Maximum entropy tomography of 4D transverse phase space distributions using 2D measurement results

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Information Entropy:

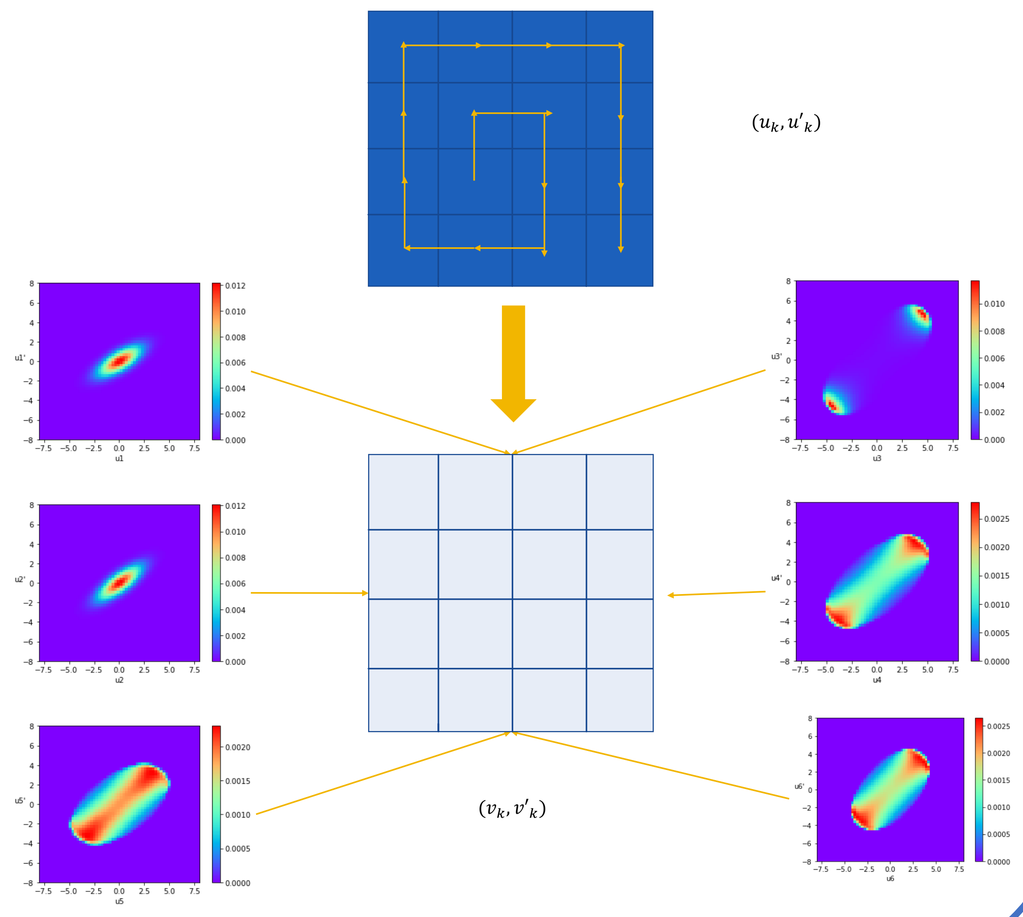
- A measure of the uncertainty of a distribution function.
- It quantifies the amount of "information" gained when an event occurs.
- For the probability of a single event, the entropy decreases as the probability increases.
- For the probabilities of multiple events, the entropies can be summed.

$$H = \sum P(x) \ln P(x)$$

$$H[\rho] = -\iiiint \rho(x, x', y, y') \ln \rho(x, x', y, y')\, dx dx' dy dy'$$

$$\rho = C \cdot \prod_{j=1}^{n} h(u_j, u_j')$$

After mapping the $h(u_j, u_j')$ function to the (v_k, v_k') plane, the value at each point is a function of (u_k, u_k')



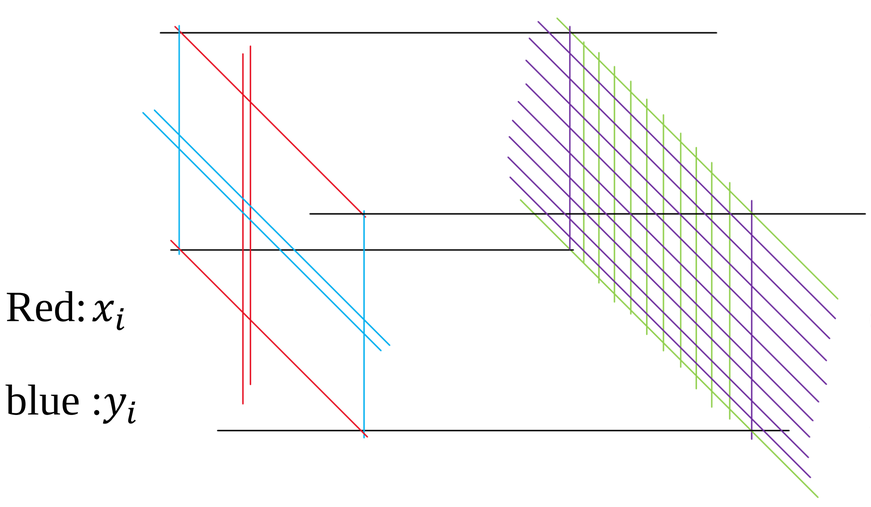
Iterative Algorithm:

$$h_k^{(m+1)}(u_k, u_k') = \frac{g_k(u_k, u_k')}{\iint C \prod_{j=1}^{k-1} h_j^{(m+1)}\left(u_j, u_j'\right) \prod_{j=k+1}^{n} h_j^{(m)}\left(u_j, u_j'\right) dv_k dv_k'}$$

$g_k(u_k, u_k')$: The k-th measured two-dimensional projection.

$h_k^{(m+1)}(u_k, u_k')$: The $h(u_j, u_j')$ iterated to the (m+1)-th iteration.

Vertical and Parallel Scanning:

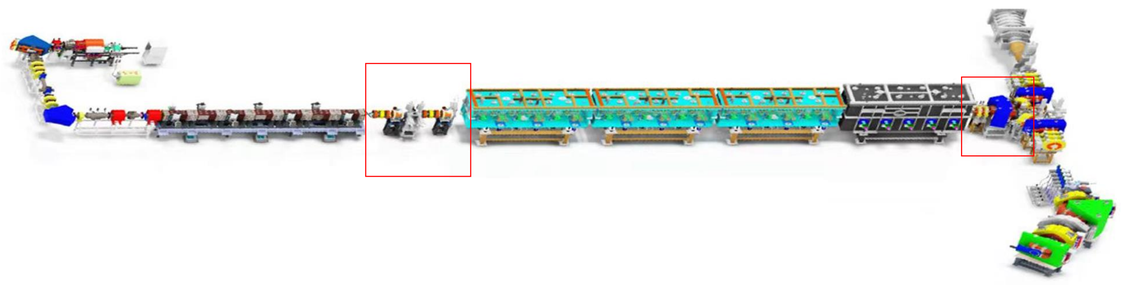


Vertical scan : (x_i, y_f) , (y_i, x_f)

Parallel scan : (x_i, x_f) , (y_i, y_f)

$$\begin{pmatrix} x_i \\ x_f \\ y_i \\ y_f \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & L & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & L \end{bmatrix} \begin{pmatrix} x_i \\ x_i' \\ y_i \\ y_i' \end{pmatrix}$$

Reconstruction of the transverse phase space distributions at the entrance and exit of the superconducting cavity:



CAFe II: a superconducting heavy ion linear accelerator.
Simulated particle: $^{40}Ar^{12}$
current :0.06 mA
particle energy at the RFQ exi :230 MeV