HZB BESSY II Light Source

ON THE OPTIMIZATION OF THE NON-LINEAR PERFORMANCE OF 4TH-GENERATION LIGHT SOURCES

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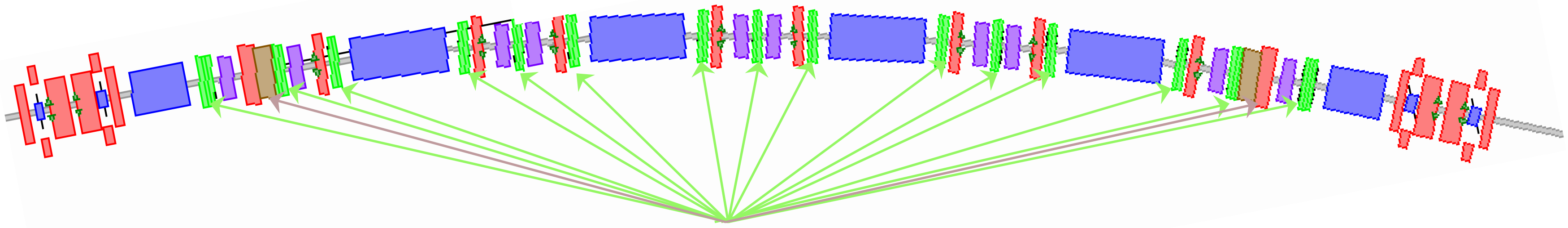
IPAC 2026, Deauville, May 20th, 2026

Cherish efficiency



1/16 of the BESSY III lattice
6-bend achromat with distributed sextupoles

4th generation storage ring light sources are usually built as **M**ulti **B**end **A**chromat lattices (MBA)

- MBA-lattices usually have many multipoles!
- proper linear lattice design can ease non-linear behaviour
  (phase matching, reverse bends, low sextupole strength) [1)]
- optimization of multipoles not trivial

Need to achieve:

- large momentum acceptance (MA) - long Touschek lifetime
- large dynamic aperture (DA) – large window for injection

1) B. Kuske and P. Goslawski, "Construction of multi-bend achromat lattices based on their substructures," Phys. Rev. Accel. Beams, 2026, accepted for publication.

2 approaches:

a) Multi-Objective Genetic Algorithms (MOGA) directly on DA and MA

‘brut force’ numeric optimization of the physical model (to best knowledge)

b) Driving Term Minimization (DTM)

Minimization effects of resonances, tune confinement

⟹ *Decide for the project phase which one to use*

Content of presentation:

- Brief introduction to driving terms
- Sketch of the DTM algorithm
- Results of DTM
- Results of MOGA calculations
- Comparison of both approaches

All calculations are performed for the BESSY III lattice (current design) and use *pyAT*[1)].

1): S. White, L. Carver, L. Farvacque, and S. Liuzzo, “Status and recent developments of python Accelerator Toolbox,” JACoW, vol. IPAC2023, WEPL031, 2023. doi:10.18429/JACoW-IPAC2023-WEPL031

After demanding mathematical treatment… [1)]

The nth order of the Hamiltonian of particle motion in a ring can be expanded as:
(using action angle variables)

$$h^{(n)} = \sum_{\substack{j,k,l,m,p>0 \\ j+k+l+m+p=n}} h_{jklmp} - (2J_x)^{\frac{(j+k)}{2}} e^{+i\phi_x(j-k)} (2J_y)^{\frac{(l+m)}{2}} e^{+i\phi_y(l-m)} \delta^p$$

Driving terms
amplitude dependence
phase dependence
energy dependence

h_{jklmp} :

- all DTs are **accumulating** sums over magnet data and linear lattice functions
- expressions can be cumbersome, see [1), 2), 3)]
- p = 0 $\Longrightarrow$ geometric DTs
- p $\neq$ 0 $\Longrightarrow$ chromatic DTs

$$\begin{aligned} h_{10201} = h^*_{01021} = \sum \Big\{ & \frac{i}{32}\bar{b}_{3i}\bar{b}_{2j}\sqrt{\beta_{xi}}\beta_{xj}\beta_{yi}\left[e^{i(\psi_{xi}+2\psi_{yi})} - e^{-i(\psi_{xi}-2\psi_{xj}-2\psi_{yi})}\right] \\ & -\frac{i}{16}\bar{b}_{3i}\bar{b}_{2j}\sqrt{\beta_{xi}}\beta_{yi}\beta_{yj}\left[e^{i(\psi_{xi}+2\psi_{yi})} - e^{i(\psi_{xi}+2\psi_{yj})}\right] \\ & +\frac{i}{16}\bar{b}_{3i}\bar{b}_{3j}\beta_{xi}\sqrt{\beta_{xj}}\beta_{yj}\eta_{xi}\left[e^{i(\psi_{xj}+2\psi_{yj})} - e^{i(2\psi_{xi}-\psi_{xj}+2\psi_{yj})}\right] \\ & +\frac{i}{8}\bar{b}_{3i}\bar{b}_{3j}\sqrt{\beta_{xj}}\beta_{yi}\beta_{yj}\eta_{xi}\left[e^{i(\psi_{xj}+2\psi_{yi})} - e^{i(\psi_{xj}+2\psi_{yj})}\right]\Big\}, \end{aligned}$$

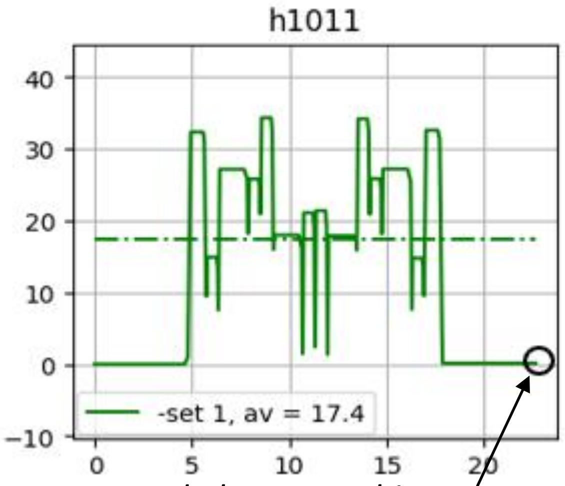
1) J. Bengtsson, “The sextupole scheme for the SLS,” 1997, http://slsbd.psi.ch/pub/slsnotes/sls0997.pdf
2) C.-X. Wang, Phys. Rev. ST Accel. Beams 12, 061001, (2009).
3) M. E. Arlandoo, “Transverse resonance island buckets in advanced light sources,” Ph.D. dissertation, Humboldt-Universität zu Berlin, 2024, doi:https://doi.org/10.18452/29179

$h^{(n)}{}_{jklmp}$:

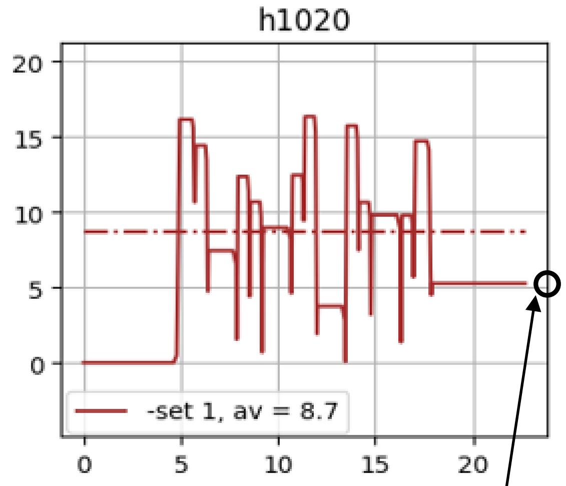
- can drive resonances, might also drive detuning, coupling, chromaticity…
- Phase dependent DTs might be cancelled by phase matching

Driving Term - examples	drives …
h_{10010}, h_{10100}	$Q_x - Q_y$, $Q_x + Q_y$
h_{11001}, h_{00111}	natural chromaticity
h_{30000}, h_{21000}, h_{10110}, h_{10020}, $h^{(2)}{}_{10200}$	3rd order resonances
h_{20001}, h_{00201}, h_{10002}	synchro-betatron resonances
h_{22000}, h_{00220}, h_{11110}	tune shift with amplitude
h_{11002}, h_{00112}	tune shift with momentum

Accumulating sum of h_{10110} and h_{10200} over one arc:



good phase matching => cancellation



bad phase matching => accumulation over ring

Minimization of driving terms: which ones?

- Restrict to: 3rd order-resonance DT
  detuning terms (TSWA, TSWM)
- Group them in 4 ‘**classes**’ of equal physical impact
- Take rms-value of each class

Two weight functions:

1. Weight function[1)] to leverage between class magnitudes
2. Option for factorial weights (factor) for individual DTs/classes
   Apply to TSWM: W = 1, 50, 100, …, 400 – 9 values

Optimization goal:

Goal: sum of 4 class rms values

DT of “HOA-case”

type	no	impact	rms value	using weight function
geometric	5	3rd order resonances	3.73	0.086
chromatic	3	sychro-betatron resonances	1.18	0.257
TSWA	3	tune shift with amplitude	118424.0	14.46
TSWM	2	tune shift with momentum	27.26	0.324
goal (sum)				15.13

1) A. Streun, OPA Documentation: Inside OPA (in-side1.pdf), https://github.com/opa-code/opa4-documentation/blob/main/inside1.pdf, Eq. 71, Accessed:2026-02-28, 2024.

Algorithm: COBYLA[1)]

Python algorithm, constraints to fix chromaticity

- Exploit potential of 8 individual sextupole families
- use 6 discrete octupole values
  O = 0, 100, …, 500 m^{-3}
- start each minimization from **'HOA-case'**:
  2 families of chromatic sextupoles, phase matching, $\xi_{x,y}=1$
- include limit on sextupole strength:
  deviation from HOA-case: 10%-40%

9 weights * 6 octupole strengths * 4 limits

⟹ 216 cases

⟹ parallelisation on HPC

~30 min / case (# minimization steps/lattice size)

1) M. J. D. Powell, *A direct search optimization method that models the objective and constraint functions by linear interpolation*, 1994.
(SciPy documentation, https://docs.scipy.org/doc/scipy/index.html, Accessed: May 08, 2025 Version: 1

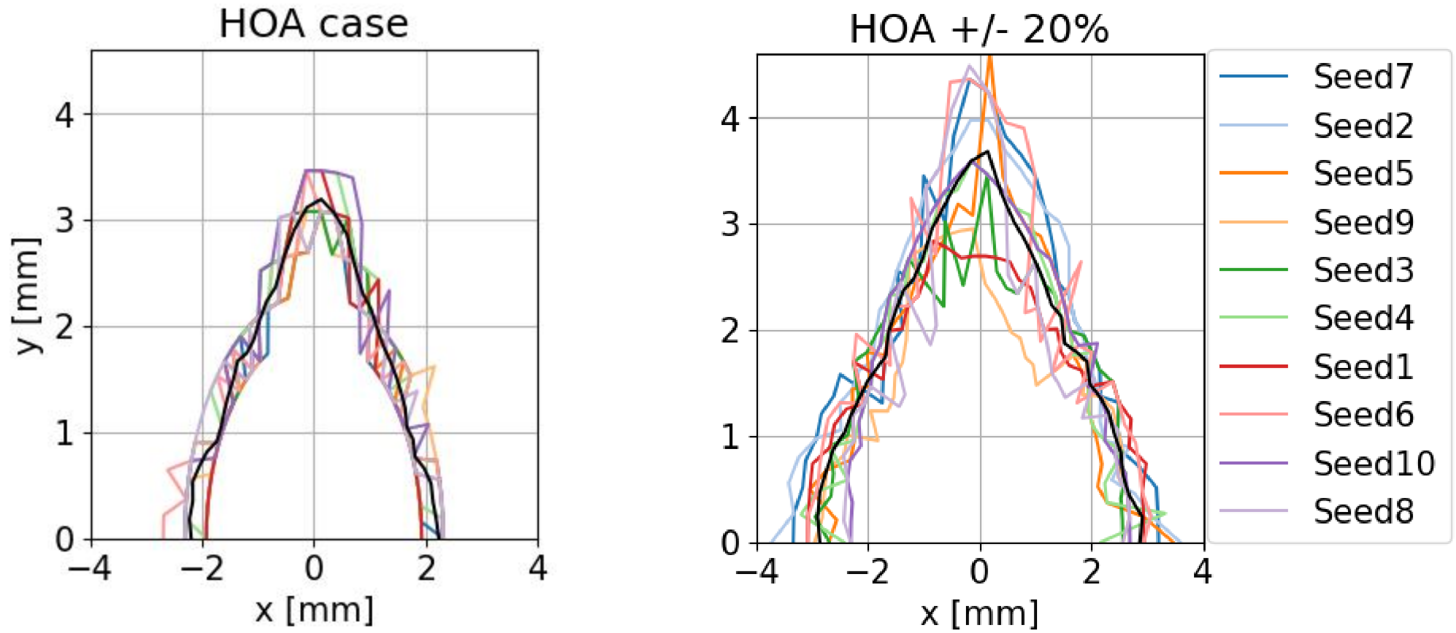
Error type	rms value
transverse misalignment (all magnets)	30 µm
roll (all magnets)	180 µrad
pitch and yaw - dipoles	60 µrad
pitch and yaw - other magnets	400 µrad
calibration	1e-4

Example 1:

a) Track HOA-case for 10 sets of magnet tolerances

b) Apply DTM allowing for 20% deviation in sextupole strength

Dynamic aperture (DA) of the BESSY III lattice for 10 sets of errors
(corrected closed orbit, no beta beat correction).



HOA-case, insensitive to errors.

DA when allowing for a 20 % deviation of sextupole strengths during DTs minimization.

Seek compromise between robustness against errors and sufficient DA and MA

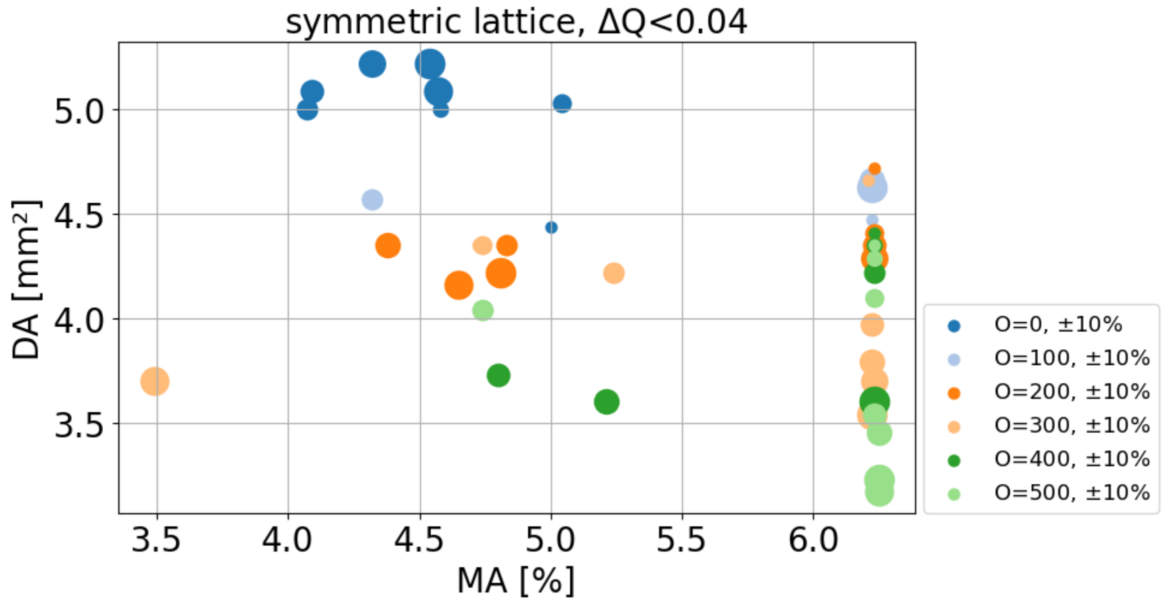
After minimization:

⟹ 216 sets of 8 sextupole and 1 octupole strengths

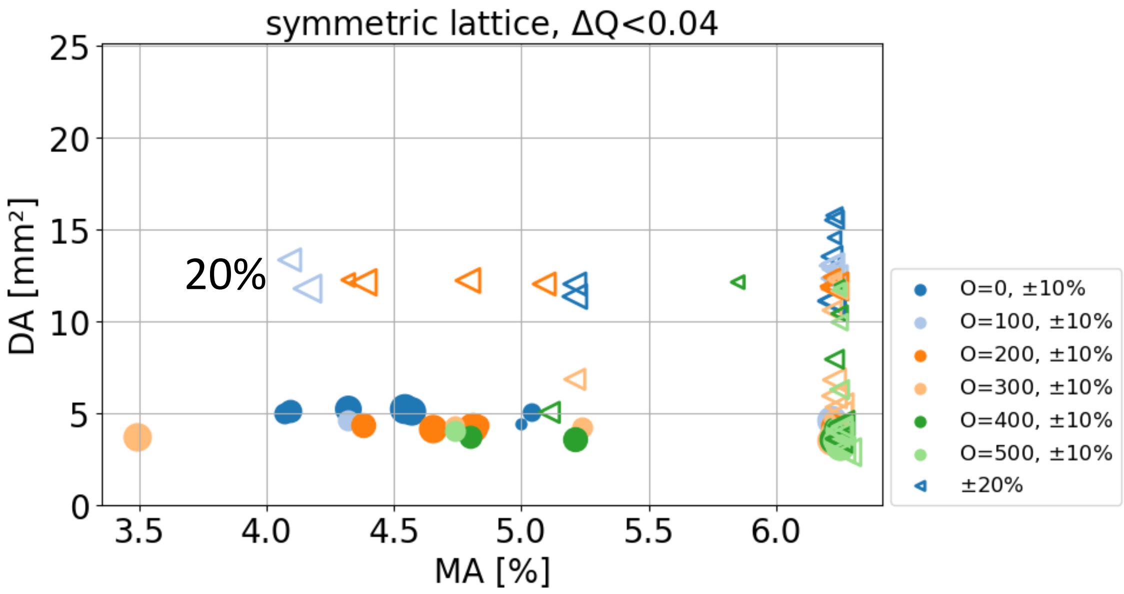
- calculate DA:
  - 6D-tracking, DA represented by its area
  - 1024 turns, no errors
  - mimic particle loss by limiting the particles tune shift $\Delta Q_{x,y} < 0.04$
- calculate MA
  - Local MA in straight section
  - apply tune shift criterion $\Delta Q_{x,y} < 0.04$

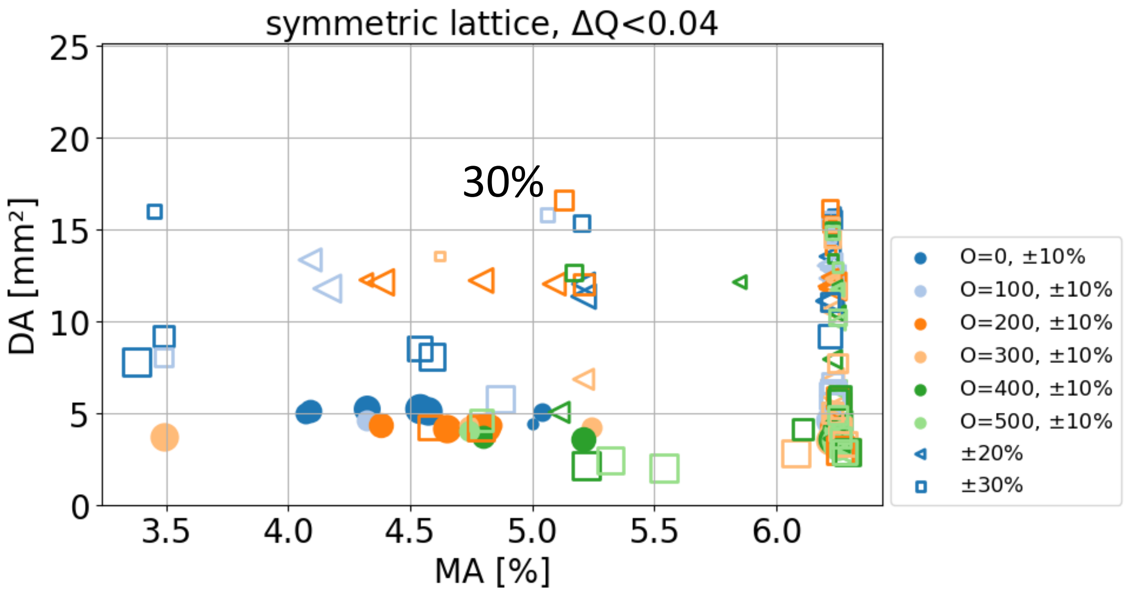
2. Example: symmetric lattice

- 10% deviation from HOA-case allowed for sextupoles
- bullet size marks weights
- colours represent octupole strength
  - DA small, decreases with increasing octupole
  - MA increases to RF acceptance

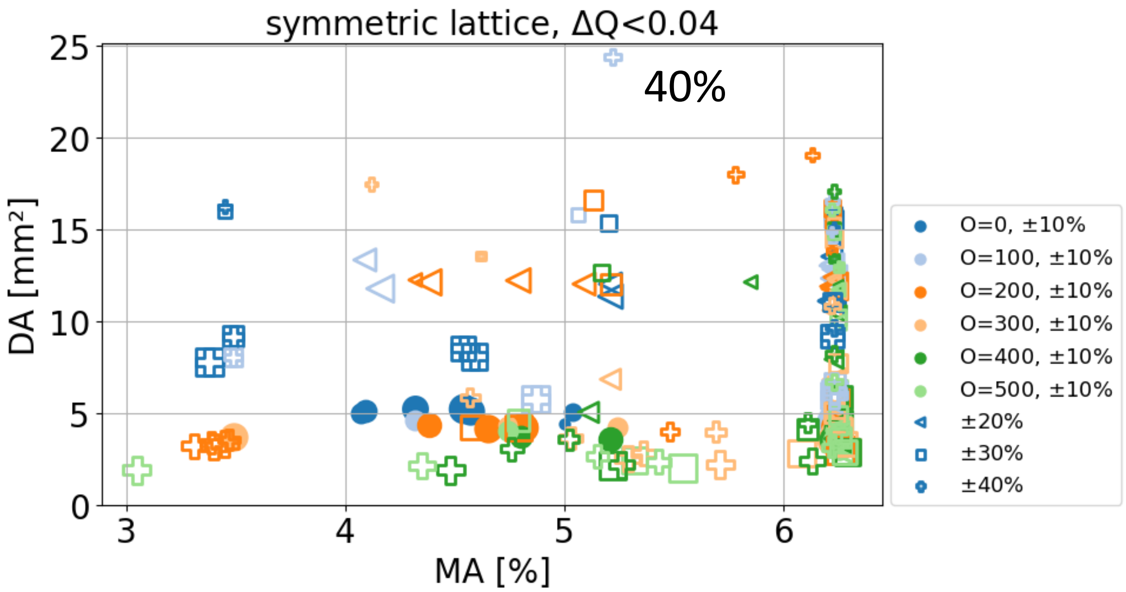


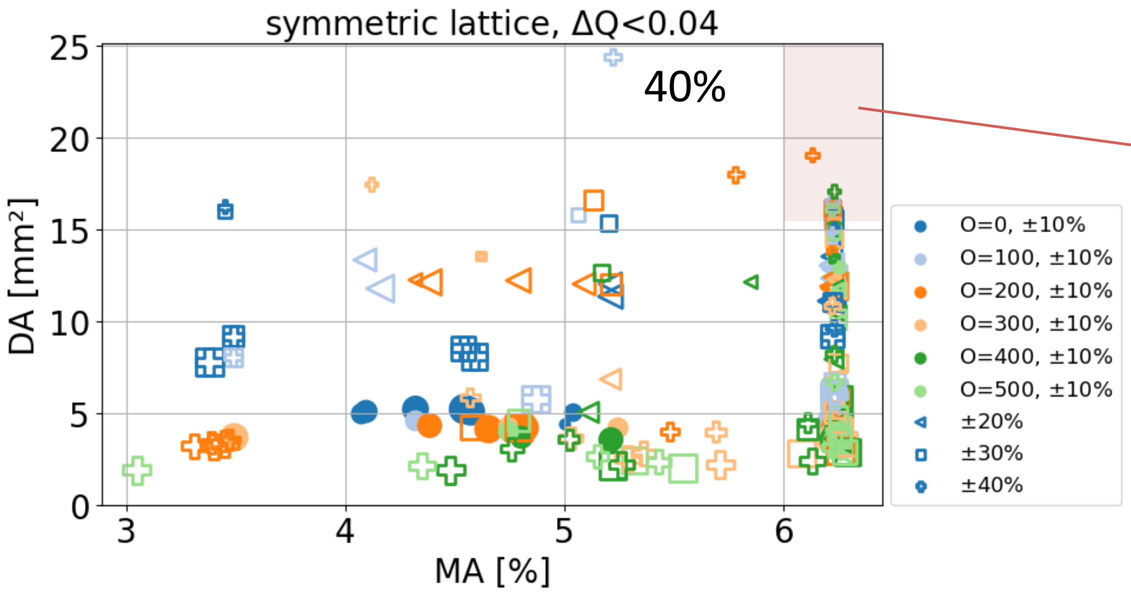
HZB BESSY II Light Source


symmetric lattice, ΔQ<0.04
DA [mm²]
MA [%]
20%
O=0, ±10%
O=100, ±10%
O=200, ±10%
O=300, ±10%
O=400, ±10%
O=500, ±10%
±20%



symmetric lattice, ΔQ<0.04
30%
DA [mm²]
MA [%]
25
20
15
10
5
0
3.5
4.0
4.5
5.0
5.5
6.0
O=0, ±10%
O=100, ±10%
O=200, ±10%
O=300, ±10%
O=400, ±10%
O=500, ±10%
±20%
±30%


symmetric lattice, ΔQ<0.04
40%
DA [mm²]
MA [%]
25
20
15
10
5
0
3
4
5
6
O=0, ±10%
O=100, ±10%
O=200, ±10%
O=300, ±10%
O=400, ±10%
O=500, ±10%
±20%
±30%
±40%



definition of ‘best’ cases (somewhat ‘arbitrary’):

- MA > 6%, 10 largest DAs

Error statistics for symmetric lattice:

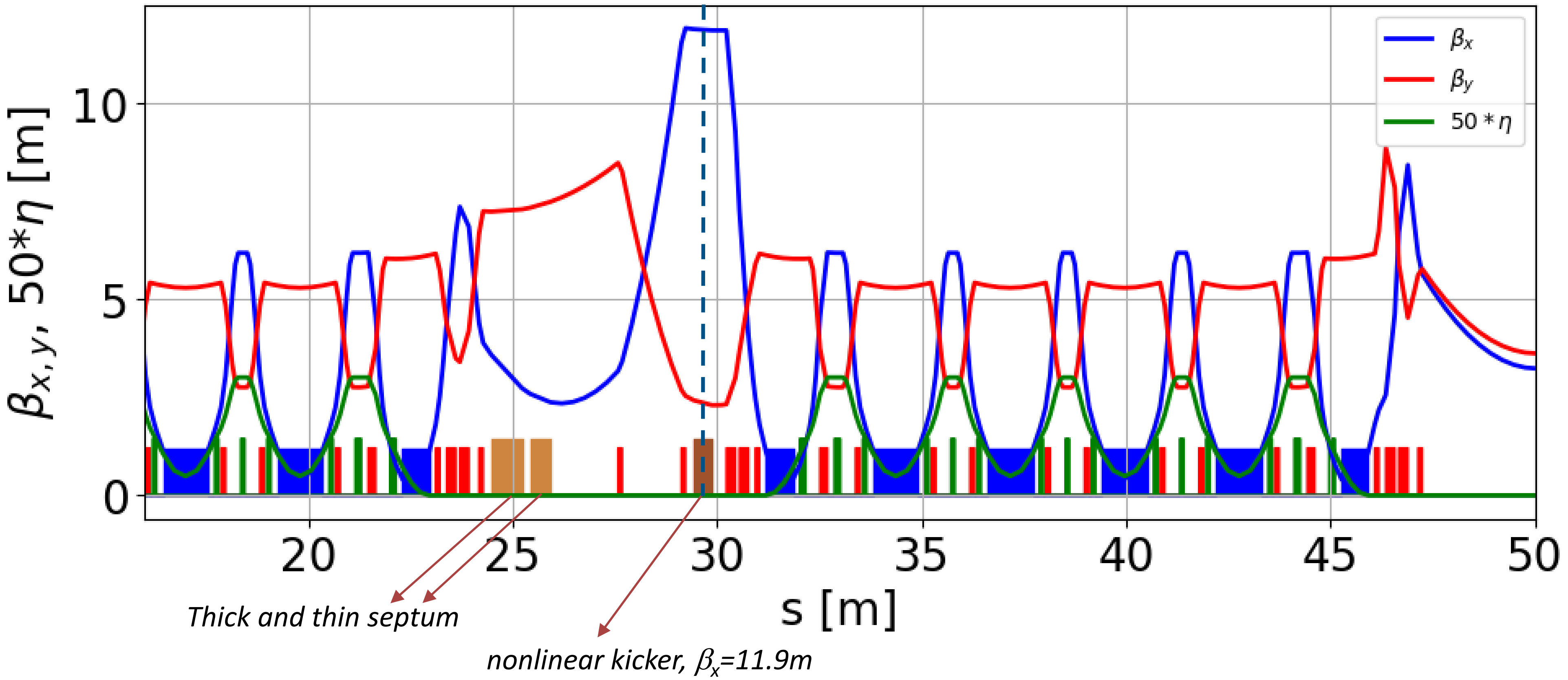
	unit	including tolerances	
		mean	std
MA	%	5.21	0.31
DA	mm^2	11.85	2.93
x_{min}	mm	-2.62	0.49

Need to assess consistency of results wrt tolerances:

- track with 10 lattices including tolerances (100 cases)
- corrected orbit, no beta beat correction

- MA > 5% ✔
- DA and x_{min} too tight for injection ✘

THP2147
"The BESSY III Injection Scheme"
M. Abo-Bakr, T. Atkinson, P. Goslawski, J. Völker



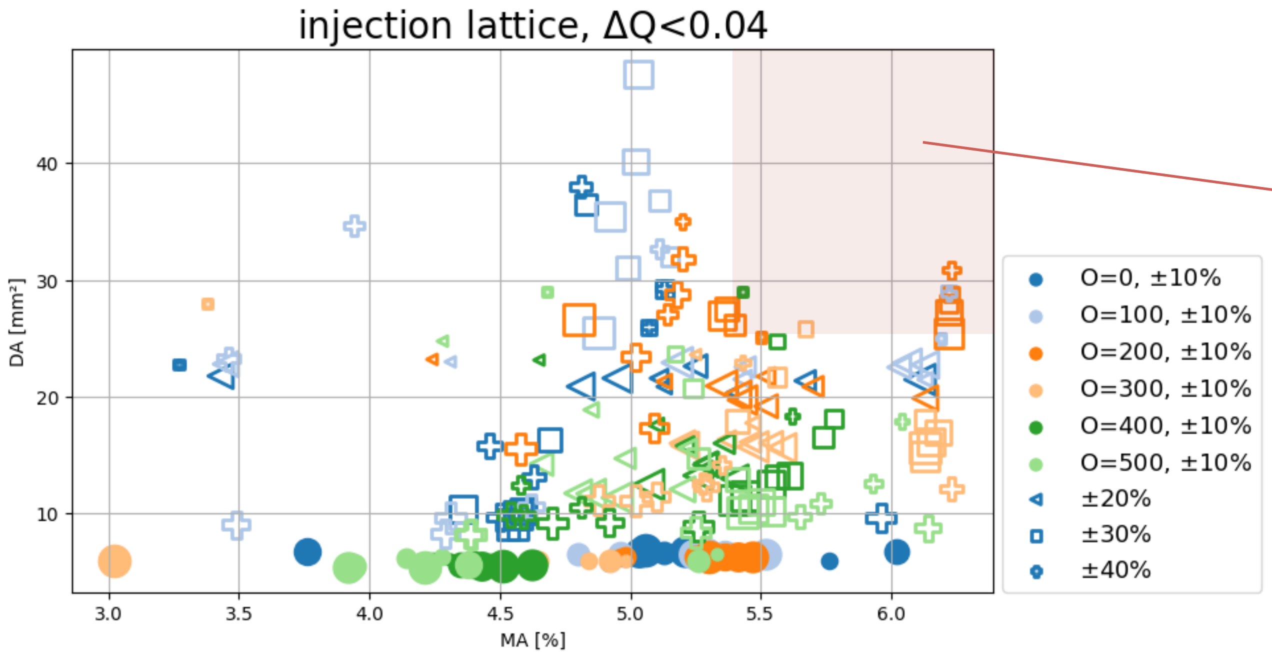
Minimal disturbance to lattice symmetry

- non-linear kicker (NLK) injection
- perturbation confined to straight
- keep phase advance over straight

chrom. DTs will change:

- DTs include K, $\beta_{x,y}$ or $d\beta_{x,y}/ds$
- periodicity = 1

⟹ different results in minimization



definition of 'best' cases (somewhat 'arbitrary'):

- MA > 5.4%, 10 largest DAs

Need to assess consistency of results wrt tolerances:

- track with 10 lattices including tolerances (100 cases)
- orbit-, beta beat- and coupling correction
- injection lattice: $\beta_{x,y}$= 11.9, 2.4m at NLK

MA > 5% ✔

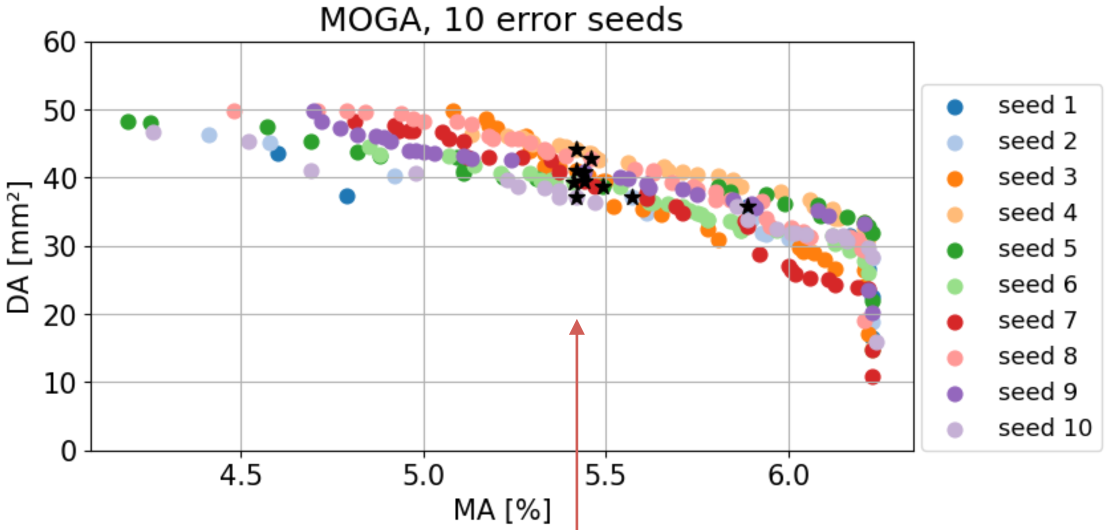
NLK max. kick at -7mm, within DA ✔

Error statistics for injection lattice:

	unit	including tolerances	
		mean	std
MA	%	5.29	0.17
DA	mm^2	39.48	3.00
x_{min}	mm	-8.60	0.66

MOGA calculations to optimize DA and MA directly

- 8 symmetric sextupole families and a single octupole
- lattice includes tolerances, orbit-, beta beat- and coupling correction
- evaluated of MA and DA area at NLK
- run takes around 20 hours on 100 CPUs of an HPC



10 best cases (black stars): largest DA for MA > 5.4%

Each MOGA result is the optimization for a specific error set – unlikely that of the real machine.

Consistency assessment necessary, also for MOGA.

10 best cases are evaluated for 10 lattices with error sets, closed orbit-, beta beat- and coupling correction.

Error statistics:

	unit	including tolerances	
		mean	std
MA	%	5.44	0.33
DA	mm^2	27.71	5.97
x_{min}	mm	-6.48	1.04

DA collapses from ~40mm² at Pareto front to 28mm² with errors

MOGA:

	unit	including tolerances	
		mean	std
MA	%	5.44	0.33
DA	mm^2	27.71	5.97
x_{min}	mm	-6.48	1.04

DTM:

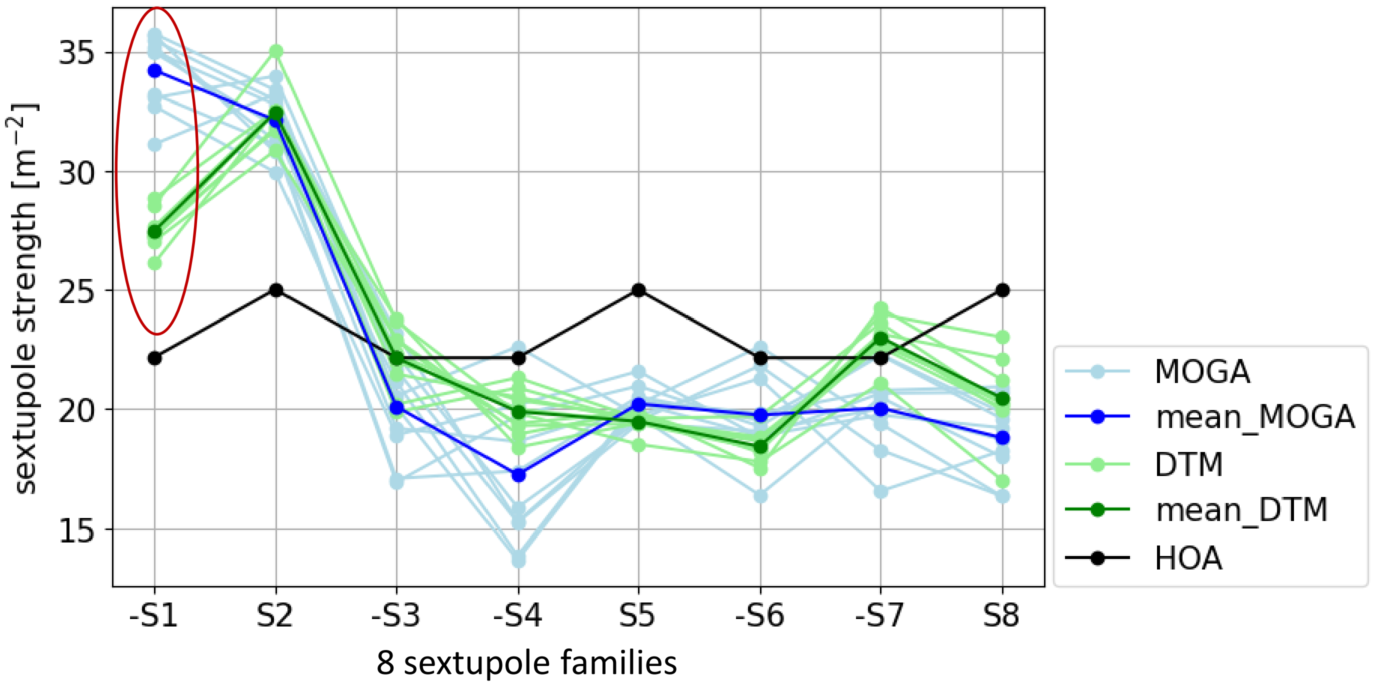
	unit	including tolerances	
		mean	std
MA	%	5.29	0.17
DA	mm^2	39.48	3.00
x_{min}	mm	-8.60	0.66

Comparison of statistical results:

- MA results comparable (+3% for MOGA)
- DA larger for DTM (+42%)
- x_{min} larger for DTM (+32%)

Results depend on the choice of 'best cases' !

Sextupole values of 'best cases'



Simulated commissioning on averaged DT from DTM: injection efficiency of **97%**

THP2035
"SIMULATED COMMISSIONING AND LATTICE ROBUSTNESS FOR BESSY III"
Benat Alberdi-Esuain, M. Abo-Bakr, P. Goslawski

- Significant part of sextupole strength shifted to the outer S1, S2, in both approaches
- Sextupole strength more confined in DTM (std: 3.9 %/ 7.8 %)
- Mean S1 is 24% stronger in MOGA (34.2 m^{-2}/27.5 m^{-2})
- Mean octupole is 10% stronger in MOGA 243 m^{-3}/220 m^{-3}

Conclusion

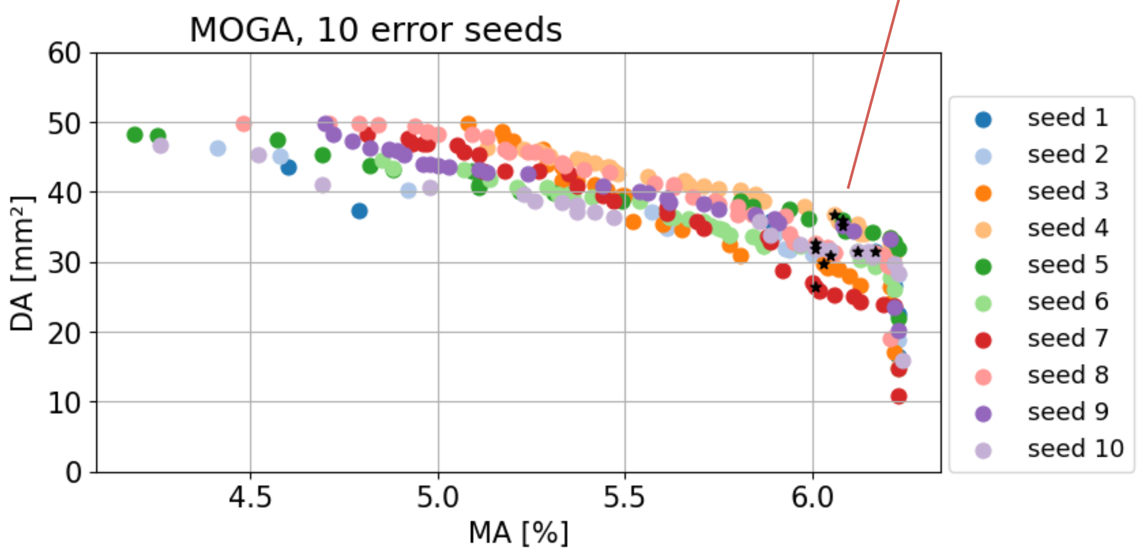
- Strategy to optimize the multipole values of MBA lattices for BESSY III
  - 4 classes of dominant DT
  - Minimizer with constraints for chromaticity
  - 2 weight functions to leverage between magnitude of the classes
  - Scan over weights, octupole, and deviation from HOA-case
- Results are show comparable MA and better DA than MOGA
- DTM takes less than 10% of computation time
- Easy to adapt non-linear configuration to
  - different working points
  - different chromaticities
  - adapt to lattice changes

Thank you for your attention!

MOGA calculations to optimize DA and MA directly

- NSGA-II algorithm of the Python pymoo package
- physics covered by pyAT and pySC
- runs start from the linear lattice
- 8 distinct, but symmetric sextupole families and a single octupole
- lattice includes tolerances, orbit-, beta beat- and coupling correction
- magnet values confined to technical limits
- constraint on chromaticity to $\xi x,y$ = 1.
- full 6D tracking, 1024 turns
- MA and DA at NLK (β_x = 11.9 m) defined by particle loss
- physical apertures ±9 mm
- population of 300 points per generation
- 100 off-springs and 100 rejected cases per generation
- 200 generations total
- evaluated of MA and DA area at NLK
- run takes around 20 hours on 100 CPUs of an HPC

10 best cases defined by: largest DA for MA > 6%



Each MOGA result is the optimization for a specific error set – unlikely that of the real machine.

Consistency assess necessary, also for MOGA.

10 best cases are evaluated for 10 lattices with error sets, closed orbit-, beta beat- and coupling correction.