad (3)$$

The natural reference for frequency is ω_c, and for field it is M_z. Choosing the cavity frequency as a reference leads to M_z becoming a variable. But the saturation magnetization is a material property, a constant. So M_z is taken as the "standard unit". We set[1] $\omega = \alpha\omega_c$ and $\omega_c = \beta \times (gM_z)$. The ferrite loaded value is less than the vacuum-filled value; and so $0 < \alpha < 1$. The solution of the quadratic is:

$$\frac{H_z}{M_z} = \frac{\alpha^2[1 + \sqrt{1 + 4(1-\alpha^2)^2(\beta/\alpha)^2}]}{2(1-\alpha^2)}\ . \qquad (4)$$

The cavity operation frequency must be below the gyromagnetic frequency, and so $\alpha\beta M_z < H_z$; this implies $\alpha < 1/\sqrt{2}$. Suppose there is an upper limit on the bias $H_z < \gamma M_z$ where $0 < \gamma < 1$. We may find an upper limit on the vacuum-filled cavity frequency $\omega_c = (gM_z)\beta$.

$$0 < \alpha < \frac{1}{\sqrt{2}} \quad \text{and} \quad 0 < \beta < \sqrt{\gamma\left[\frac{1}{\alpha^2 - 1} + \frac{\gamma}{\alpha^2}\right]}\ .$$

The tuning range parameter α is progressively limited to smaller values as β rises or γ falls. The properties of ferrite imply $\gamma \ll 1$.

[1] Here α is NOT the LLG relaxation parameter.

Magnetic Quality Factor

The magnetic quality factor Q_m of the ferrite material is the per cycle energy stored divided by the energy dissipated:

$$\begin{aligned} Q_m &= \frac{\mu'}{\mu''} = \frac{1 + \Re[\chi_{yy}]}{\epsilon + \Im[\chi_{yy}]} \qquad (5)\\ &\approx \frac{[(gH_z)^2 - \omega^2][g^2H_z(H_z+M_z) - \omega^2]}{\epsilon \times [(gH_z)^2 - \omega^2]^2 + gM_z[(gH_z)^2 + \omega^2]\omega\delta}\ .\end{aligned}$$

Strictly speaking $\epsilon \equiv 0$; however, the need for it will be come apparent. Q_m evaluated in the DC limit $\omega \to 0$ becomes $(1 + M_z/H_z)/\epsilon$. Thus ϵ regularizes Q_m to evade the singularity at $\omega = 0$. In practice, the ratio of measured values of μ', μ'' at low frequency is very large; and therefore $\epsilon \ll 1$ and can be omitted (set to zero) when Q_m is evaluated in the MHz range and above. The quality factor becomes zero at two values of angular frequency ω equal gH_z and $g\sqrt{H_z(H_z+M_z)}$.

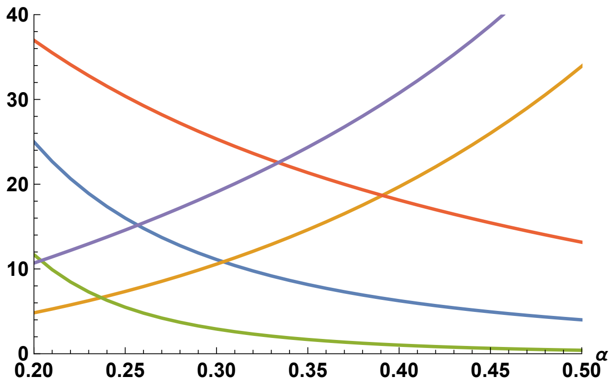

Figure 2: Perpendicular bias tuning program versus fractional frequency $\alpha = \omega/\omega_c$. μ_Δ shown blue; H_z/M_z as percentage shown gold; $S = \omega/\omega_0$ as percentage shown coral; $\chi''_{yy}/(\delta)$ shown green; and $5 \times Q_m\delta$ shown purple.

EXAMPLE

The example is taken from the Fermilab Booster cavity [3]. Suppose we employ a garnet ferrite with saturation magnetization 800 Oersted (63662 A/m). The equivalent frequency $gM_z/(2\pi)$ is 2.24 GHz. Suppose that we wish to use a vacuum-filled cavity with 200 MHz resonance frequency. The equivalent field is $H_c = 71.4$ Oersted (5682 A/m). Suppose that we fill the cavity with ferrite, and wish operate it over the range 40-100 MHz. What is the range of bias field required? We evaluate Eq. (4) with $\beta = H_c/M_z = 0.08925$ and α ranging over $[0.2, 0.5]$. The results are plotted in Fig. 2. The incremental permeability μ_Δ starts high at low frequency and falls.

The parameter $S = \omega/\omega_0 = \alpha\beta/(H_z/M_z)$. For values $S < 1/2$ the real component of the complex susceptibility χ is almost independent of the loss parameter $\delta = \alpha_G \times (M_z/M)$ where α_G is the Gilbert damping factor. Thus the calculation has been self-consistent: we presumed $\delta = 0$, and we have found a solution regime where a non-zero value of δ would make (almost) no difference to the solution values. This is always the case when $\beta \ll 1$. Contrastingly, if $\beta \simeq 1$ or larger, as is the case if we had chosen a large value for ω_c, the permeability, bias field and S-values and FMR power losses all grow to large values.

It is noteworthy that the magnetic quality factor appears to rise with cavity resonance frequency. Contrastingly, at fixed bias field, we know Q_m to be largest at low frequency. The explanation is that we have plotted $Q_m(\alpha, H_z)$; and both α and $H_z(\alpha)$ vary in order to satisfy Eq. (4). Note again that $\alpha = \omega/\omega_c$ is a scale parameter, not the Gilbert loss factor.

CONCLUSION

Starting from a vacuum-filled RF cavity with resonance frequency ω_c, and filling the cavity with YIG ferrite, and employing the components of the Polder tensor, we have found the new resonance frequency ω and magnetic quality factor as a function of the externally applied bias field. The magnetic bias and frequency have been found self-consistently; and this explains (without additional assumptions) why the Q_m is found to vary if the tuning range is large.
Further details, results and explanation are presented by Koscielniak [4].

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