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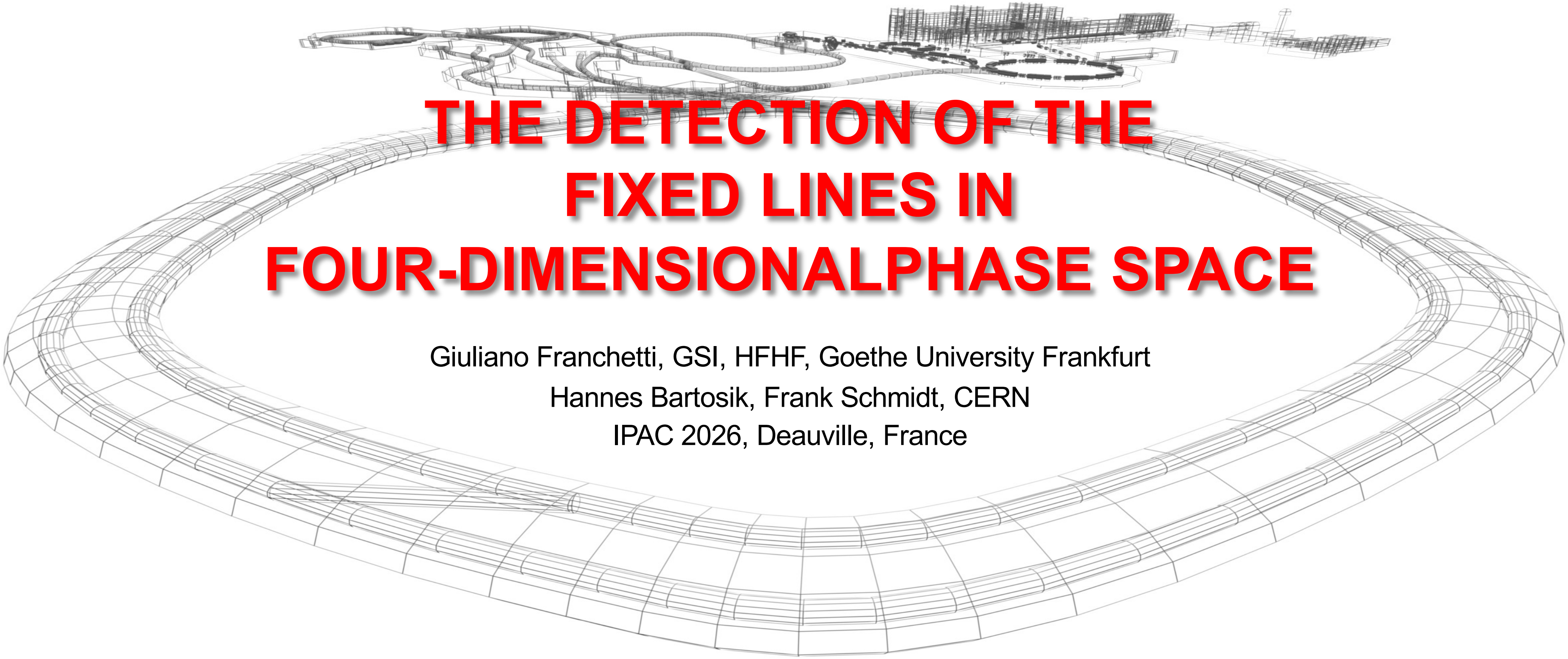

# THE DETECTION OF THE FIXED LINES IN FOUR-DIMENSIONALPHASE SPACE

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Hannes Bartosik, Frank Schmidt, CERN

IPAC 2026, Deauville, France

# Acknowledgements

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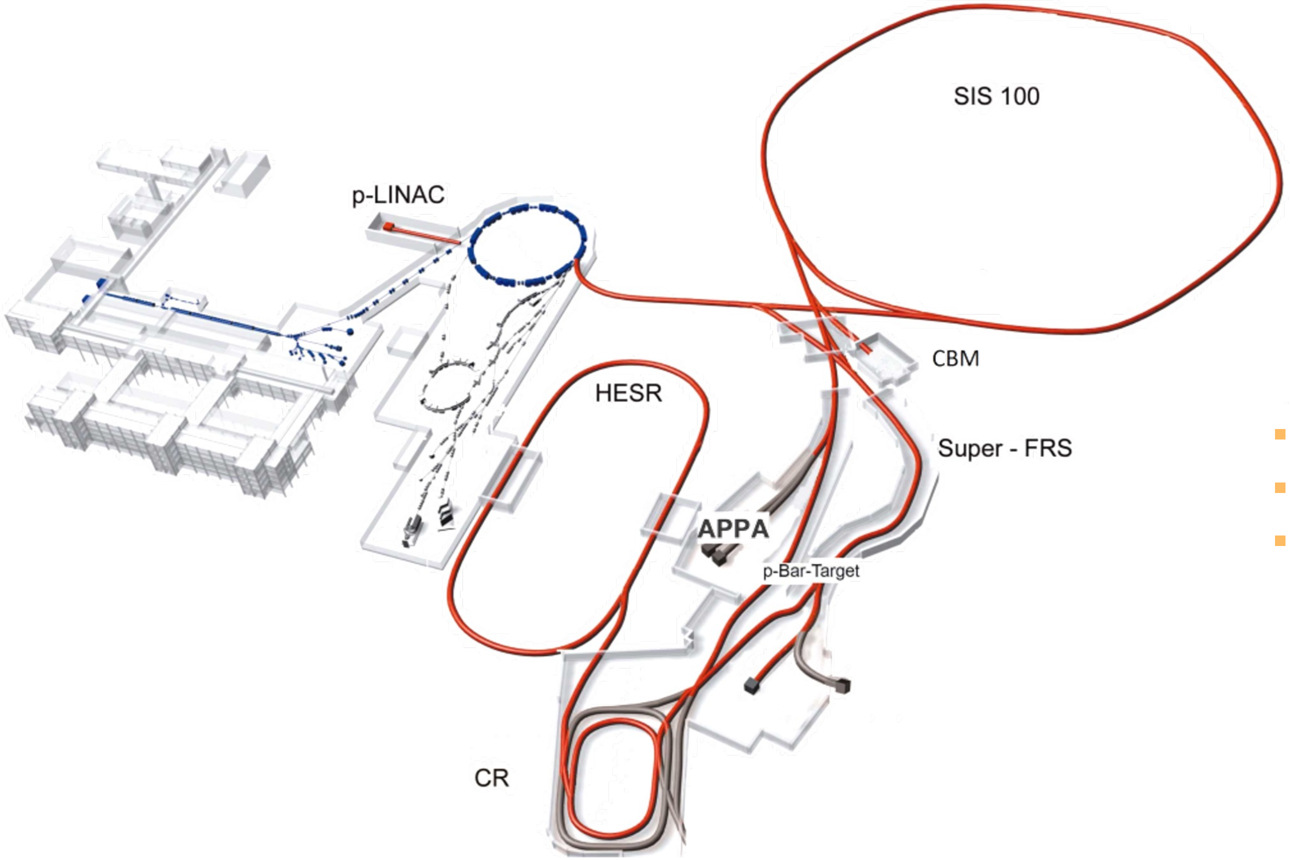
R. Jones, V. Kain, Y. Papaphilippou, and F. Zimmermann (CERN)

The operation team at CERN for their support at various stages of this study.

Thanks to Ivonne Leifels, Ralph Assmann, Kathrin Grosse (GSI)

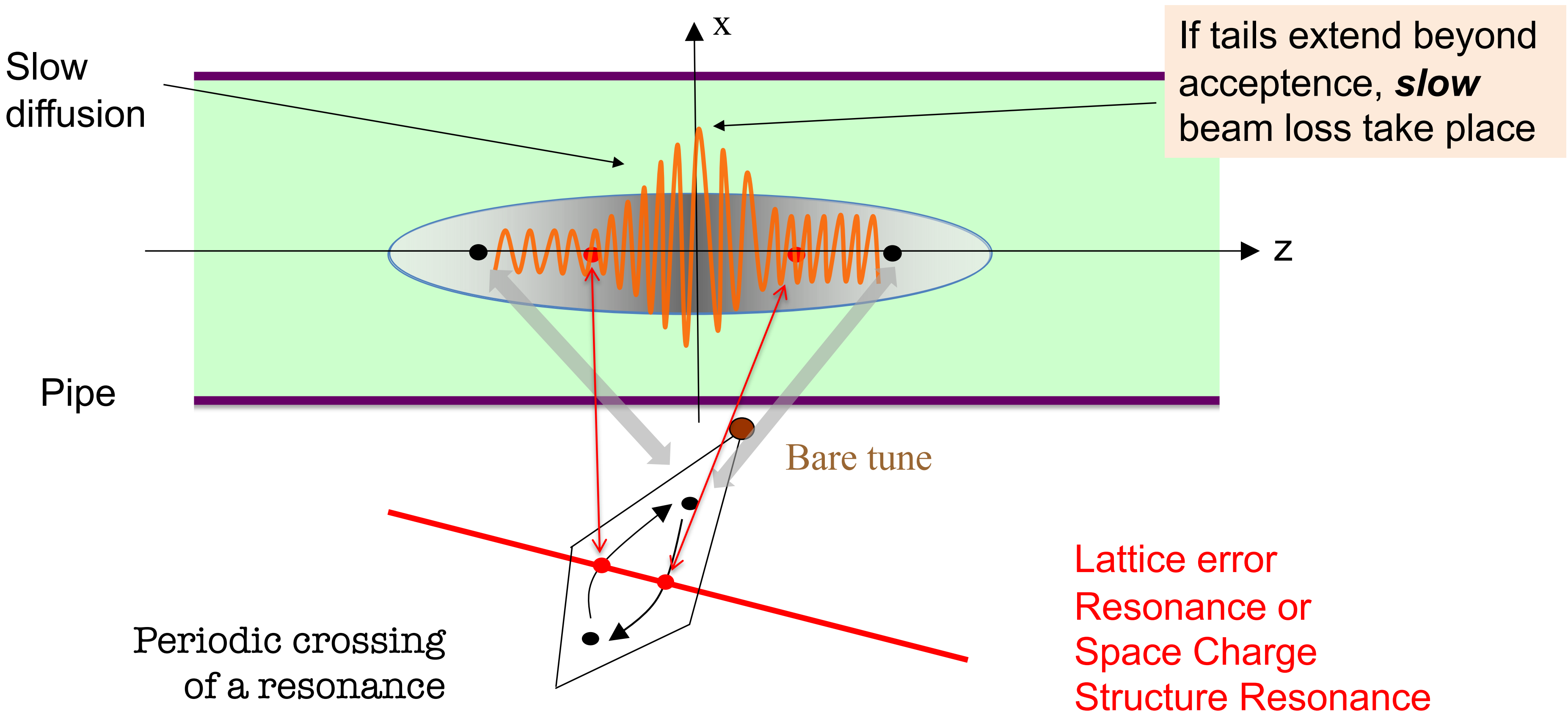
# Motivation: the FAIR project



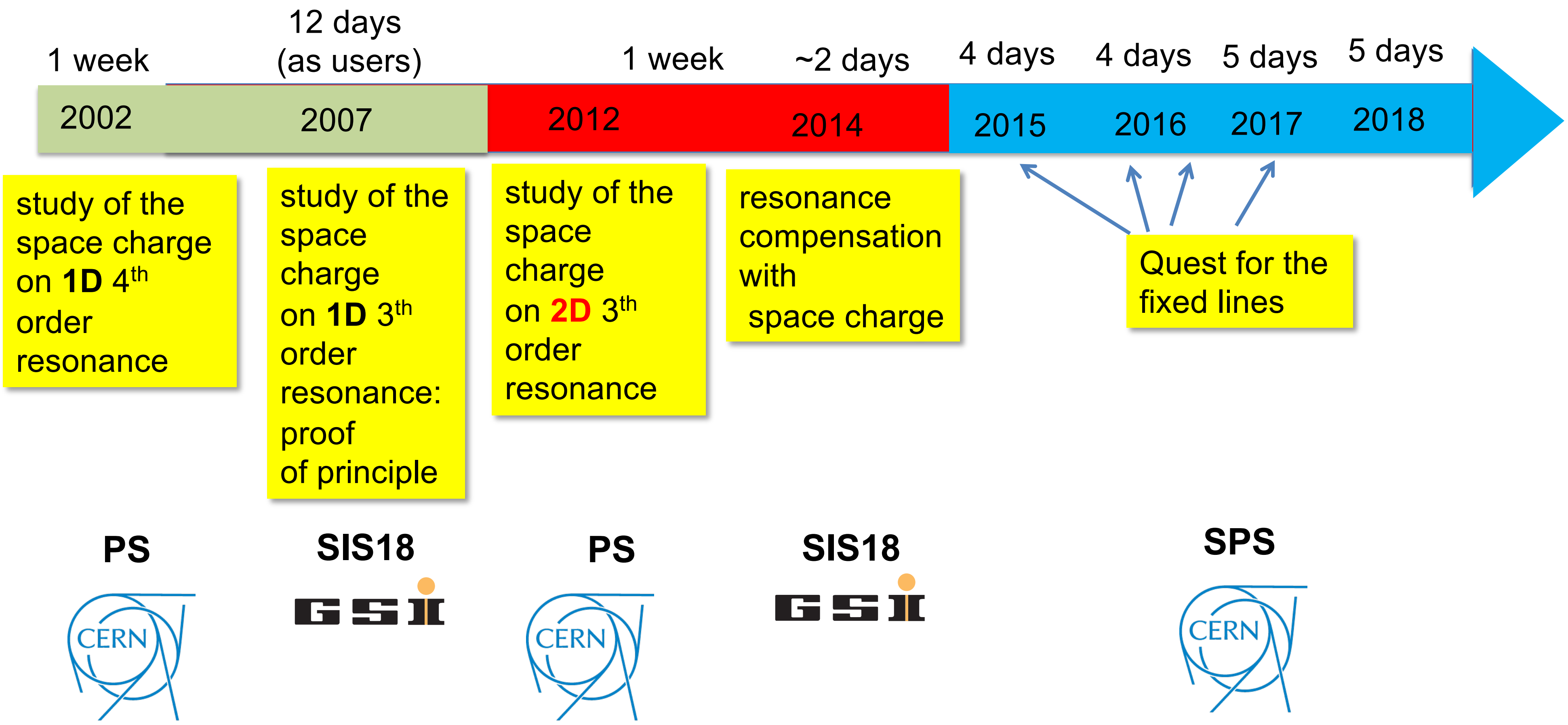

- SIS100 beam parameters:
- Every ion from p to U
- $U^{28+}$ -ions for RIB production:
  - $5x10^{11}$ / cycle
  - Rep. rate: 0.5 Hz
  - Energy: 400–2715 MeV/u

https://www.gsi.de/forschungbeschleuniger/fair

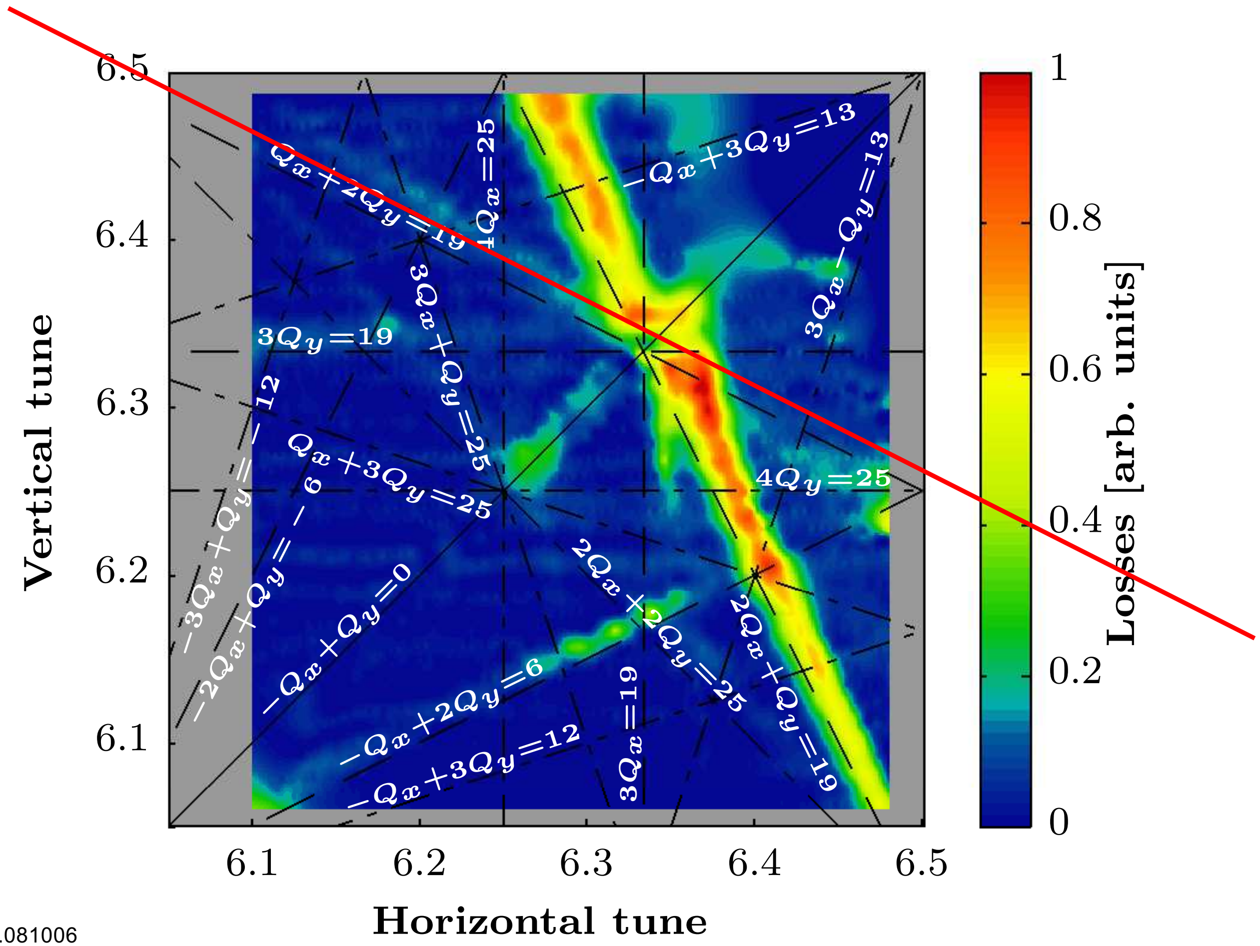
# Space charge and resonances



# The incoherent effects of space charge

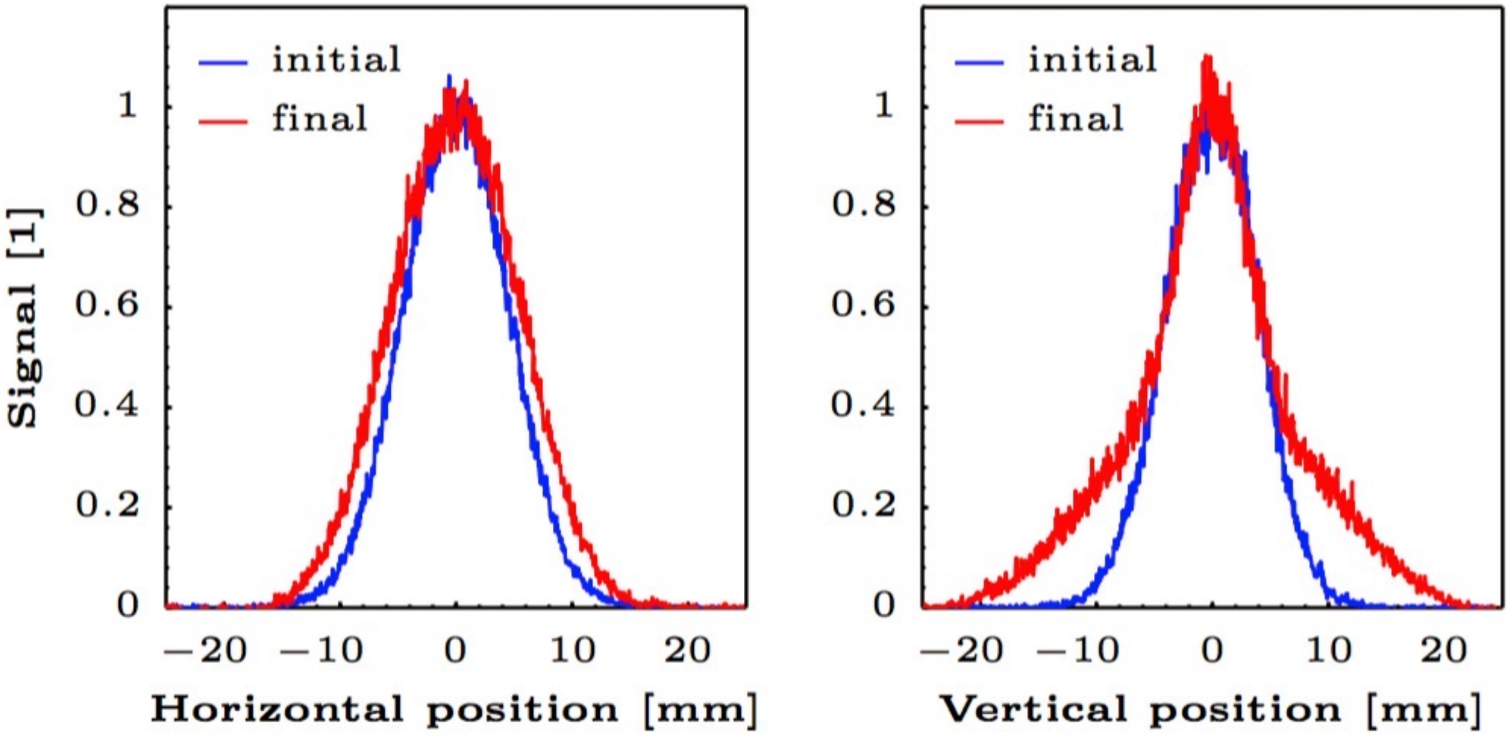


# The 2012 CERN-PS measurement campaign




Phys.Rev.Accel.Beams,vol.20,no.8,p.081006,
Aug. 2017. doi:10.1103/PhysRevAccelBeams.20.081006

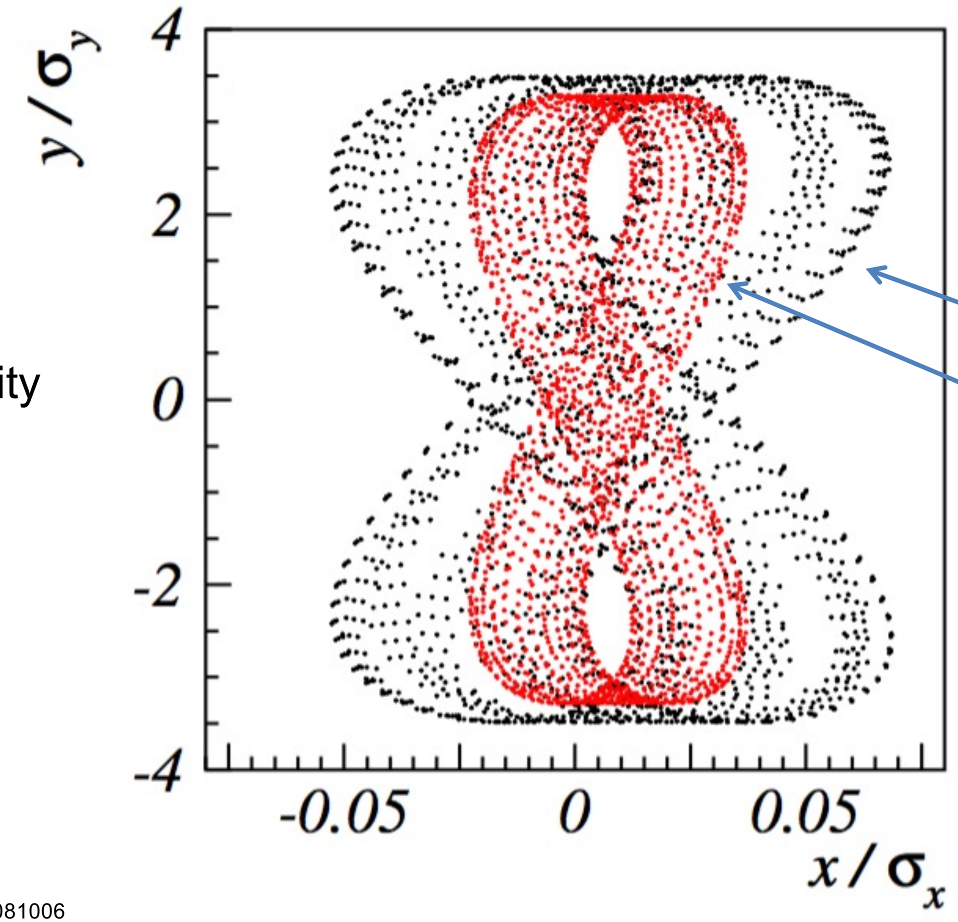
# Beam Profiles for $Q_{x0} = 6.104$




Phys.Rev.Accel.Beams,vol.20,no.8,p.081006,
Aug. 2017. doi:10.1103/PhysRevAccelBeams.20.081006

# Largest resonant orbits at $z/\sigma_z = 0$

No chromaticity




The two larger resonant orbits…

They have a structure of a Lissajous orbit

Phys.Rev.Accel.Beams,vol.20,no.8,p.081006,
Aug. 2017. doi:10.1103/PhysRevAccelBeams.20.081006

## The theory framework

$$H = H_0 + H_1$$ ← Nonlinear errors

↑ Quadratic

Solution of system with $H_0$ → $x = \sqrt{\beta_x \epsilon_x} \cos(\psi_x(s) + \phi_0)$

Solution of the perturbed system with H → $x = \sqrt{\beta_x a_x} \cos(\psi_x(s) + \varphi_x)$

New canonical dynamical variables

Instead of discussing x,x',y,y' we discuss $a_x, \varphi_x, a_y, \varphi_y$

# Resonant dynamics

$$-\tilde{a}'_x = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{\varphi}_x} = -2\sqrt{\tilde{a}_x}\tilde{a}_y \Lambda \sin(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha)$$

$$\tilde{\varphi}'_x = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{a}_x} = 2\frac{1}{2\sqrt{\tilde{a}_x}}\tilde{a}_y \Lambda \cos(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha) + t_x \frac{2\pi\Delta_r}{L}$$

$$-\tilde{a}'_y = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{\varphi}_y} = -4\sqrt{\tilde{a}_x}\tilde{a}_y \Lambda \sin(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha)$$

$$\tilde{\varphi}'_y = 2\frac{\partial \tilde{H}_{s1}}{\partial \tilde{a}_y} = 2\sqrt{\tilde{a}_x}\Lambda \cos(\tilde{\varphi}_x + 2\tilde{\varphi}_y + \alpha) + t_y \frac{2\pi\Delta_r}{L}$$

$\Lambda$ Is the amplitude driving term

$\alpha$ Is the phase of the driving term

$$t_x + 2t_y = 1$$

$$C = N_y \hat{a}_x - N_x \hat{a}_y$$

This is an invariant of motion
(in the slow harmonics approximation)

# Stationary solution

Resonance or fixed line → $\tilde{a}'_x = 0, \tilde{\varphi}'_x = 0, \tilde{a}'_y = 0, \tilde{\varphi}'_x = 0$

Back to Courant-Snyder coordinates, the resonant orbits in the Poincare' section

$$\hat{x}(t) = \sqrt{a_x}\cos(-2t - \alpha + \pi M),$$

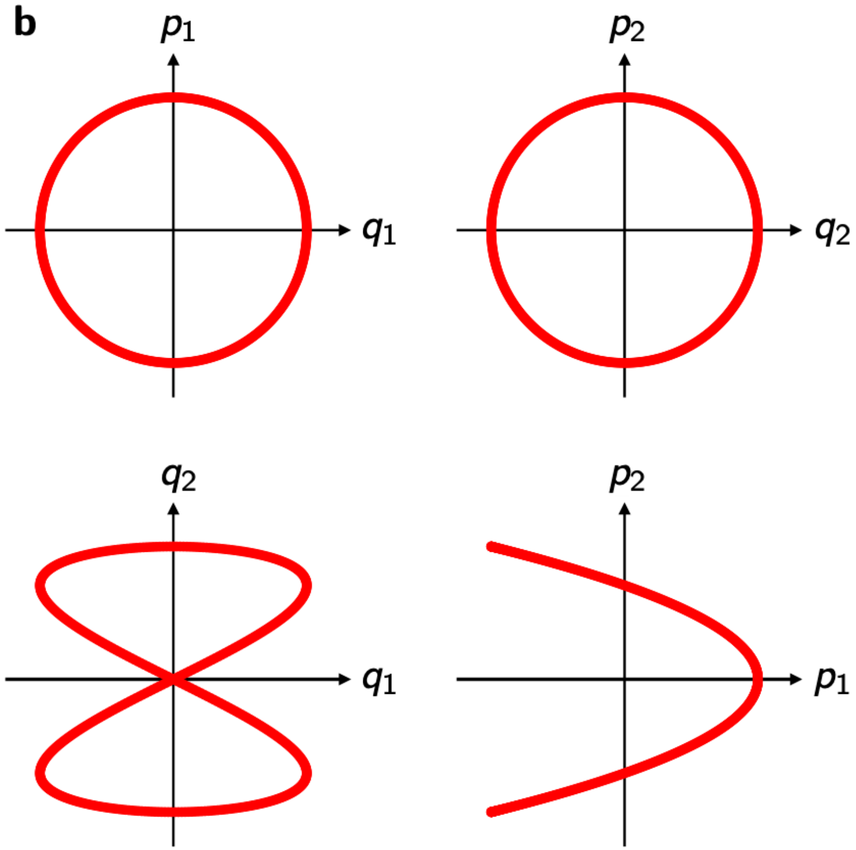
$$\hat{y}(t) = \sqrt{a_y}\cos(t),$$

$$\hat{p}_x(t) = -\sqrt{a_x}\sin(-2t - \alpha + \pi M),$$

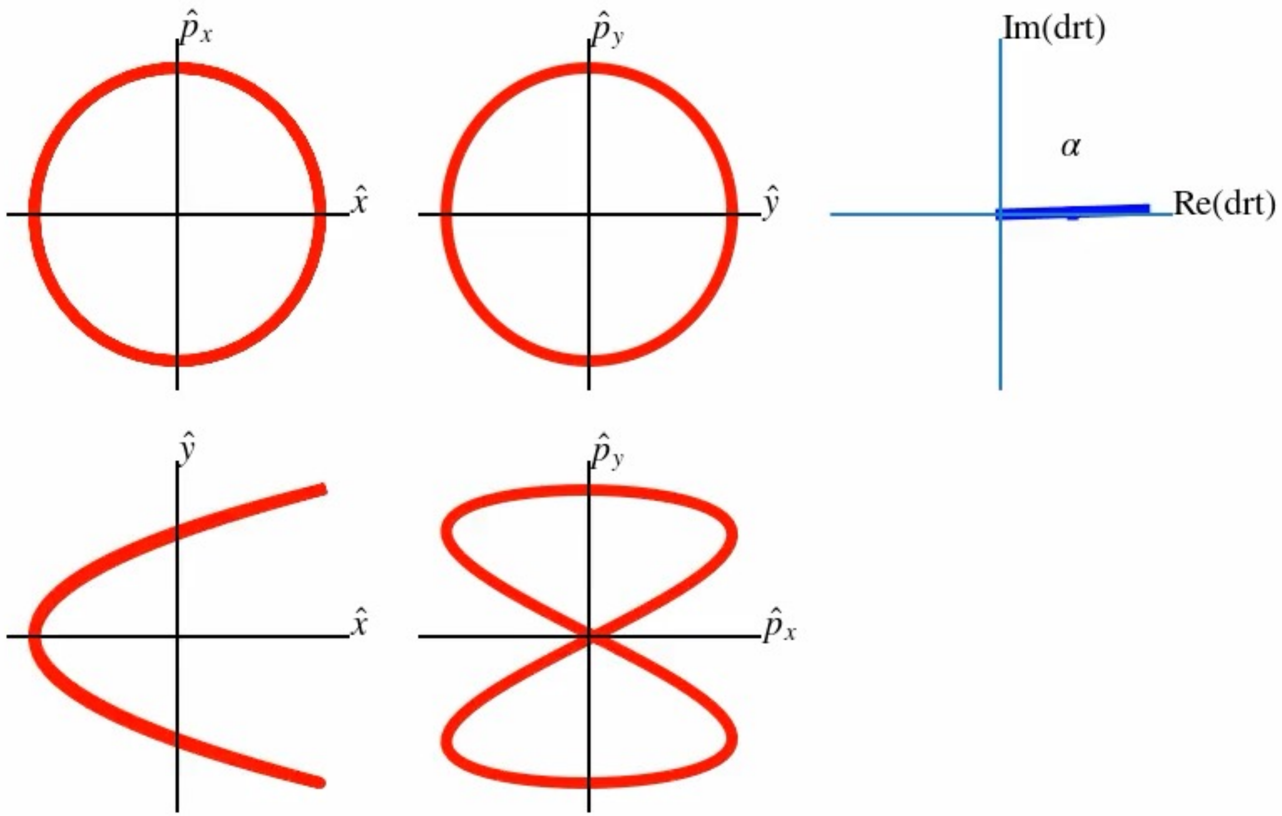
$$\hat{p}_y(t) = -\sqrt{a_y}\sin(t),$$

Phys. Rev. Lett., vol. 114, no. 23, p.234801, Jun. 2015.
doi:10.1103/PhysRevLett.114.234801

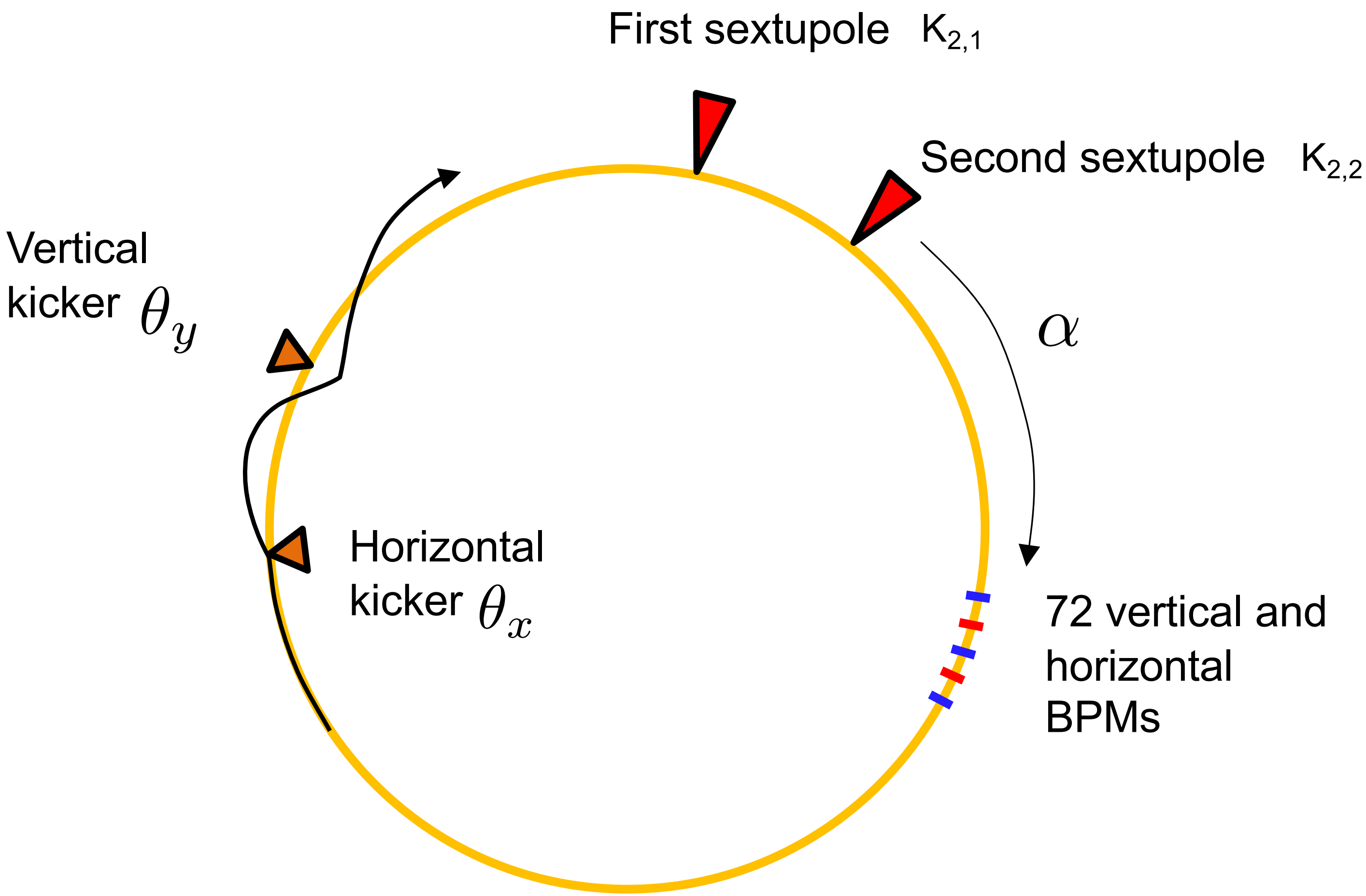
# Two-dimensional resonance



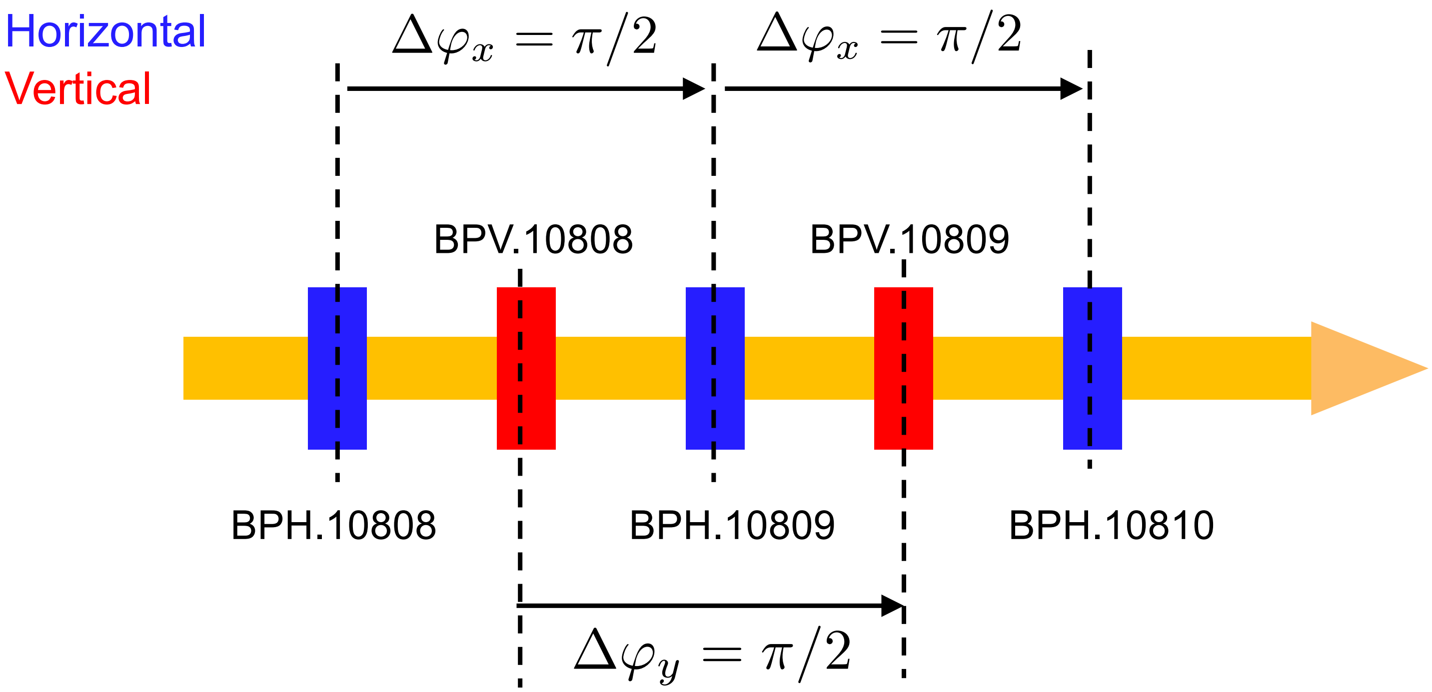

Nature Physics, vol. 20, no. 6, pp. 928–
933, 2024. doi:10.1038/s41567-023-02338-3


$\hat{p}_x$
$\hat{x}$
$\hat{p}_y$
$\hat{y}$
Im(drt)
$\alpha$
Re(drt)
$\hat{y}$
$\hat{x}$
$\hat{p}_y$
$\hat{p}_x$

# Experiment set-up

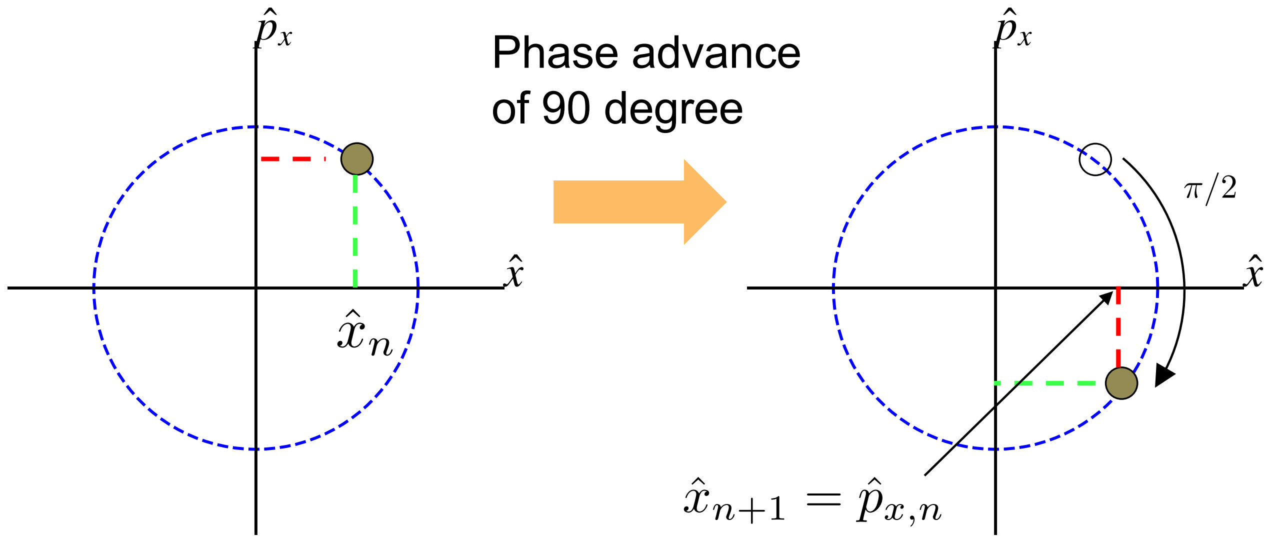


# Phase space reconstruction



# Phase space reconstruction


In Courant-Snyder coordinates



# First approach to find the fixed lines (2015)

1) Prepare a sextupoles setting to excite the resonance properly
2) Set the SPS ($Q_x$, $Q_y$) close to the resonance
3) Try kicking in x/y the beam with many attempts
4) Reconstructing the phase space, hunting the fixed line

Really hard, as the machine modeling always leaves out small Features of SPS

By knowing what we expect to see in phase space, we literally **hunted the fixed lines**

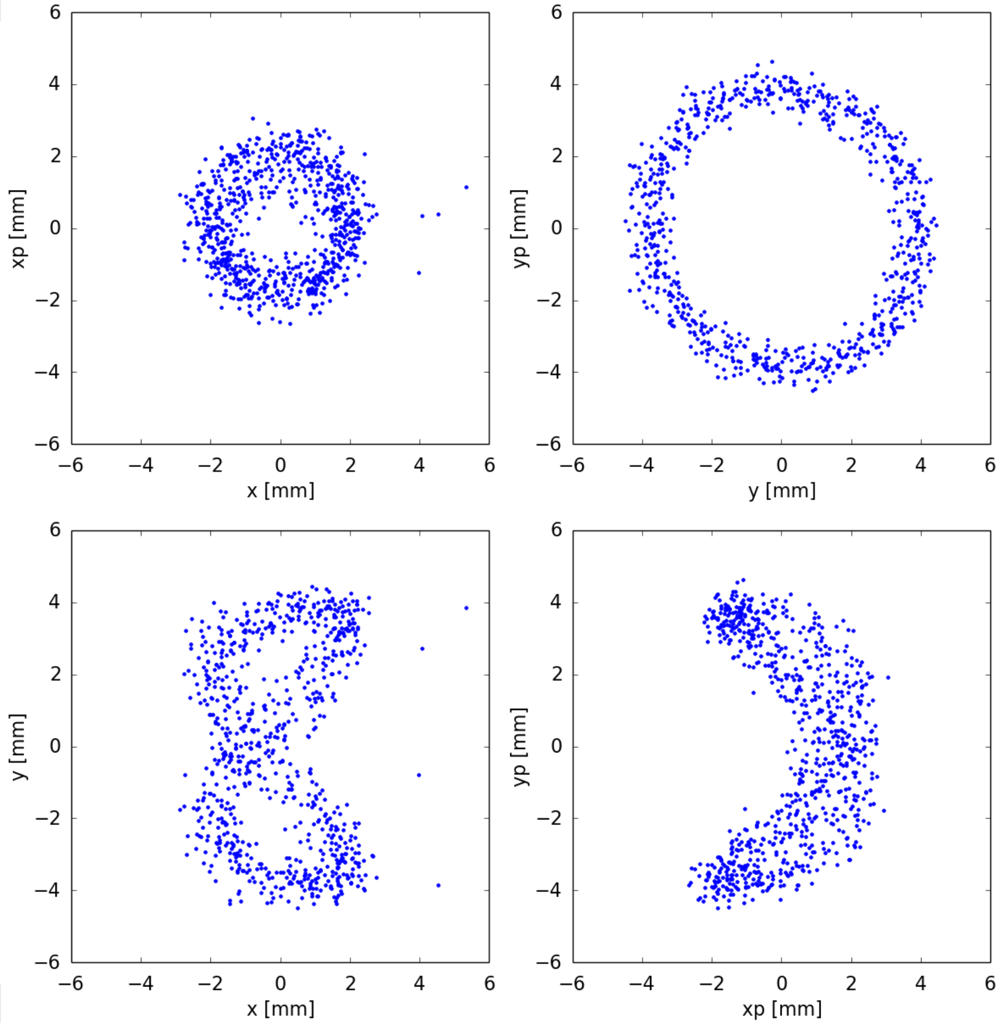
## SPS campaign on May 2015

# Measurement @ SPS

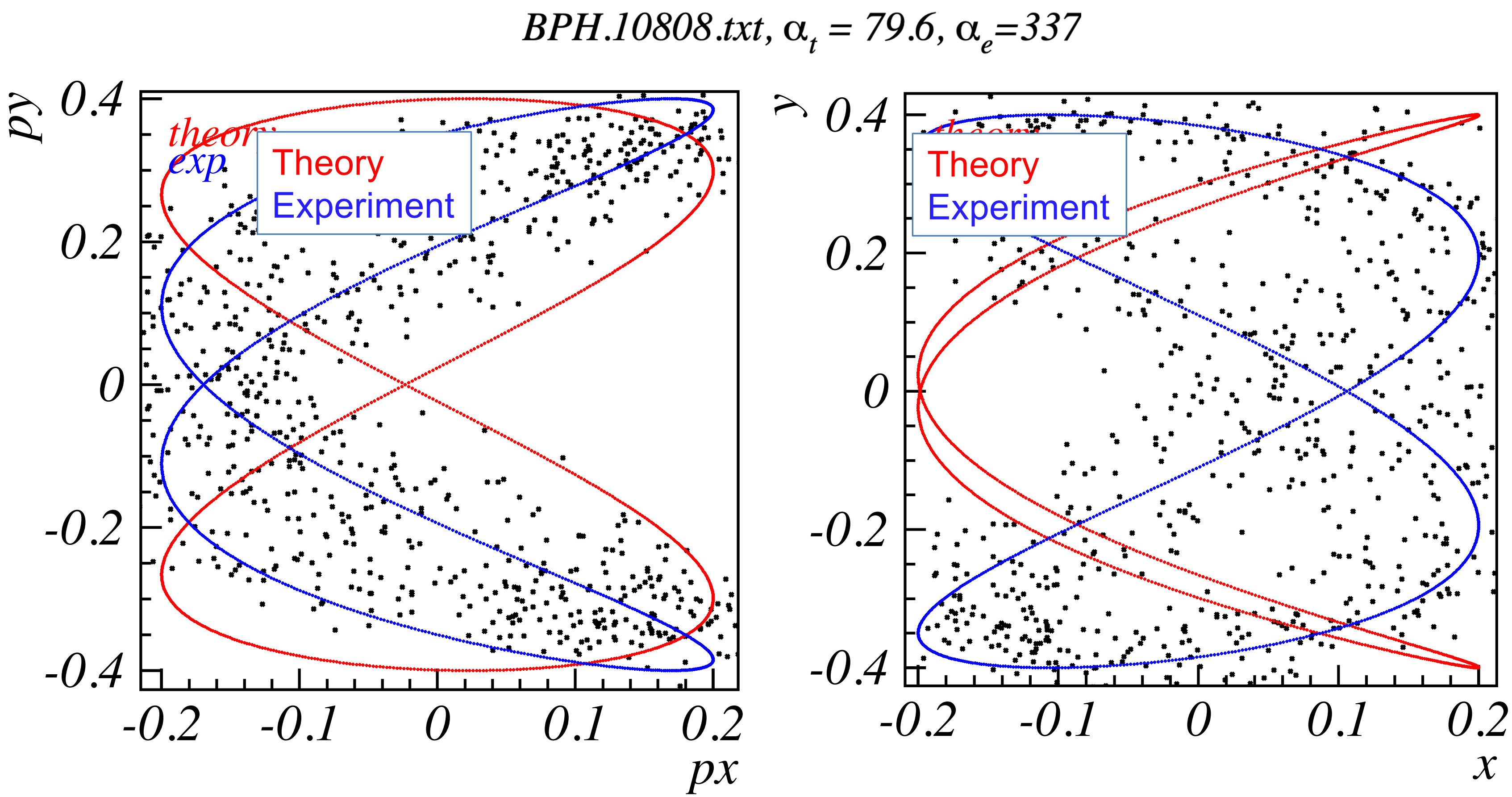
2015-05-24 00:03 –
start at BPM 20
(800 turns starting at 85)

Energy 26 GeV
$Q_x = 26.1113$
$Q_y = 26.4453$
$Q_x + 2\,Q_y = 79.001$
H kick 2 kV
V kick 6 kV

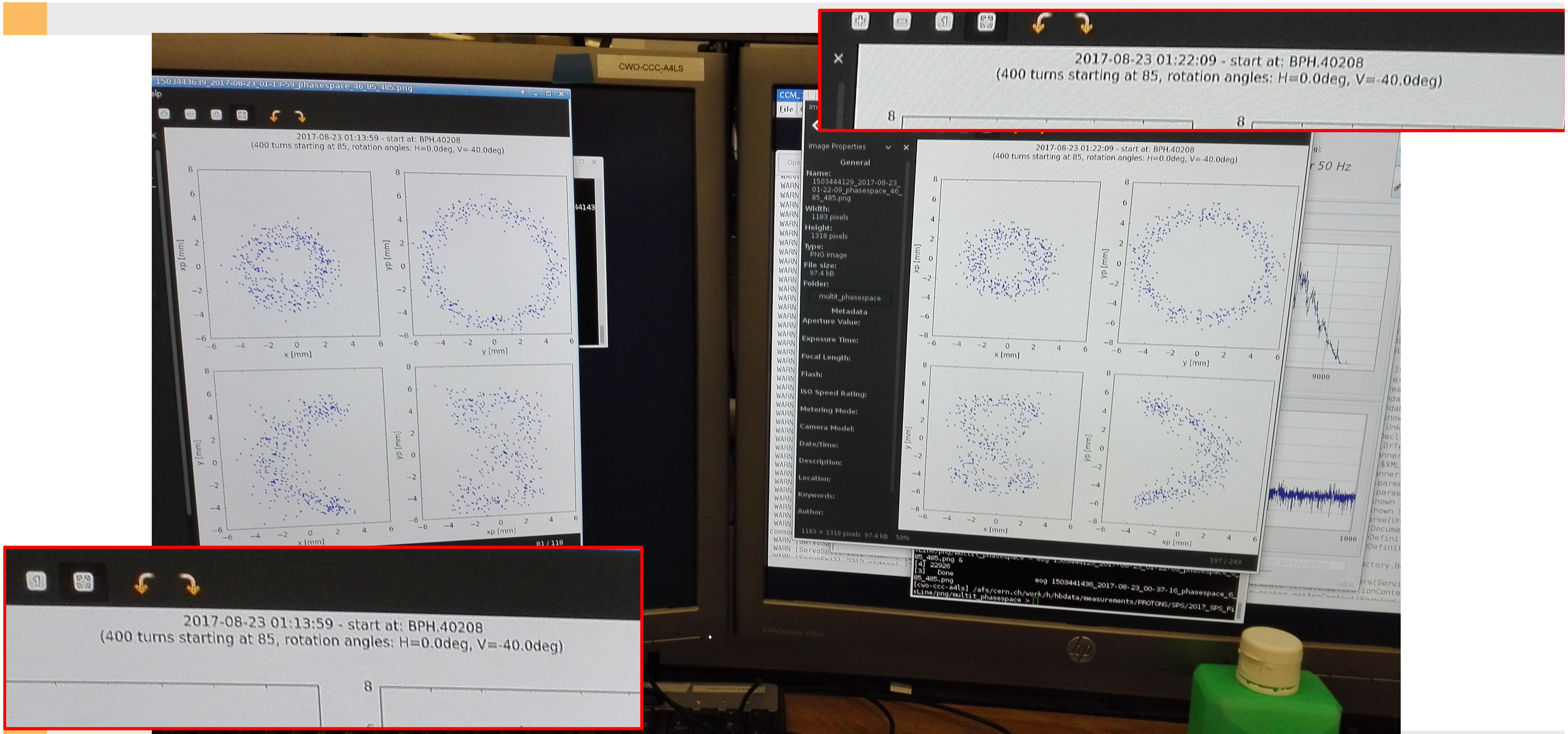
H. Bartosik,
G. Franchetti,
F. Schmidt,
M. Titze



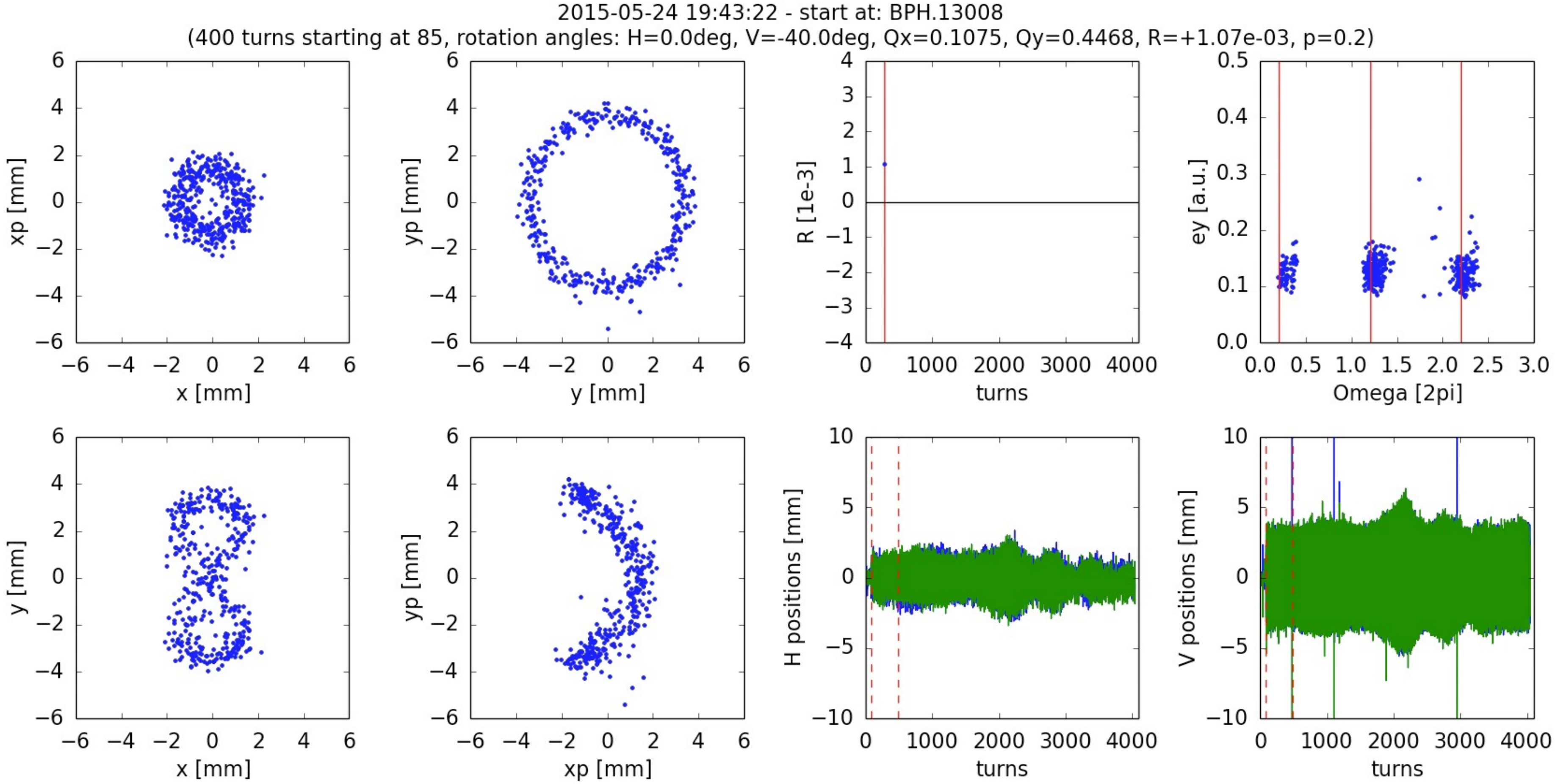
# Something was wrong…



# The campaign was repeated in 2017, but something was wrong again…



# What was wrong?



# Recognized difficulties

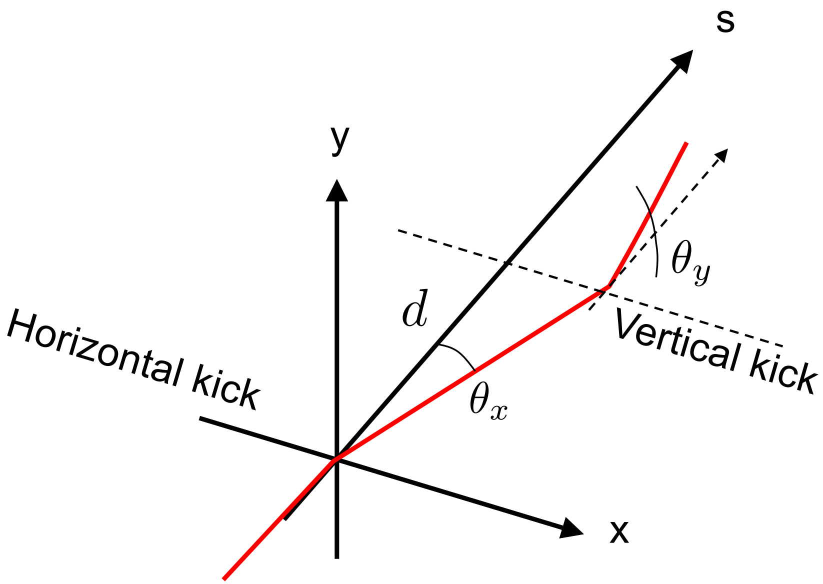
1. The effect is intrinsically fragile: tune modulation caused by power converter ripple and magnetic-field fluctuations.
   → Beam was accelerated to 100 GeV/c before excitation.
   Machine settings optimized.

2. Quadrupole tolerances generate up to ~5% “beta beating.”

3. The SPS provides only **one** horizontal and **one** vertical kicker suitable for the experiment

4. The synchronization of the kickers had to be carefully controlled.
   Only one kick in the correct order, and not multiple kicks…

# Kicking the beam onto a fixed line: can we?




After the vertical kick

$$(\theta_x, \theta_y) \rightarrow (d \tan\theta_x, \theta_x, 0, \theta_y)$$

This is a 2-dimensional surface embedded in the 4-dimensional phase space

The beam can be transferred onto a fixed line only if the angle $\alpha$ of the driving term is consistent with the combined effect of the kickers and the lattice between them

# New campaign strategy

The difficulty required a full exploration via scanning with LSA of

a) the sextupole strengths $K_{2,1}$,$K_{2,2;}$
b) the distance from resonance $\Delta_r$
c) the kicker amplitudes $\theta_x$, $\theta_y$.

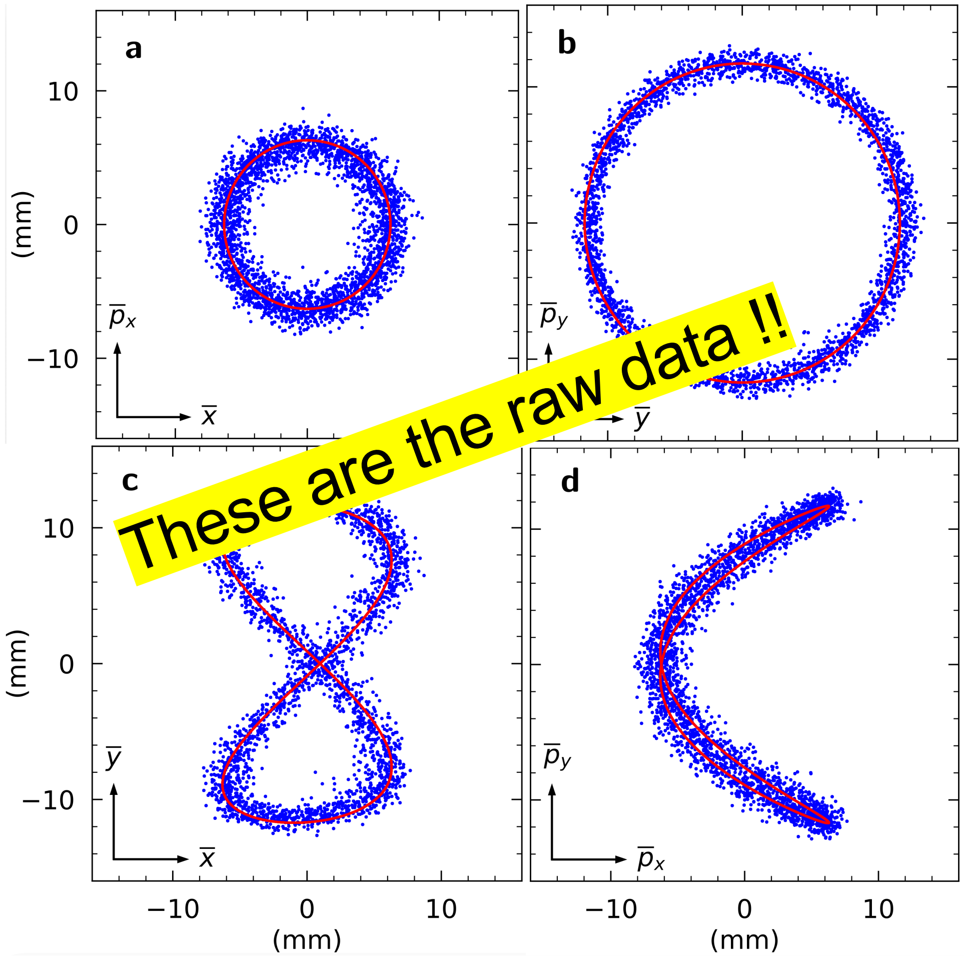
*Strategy employed:*

Kicking the beam on the fixed line: fixed lines were identified by scanning $\alpha$ to find suitable conditions.

Once evidence of a fixed line was found, scans of $\Delta_r$, sextupole strengths, and kicker amplitudes were performed.

For every measurement, turn-by-turn beam positions acquired from all available BPMs were recorded.

# Measurement of the orbit in the phase space




Nature Physics, vol. 20, no. 6, pp. 928–
933, 2024. doi:10.1038/s41567-023-02338-3

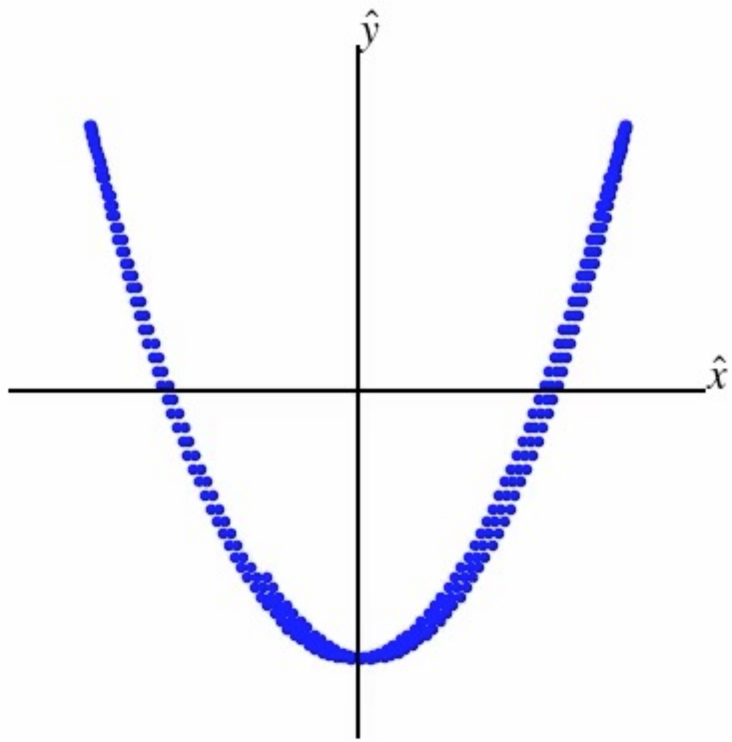
# Is it truly a fixed line?

Lissajous patterns happen even if there is no resonance

$$\hat{x} = \cos(2\pi Q_x n)$$
$$\hat{y} = \cos(2\pi Q_y n)$$

$$Q_x - 2Q_y = N$$

$$\Delta_r = 2 \times 10^{-4}$$



# Back to the dynamics

$$C = N_y \hat{a}_x - N_x \hat{a}_y$$

Defining

$$\Phi = \phi_x + 2\phi_y$$

$$\mathbf{a} = a_x$$

$$\mathsf{a}' = \frac{4\rho_s}{R} N_x \mathsf{a}^{n_x/2} \mathsf{a}_y^{n_y/2} \sin(\Phi),$$

$$\Phi' = \frac{2\rho_s}{R} \mathsf{a}^{n_x/2} \mathsf{a}_y^{n_y/2} \left( N_x \frac{n_x}{\mathsf{a}} + N_y \frac{n_y}{\mathsf{a}_y} \right) \cos(\Phi) + \frac{\Delta_{r0}}{R} + N_x \frac{d\mathsf{V}}{d\mathsf{a}}.$$

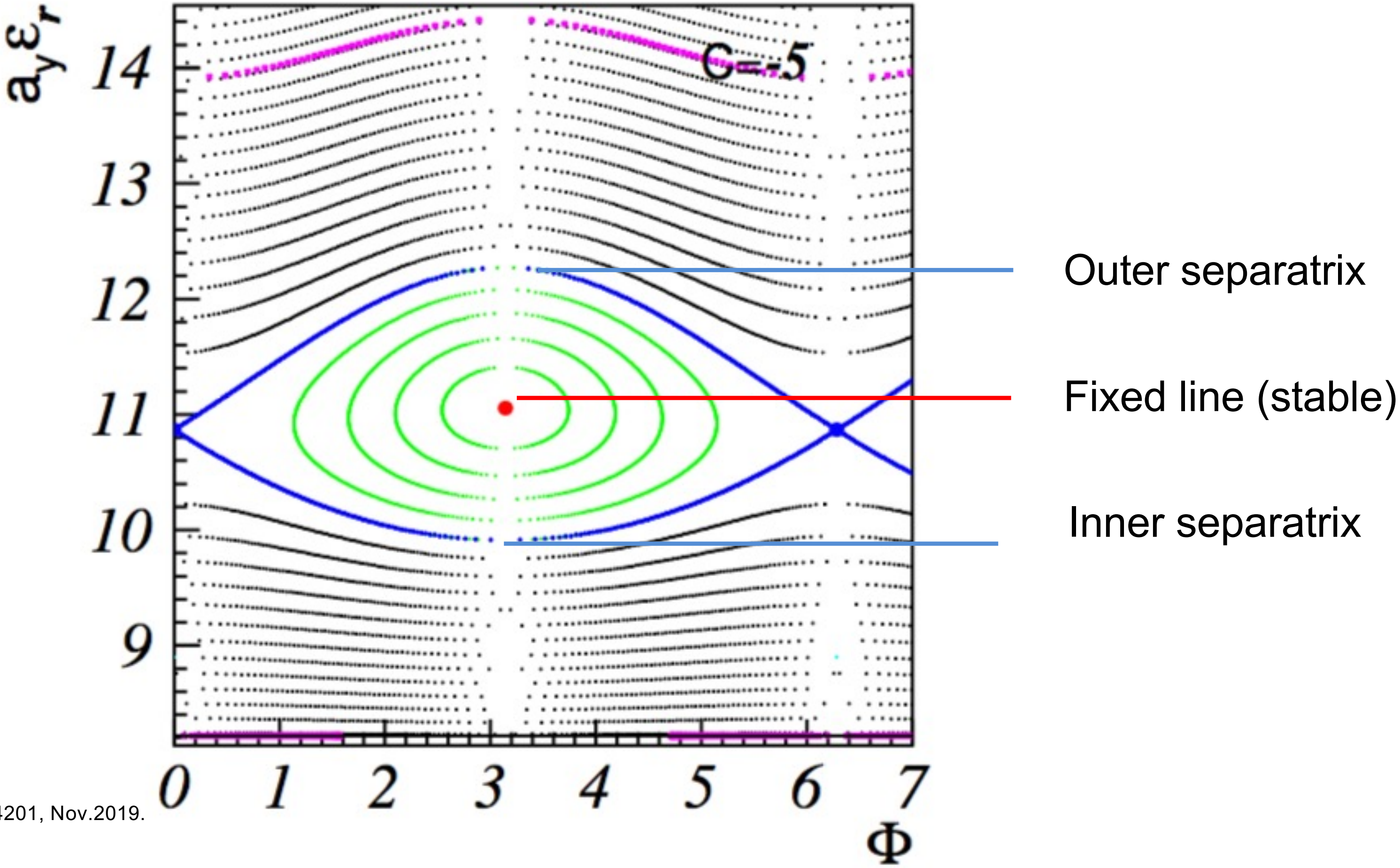
If a particle is not on the resonance $(\mathbf{a}, \Phi)$ oscillates

Phys. Rev. Accel. Beams, vol. 22, no. 11, p.114201, Nov.2019.
doi:10.1103/PhysRevAccelBeams. 22.114201

# Resonant dynamics in the space $(\mathbf{a}, \Phi)$

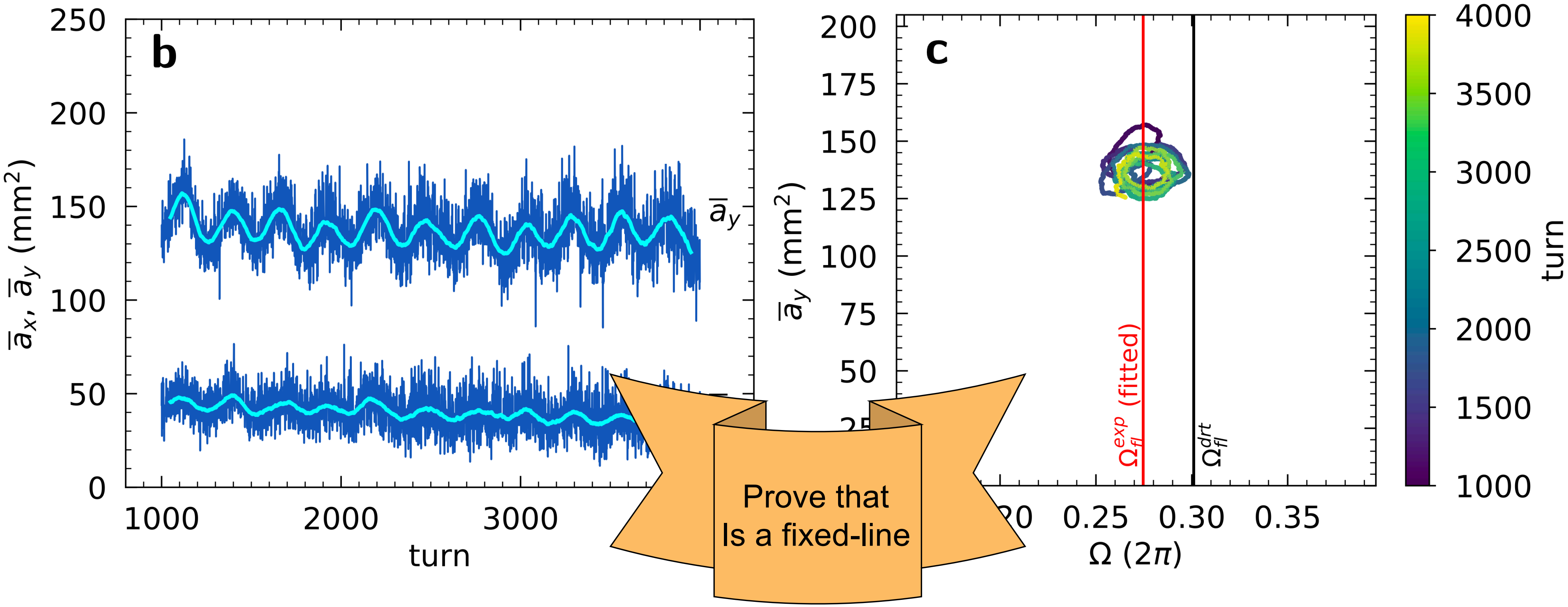
Simulation example:

Qx + 2Qy = 19



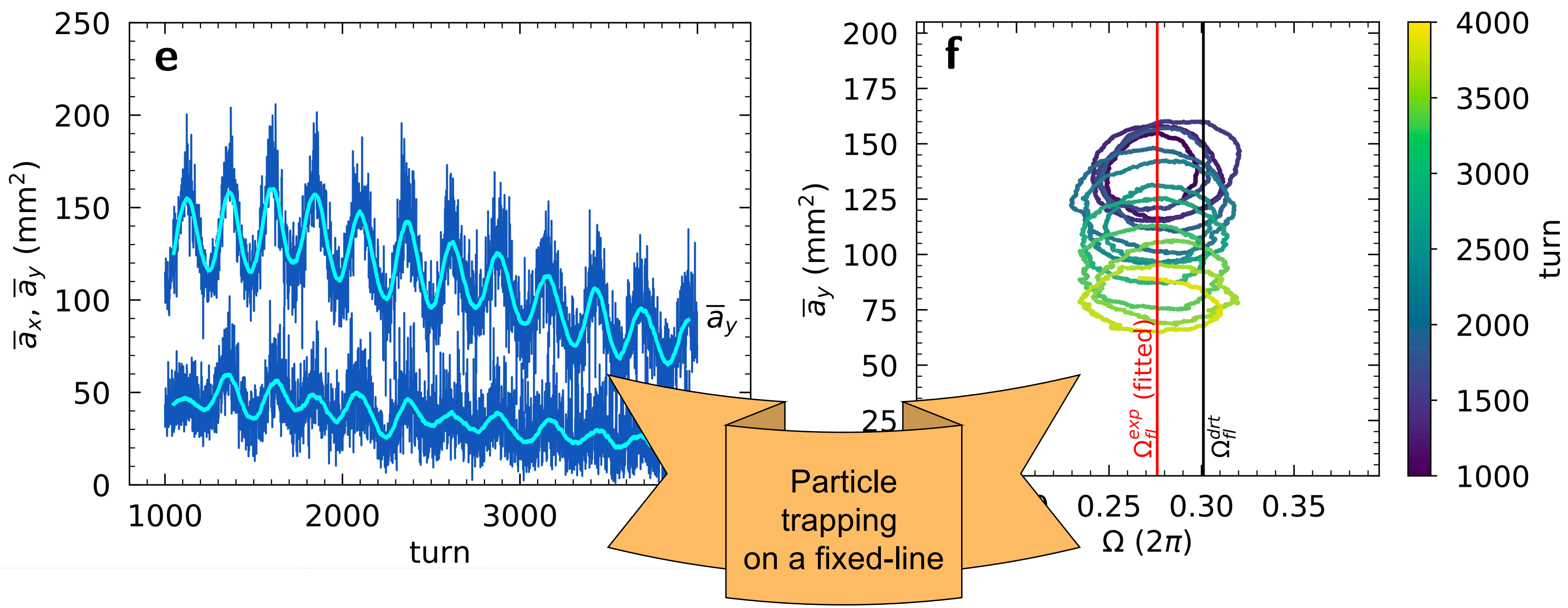

Phys. Rev. Accel. Beams, vol. 22, no. 11, p.114201, Nov.2019.
doi:10.1103/PhysRevAccelBeams. 22.114201

# From the measurements




Nature Physics, vol. 20, no. 6, pp. 928–933, 2024. doi:10.1038/s41567-023-02338-3

# From the measurements: unwanted drift

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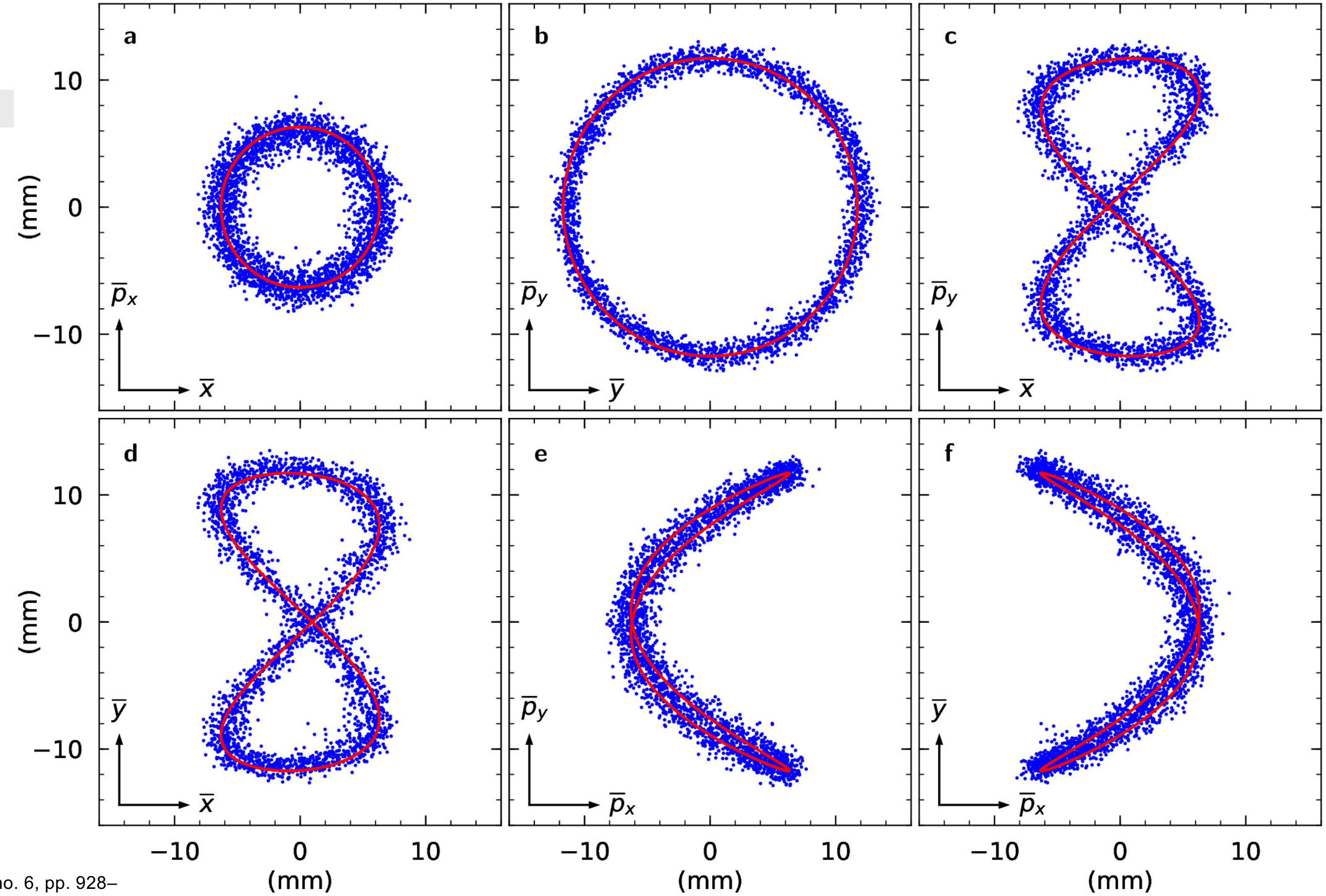


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# Outlook

Due to periodic resonance crossing, the particle diffuses along the invariant lines, → the edge of the halo formation can be predicted, and beam emittances can be optimized

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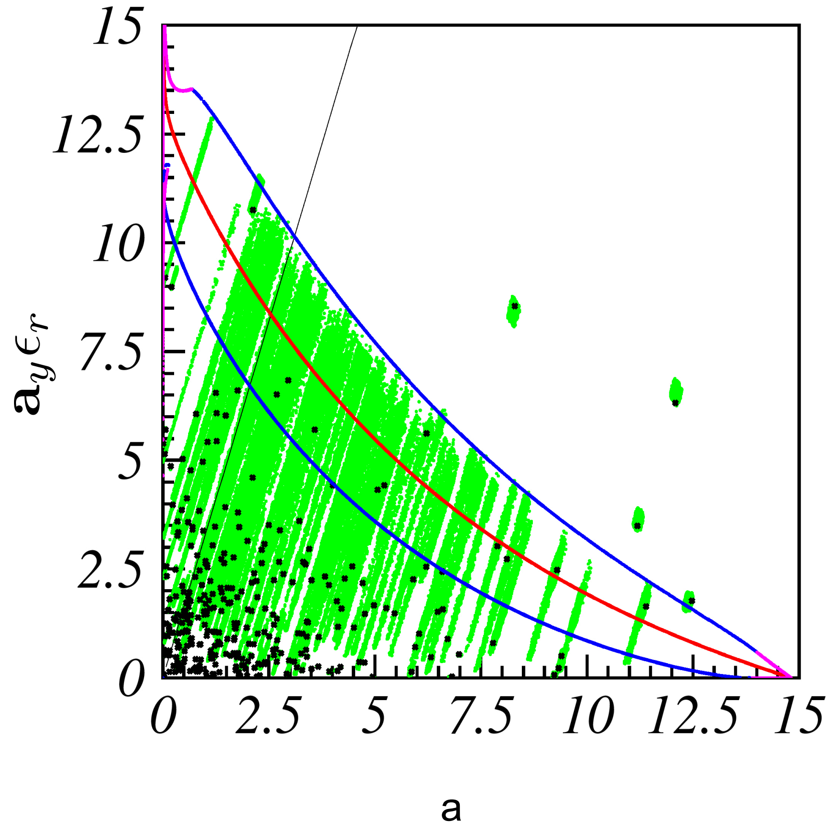


Advanced beam manipulation

Creating a cross-plane beam correlation using the fixed lines
→ see the work of Eleanor Lamb *et al.*

# Further acknowledgments

| | |
|---|---|
| GSI | O. Choriny, W. Bayer, O. Boine-Frankenheim, C. Omet, B. Franczak, P. Forck, T. Giacomini, I. Hofmann, F. Kesting, M. Kirk, H. Kollmus, T. Mohite, A. Parfenova, P. Schuett, P. Spiller |
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