



BESIII



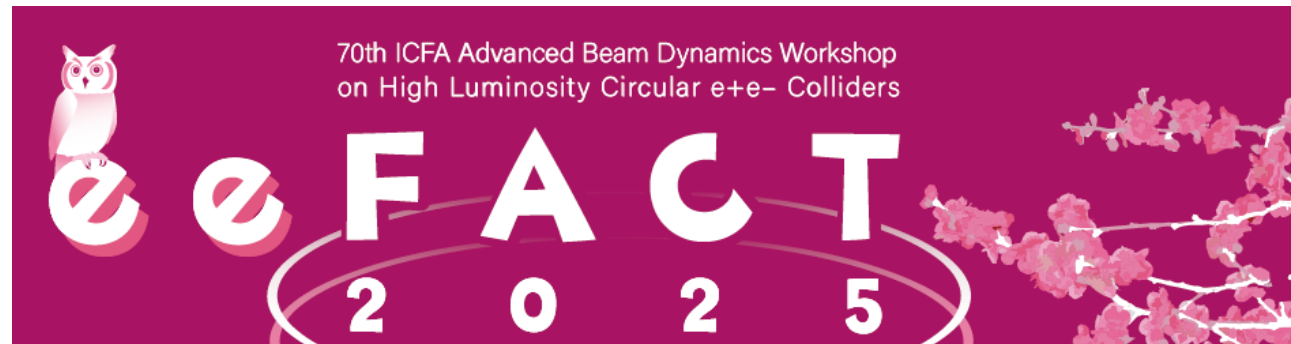
超级陶粲装置  
Super Tau-Charm Facility

# Charm physics studies at BESIII, Belle (II) and STCF

Yu Zhang

(On behalf of the BESIII Collaboration)

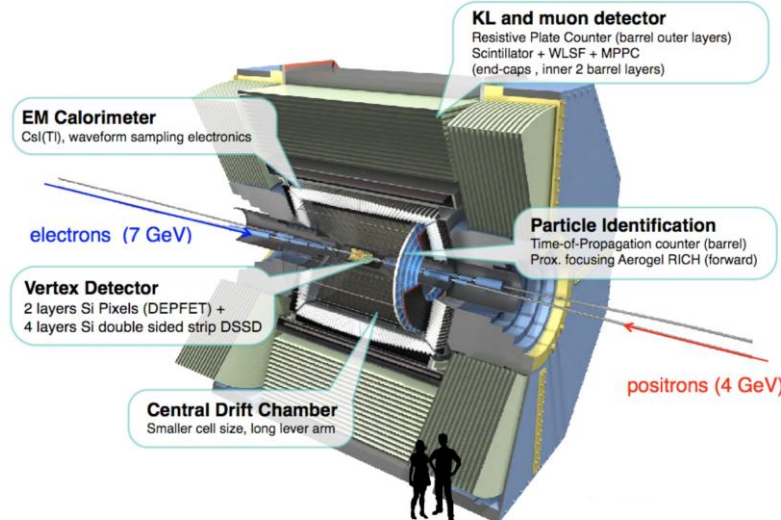
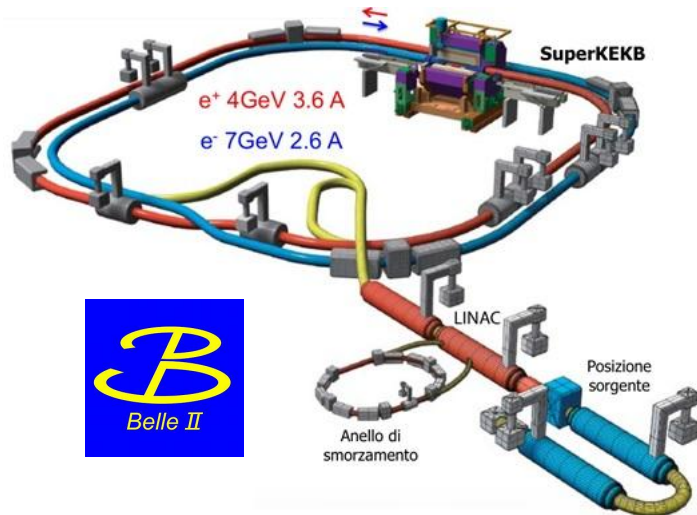
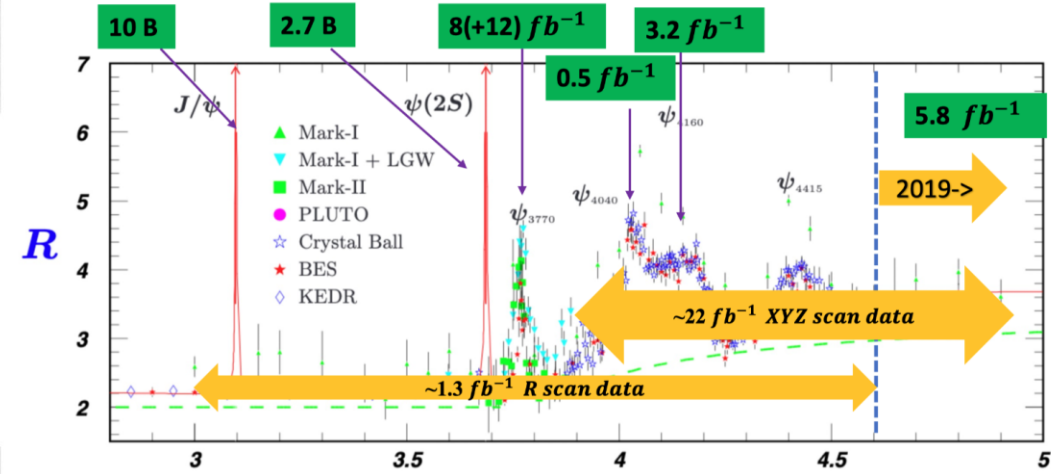
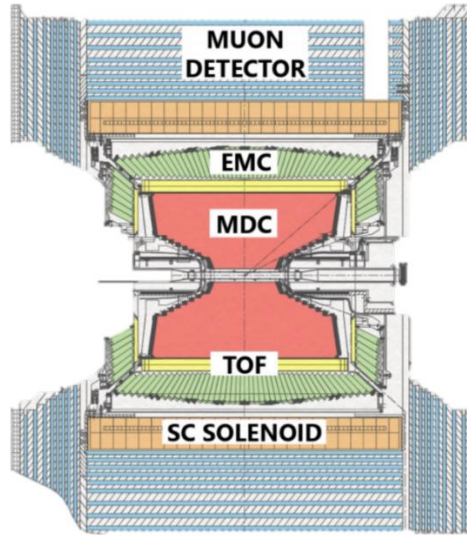
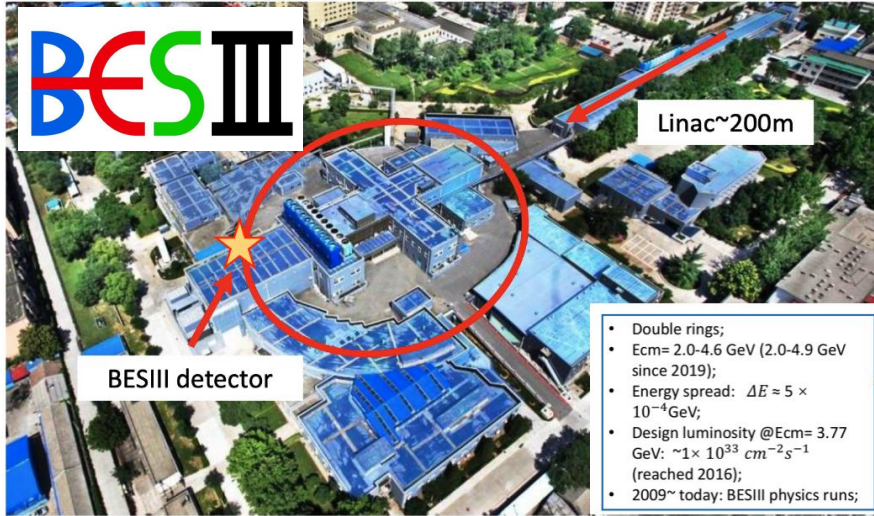
University of South China



# Outline

- Introduction
- Charm mixing and CPV studies
- Precision determination of  $|V_{cs(d)}|$
- Summary

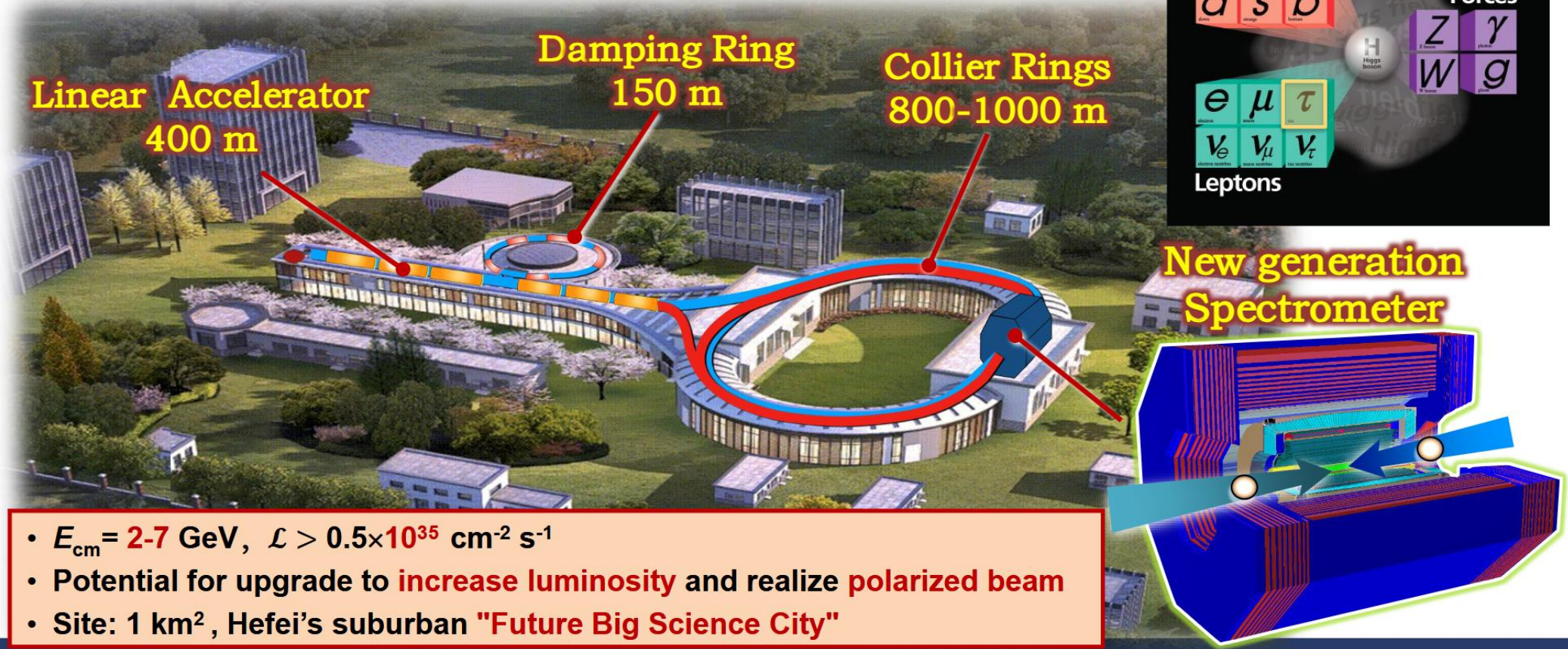
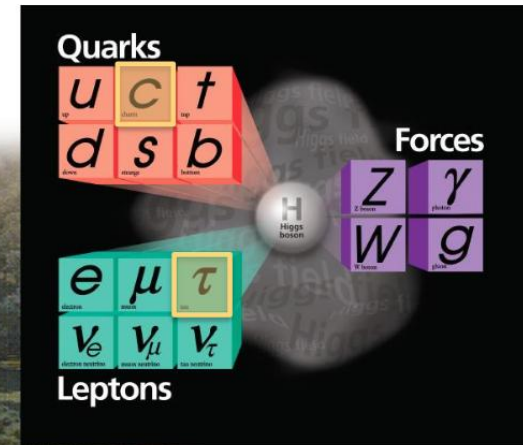
# Electron-positron experiments for charm studies



$e^+e^-$  asymmetric collider at  $\Upsilon(4S)$   
 To accumulate a dataset of  $\int L dt = 50\text{ ab}^{-1}$

# Future experiment—Super Tau Charm Facility

A factory producing massive **tau lepton** and **hadrons**, to unravel the mystery of **how quarks form matter** and the **symmetries** of fundamental interactions



- $E_{\text{cm}} = 2-7 \text{ GeV}$ ,  $\mathcal{L} > 0.5 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- Potential for upgrade to **increase luminosity** and realize **polarized beam**
- Site: 1 km<sup>2</sup>, Hefei's suburban "Future Big Science City"

From Jingyu Tang's talk

Expect to accumulate datasets for charm hadrons with an integrated luminosity 100 times that of BESIII

# Charm mixing

Mass eigenstates:  $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$

Time evolution of an initially flavour eigenstate  $D$  meson:

$$|D_{phys}^0(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\bar{D}^0\rangle, \quad |\bar{D}_{phys}^0(t)\rangle = g_+(t)|\bar{D}^0\rangle + \frac{q}{p}g_-(t)|D^0\rangle$$

$$g_+(t) = \exp(-(im + \Gamma/2)t) \cosh((i\Delta m - \Gamma/2)t/2) \quad m \equiv (m_1 + m_2)/2, \Delta m \equiv m_2 - m_1$$

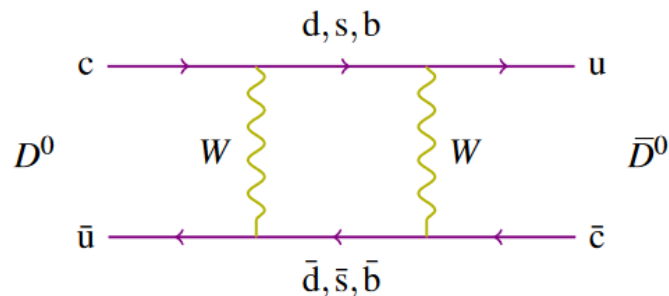
$$g_-(t) = \exp(-(im + \Gamma/2)t) \sinh((i\Delta m - \Gamma/2)t/2) \quad \Gamma \equiv (\Gamma_1 + \Gamma_2)/2, \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$

Charm mixing parameters:  $x \equiv \frac{\Delta m}{\Gamma}, y \equiv \frac{\Delta\Gamma}{2\Gamma}$

Short distance contributions

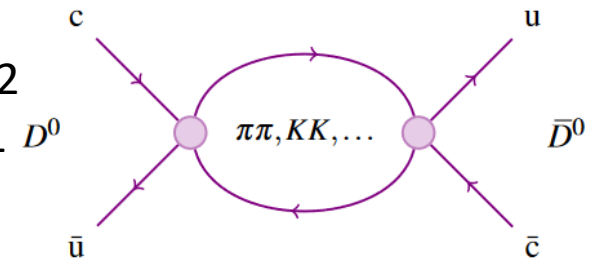
Long distance contributions

loop level  $(x, y) \sim 10^{-7}$



$(x, y) \sim 10^{-3}$

- Phys. Lett. B 810 (2020) 135802
- Chin. Phys. C 42 (2018) 063101
- Phys. Rev. D 81 (2010) 114020
- Phys. Rev. D 65 (2002) 054034



Plots from arXiv:1503.00032

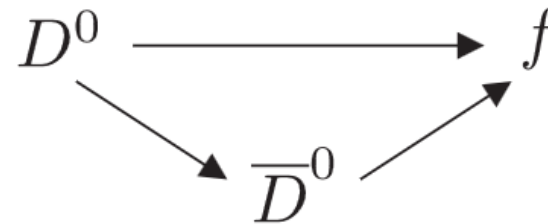
# Indirect CPV associated with charm mixing

- Type 1. CP violation in mixing  $|p| \neq |q|$ 
  - Mass eigenstates are not CP eigenstates

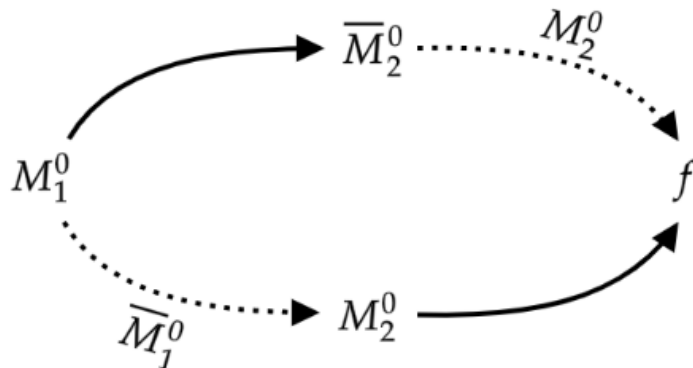
- Type 2. CPV due to interference between mixing and decay

$$\text{Im} \left( \frac{q A(\bar{D}^0 \rightarrow f)}{p A(D^0 \rightarrow f)} \right) \neq \text{Im} \left( \frac{p A(D^0 \rightarrow \bar{f})}{q A(\bar{D}^0 \rightarrow \bar{f})} \right)$$

- Non-zero  $\phi$  for  $\frac{q}{p} = \left| \frac{q}{p} \right| e^{i\phi}$



- Type 3. Interference between charm mixing and kaon mixing



[PRD 110 (2024) L031301]

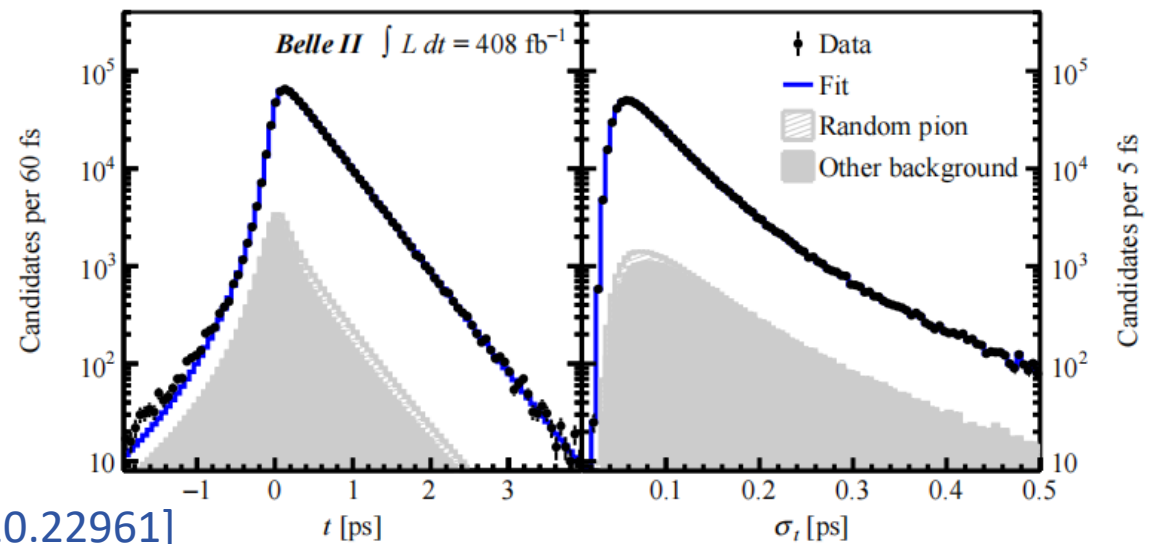
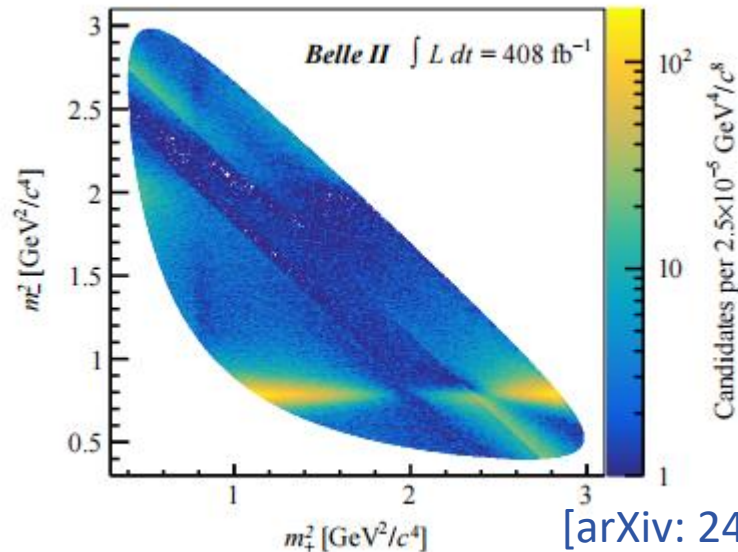
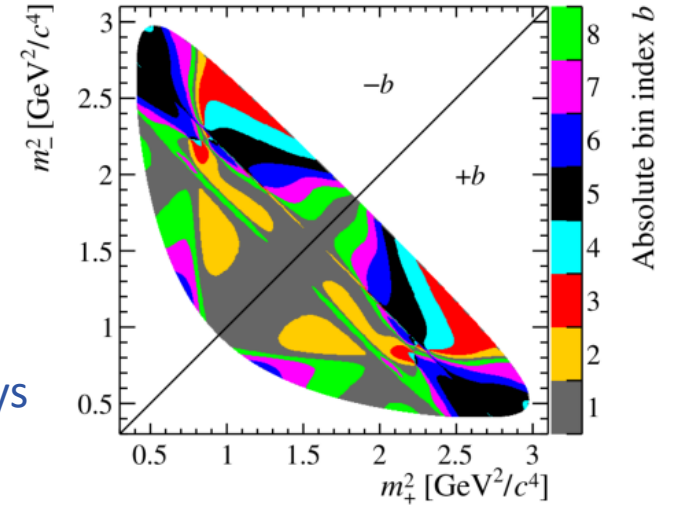
# Determination of mixing parameters with Belle+Belle II data

- Flavour-tagged  $D$  mesons from  $D^* \rightarrow D(K_S^0 \pi^+ \pi^-) \pi$  decays
- Time-dependent distribution of decay rate

$$p_{\pm b}(t) \propto g_{\pm}^2(t) + r_b g_{\mp}^2(t) + 2\sqrt{r_b} \text{Re}[X_{\pm b} g_{\pm}^*(t) g_{\mp}(t)]$$

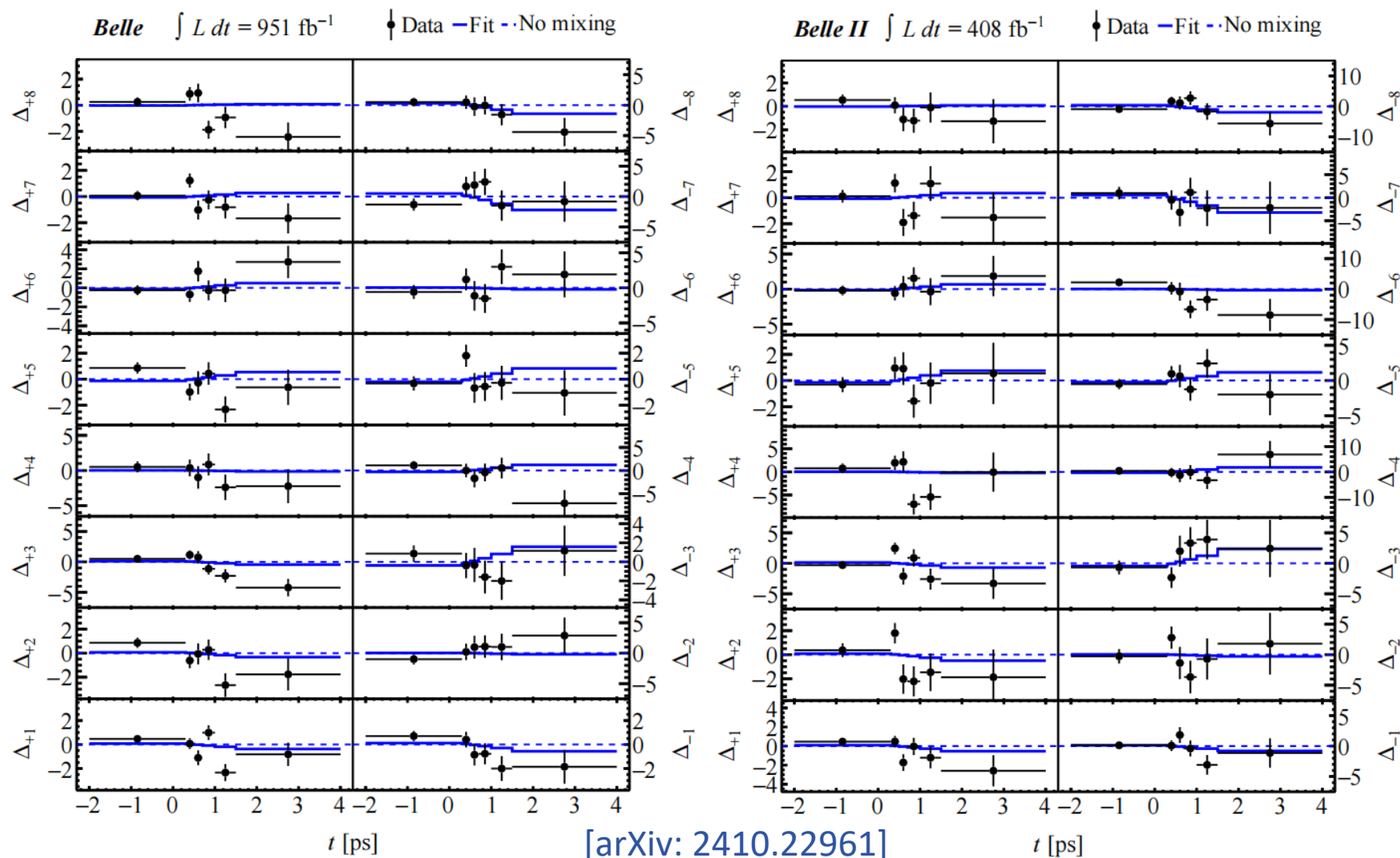
Charm mixing

$X_{\pm b}$ : Strong-phase difference between  $D^0$  and  $\bar{D}^0$  decays  
 $r_b$ : Ratio of  $D^0$  and  $\bar{D}^0$  decay amplitude square



[arXiv: 2410.22961]

# Binned time-dependent decay rate



Assuming no CPV

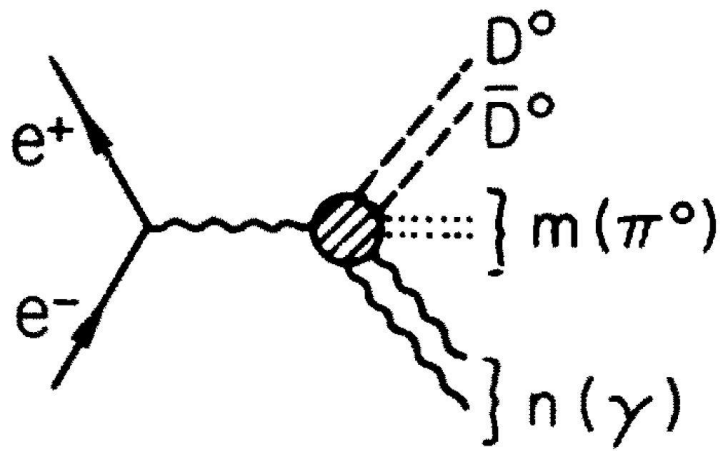
$$x = (4.0 \pm 1.7 \pm 0.4) \times 10^{-3}$$

$$y = (2.9 \pm 1.4 \pm 0.3) \times 10^{-3}$$

Stat. Significance of  $2.7\sigma$



# Charm mixing study with quantum correlated $D\bar{D}$



[Phys. Rev. D 15 (1997) 1254]

- $D^0\bar{D}^0$  pairs produced by  $e^+e^-$  annihilations near threshold at BESIII/STCF are in QC state with  $C = (-1)^{n+1}$ ,  $n$  is the number of accompany photons
- @3770 MeV  
 $C$  odd for  $e^+e^- \rightarrow D^0\bar{D}^0$
- @Higher energy points  
 $C$  even for  $e^+e^- \rightarrow D^0\bar{D}^{*0} + c.c. \rightarrow \gamma D^0\bar{D}^0$   
 $C$  even for  $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} + c.c. \rightarrow \gamma\pi^0 D^0\bar{D}^0$   
 $C$  odd for  $e^+e^- \rightarrow D^0\bar{D}^{*0} + c.c. \rightarrow \pi^0 D^0\bar{D}^0$   
 $C$  odd for  $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} + c.c. \rightarrow \gamma\gamma D^0\bar{D}^0$   
 $C$  odd for  $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} + c.c. \rightarrow \pi^0\pi^0 D^0\bar{D}^0$

# Quantum-correlated double decay rates

C-even

$$\begin{aligned} \Gamma(f_1, f_2) \propto & 3(x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 + 2R_{D^0}R_{\bar{D}^0}\lambda_{D^0}\lambda_{\bar{D}^0}) \\ & + (2 - 3(x^2 - y^2))(1 + 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2) \\ & - 4\mathbf{y}[R_{\bar{D}^0}(1 + |\lambda_{D^0}|^2)\text{Re}(\lambda_{\bar{D}^0}) + R_{D^0}(1 + |\lambda_{\bar{D}^0}|^2)\text{Re}(\lambda_{D^0})] \\ & - 4\mathbf{x}[R_{\bar{D}^0}(1 - |\lambda_{D^0}|^2)\text{Im}(\lambda_{\bar{D}^0}) + R_{D^0}(1 - |\lambda_{\bar{D}^0}|^2)\text{Im}(\lambda_{D^0})] \end{aligned}$$

C-odd

$$\begin{aligned} \Gamma(f_1, f_2) \propto & (x^2 + y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0})) \\ & + (2 - (x^2 - y^2))(1 - 2R_{D^0}R_{\bar{D}^0}\text{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2) \end{aligned}$$

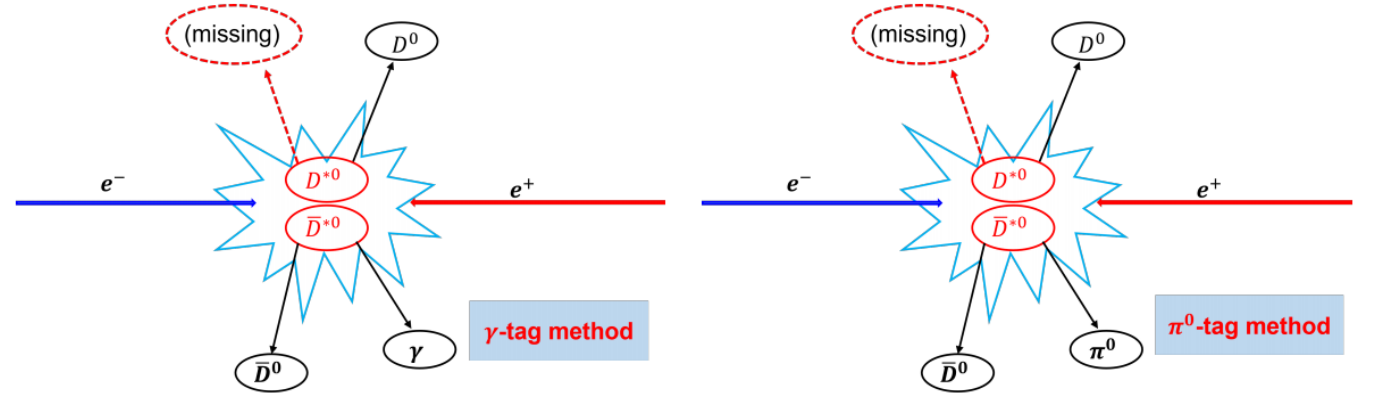
$$\lambda_{D^0} = r_{f_1} \left| \frac{q}{p} \right| e^{-i(\delta_D^{f_1} + \phi)} \quad \lambda_{\bar{D}^0} = 1/r_{f_2} \left| \frac{p}{q} \right| e^{i(\delta_D^{f_2} + \phi)}$$

$\delta_D^f$ : CP-conserving phase,  $\phi$ : indirect CPV phase,  $\left| \frac{p}{q} \right|$ : indirect CPV

Exploring the QC in  $D^0\bar{D}^0$  decays allows to extract the mixing and CPV parameters along with CP-conserving strong-phase differences with time-integrated decay rates

# Analysis strategy

$K^\mp \pi^\pm \pi^0$		tag modes
Flavour	OS	$K^\pm \pi^\mp \pi^+ \pi^-, K^\pm \pi^\mp \pi^0, K^\pm \pi^\mp$
	LS	$K^\mp \pi^\pm \pi^+ \pi^-, K^\mp \pi^\pm \pi^0, K^\mp \pi^\pm$
CP	even	$\pi^+ \pi^- \pi^0, \pi^+ \pi^-, K^+ K^-$
	odd	$K_S^0 \pi^0, K_S^0 \eta'_{\gamma \pi^+ \pi^-}$
Self-conjugate		$K_S^0 \pi^+ \pi^-$



$$e^+ e^- \rightarrow D^0 \bar{D}^{*0} + c.c.$$

$$N_{S,T}^+ = 2N_{D^{*0} \bar{D}^0} \Gamma_+(S,T) \epsilon_{S,T}^+ BF(D^{*0} \rightarrow \gamma D^0)$$

$$N_{S,T}^- = 2N_{D^{*0} \bar{D}^0} \Gamma_-(S,T) \epsilon_{S,T}^- BF(D^{*0} \rightarrow \pi^0 D^0)$$

$$\frac{\Gamma_+(S,T)}{\Gamma_-(S,T)} = \frac{N_{S,T}^+ \epsilon_{S,T}^+ BF(D^{*0} \rightarrow \gamma D^0)}{N_{S,T}^- \epsilon_{S,T}^- BF(D^{*0} \rightarrow \pi^0 D^0)}$$

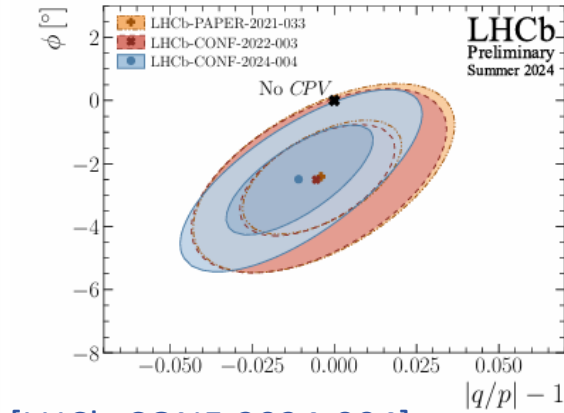
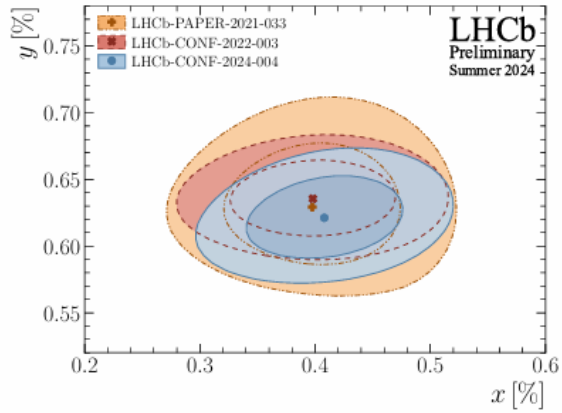
$$e^+ e^- \rightarrow D^{*0} \bar{D}^{*0} + c.c.$$

$$N_{S,T}^+ = 2N_{D^{*0} \bar{D}^0} \Gamma_+(S,T) \epsilon_{S,T}^+ BF(D^{*0} \rightarrow \gamma D^0) BF(D^{*0} \rightarrow \pi^0 D^0)$$

$$N_{S,T}^- = N_{D^{*0} \bar{D}^0} \Gamma_-(S,T) \epsilon_{S,T}^- BF(D^{*0} \rightarrow \pi^0 D^0) BF(D^{*0} \rightarrow \pi^0 D^0)$$

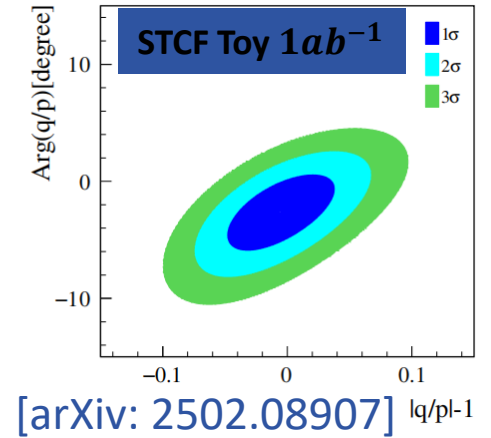
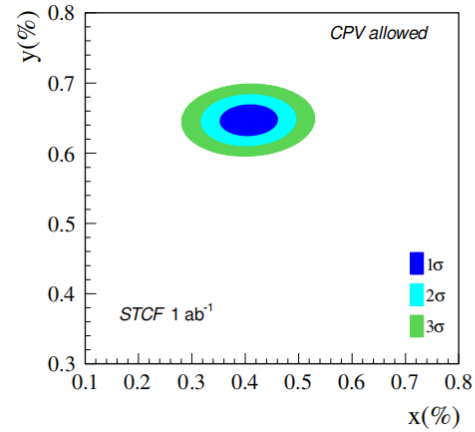
$$\frac{\Gamma_+(S,T)}{\Gamma_-(S,T)} = \frac{N_{S,T}^+ \epsilon_{S,T}^+ BF(D^{*0} \rightarrow \pi^0 D^0)}{2N_{S,T}^- \epsilon_{S,T}^- BF(D^{*0} \rightarrow \gamma D^0)}$$

# Status and prospects of charm mixing and indirect CPV



[LHCb-CONF-2024-004]

$$\begin{aligned}
 x &= (4.1 \pm 0.5) \times 10^{-3} \\
 y &= (6.19 \pm 0.21) \times 10^{-3} \\
 |q/p| &= 0.984^{+0.014}_{-0.015} \\
 \arg(q/p) &= (-1.6^{+1.1}_{-1.2})^\circ
 \end{aligned}$$



$$\begin{aligned}
 x &= (? \pm 0.36) \times 10^{-3} \\
 y &= (? \pm 0.15) \times 10^{-3} \\
 |q/p| &=? \pm 0.028 \\
 \arg(q/p) &= (? \pm 2.14)^\circ
 \end{aligned}$$

- Belle II could finally reach  $\sigma(x) \sim 0.028\%$  and  $\sigma(y) \sim 0.023\%$  with  $K_S^0 \pi^+ \pi^-$  decay only by scaling the existing measurement results, see also in arXiv: 1808.10567
- As a comparison,  $\sigma(x) < 0.005\%$ ,  $\sigma(y) < 0.005\%$ ,  $\sigma(|q/p|) < 0.004$  and  $\sigma(\arg(q/p)) < 0.18^\circ$  with  $K_S^0 \pi^+ \pi^-$  decay for LHCb Upgrade II, see also in arXiv: 1808.08865
- By then, STCF could still be critical to provide **inputs of strong-phase differences** between  $D^0$  and  $\bar{D}^0$  decays
- Currently, BESIII provide these inputs to charm mixing and indirect CPV studies in Belle (II) and LHCb

# Status and prospects of strong-phase measurements

C-odd correlated  $D^0\bar{D}^0$  pairs at BESIII (STCF) is an ideal place to determine the strong-phase parameters and provide best constraints

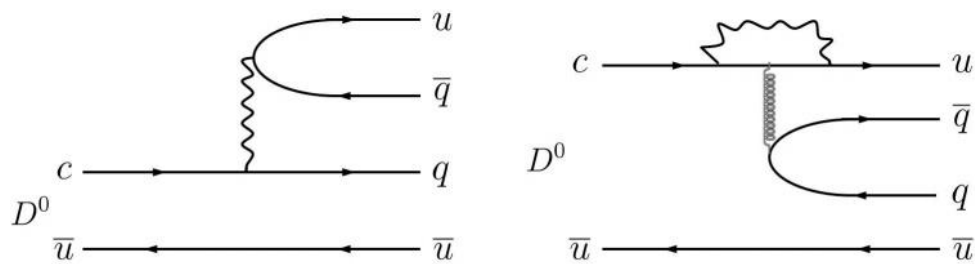
Decay	Strong-phase parameters	2.93 fb <sup>-1</sup> data	Status
$K_{S,L}^0\pi^+\pi^-$	$c_i, s_i$	PRL 124 (2020) 241802 PRD 101 (2020) 112002	8 and 20 fb <sup>-1</sup> ongoing Unbinned phase correction: ongoing
$K_{S,L}^0K^+K^-$	$c_i, s_i$	PRD 102 (2020) 052008	20 fb <sup>-1</sup> ongoing Unbinned phase correction: ongoing
$K^\pm\pi^\mp\pi^+\pi^-$	$\delta_D, R_D$	JHEP 05 (2021) 164	8 and 20 fb <sup>-1</sup> ongoing
$\pi^+\pi^-\pi^+\pi^-$	$F^+/c_i, s_i$	PRD 106 (2022) 092004 PRD 110 (2024) 112008	20 fb <sup>-1</sup> ongoing
$K^+K^-\pi^+\pi^-$	$F^+/c_i, s_i$	PRD 107 (2023) 032009	$c_i, s_i$ : arXiv: 2502.12873
$K_S^0\pi^+\pi^-\pi^0$	$F^+/c_i, s_i$	PRD 108 (2023) 032003	$c_i, s_i$ : ongoing
$K^\pm\pi^\mp\pi^0$	$\delta_D, R_D$	JHEP 05 (2021) 164	8 and 20 fb <sup>-1</sup> ongoing
$K^\pm\pi^\mp$	$\delta_D$	EPJC 82 (2022) 1009	20 fb <sup>-1</sup> ongoing
$K_S^0K^\pm\pi^\mp$	$\delta_D, R_D$	—	20 fb <sup>-1</sup> ongoing
$\pi^+\pi^-\pi^0$ , $K^+K^-\pi^0$	$F^+$	PRD 111 (2025) 012007	20 fb <sup>-1</sup> ongoing

- Integrated luminosity of  $\psi(3770)$  data at BESIII has reached 20 fb<sup>-1</sup> by the year of 2024
- The uncertainty of strong-phase inputs will be further reduced
- STCF will provide more precise inputs for Belle II and LHCb Upgrade

# CP Violation in direct charm-meson decays

- CPV in direct decays  $A$  and CP-transformed  $\bar{A}$
- Non-zero CP-violating phase difference  $\xi_j - \xi_k$  and CP-conserving phase difference  $\delta_j - \delta_k$

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = - \frac{2 \sum |A_j| |A_k| \sin(\xi_j - \xi_k) \sin(\delta_j - \delta_k)}{\sum |A_j|^2 + 2 \sum |A_j| |A_k| \cos(\xi_j - \xi_k) \cos(\delta_j - \delta_k)}$$



LHCb made the first observation

$$\Delta a_{CP}^{\text{dir}} = a_{K^+K^-}^{\text{dir}} - a_{\pi^+\pi^-}^{\text{dir}} = (-1.57 \pm 0.29) \times 10^{-3}$$

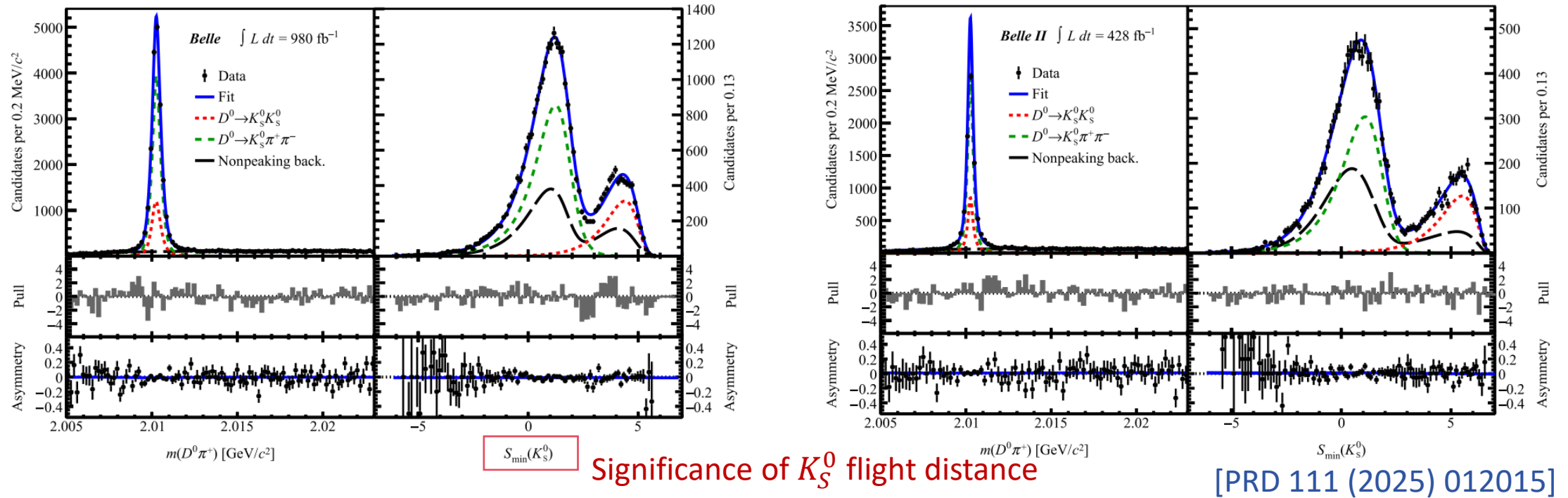
[PRL 122 (2019) 211803]

and first evidence for single decay

$$a_{K^+K^-}^{\text{dir}} = (7.7 \pm 5.7) \times 10^{-4}, \quad a_{\pi^+\pi^-}^{\text{dir}} = (23.2 \pm 6.1) \times 10^{-4}$$

[PRL 131 (2023) 091802]

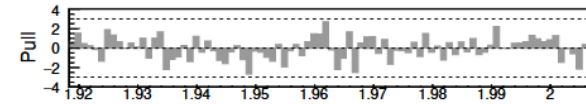
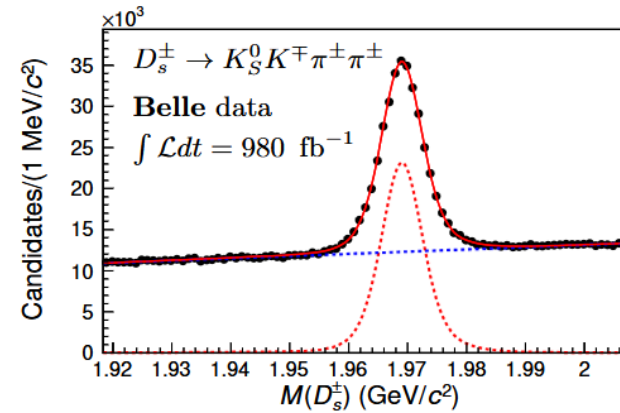
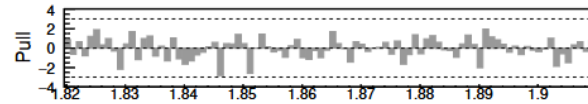
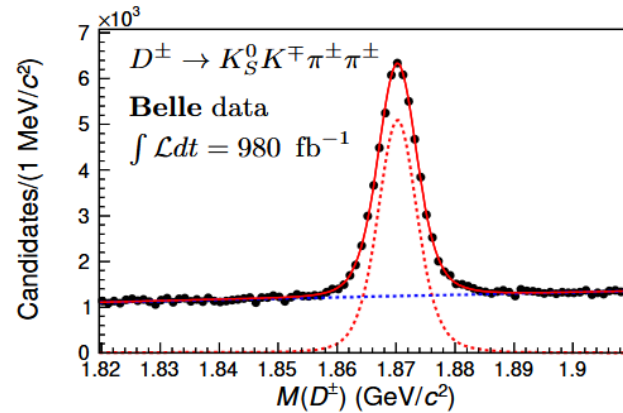
# Measurement of direct CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ with Belle and Belle II data



$$A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = \frac{\Gamma(D^0 \rightarrow K_S^0 K_S^0) - \Gamma(\bar{D}^0 \rightarrow K_S^0 K_S^0)}{\Gamma(D^0 \rightarrow K_S^0 K_S^0) + \Gamma(\bar{D}^0 \rightarrow K_S^0 K_S^0)} = (-1.4 \pm 1.3 \pm 0.1)\%$$

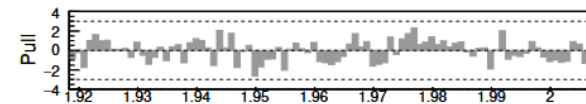
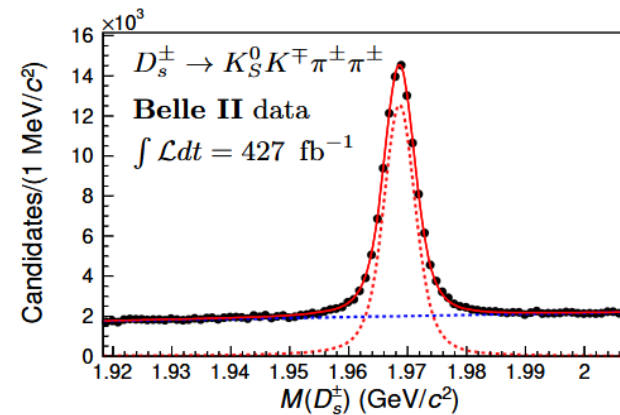
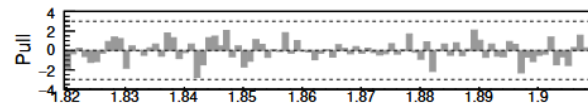
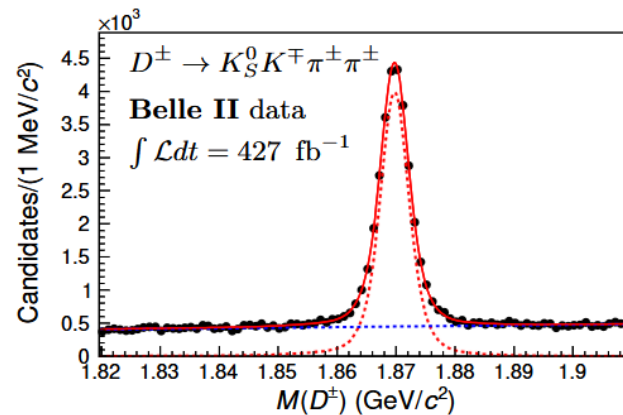
- Consistent with no CP violation
- $A_{CP}(D^0 \rightarrow K_S^0 K_S^0) = (-3.1 \pm 1.2 \pm 0.4 \pm 0.2)$  by LHCb [PRD 104 (2021) L031102] with a significance of  $2.4\sigma$

# Search for CPV in $D_{(s)}^{\pm} \rightarrow K_S^0 K^{\mp} \pi^{\pm} \pi^{\pm}$ using triple and quadruple products with Belle and Belle II data



Singly Cabibbo suppressed

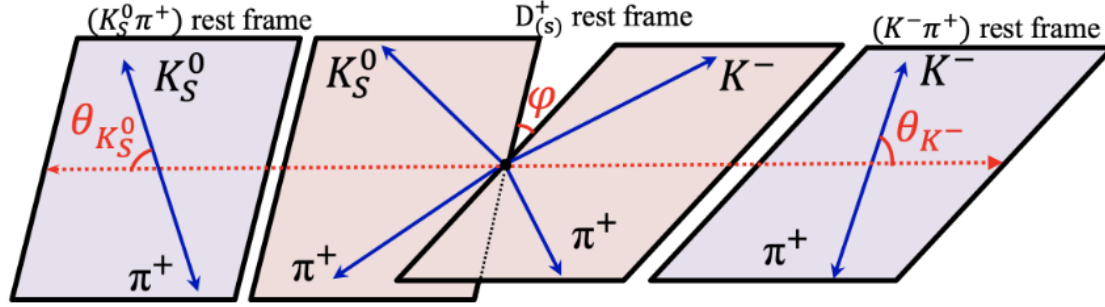
Cabibbo favored  
No CPV in the SM



[arXiv: 2409.15777]



# Search for CPV in $D_{(s)}^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ using triple and quadruple products with Belle and Belle II data



$$C_{\text{TP}} = (\vec{p}_{K^-} \times \vec{p}_{\pi^+}) \cdot \vec{p}_{K_S^0},$$

$$C_{\text{QP}} = (\vec{p}_{K^-} \times \vec{p}_{\pi^+}) \cdot (\vec{p}_{K_S^0} \times \vec{p}_{\pi^+})$$

$$A_X(D_{(s)}^+) \equiv \frac{N(D_{(s)}^+, X > 0) - N(D_{(s)}^+, X < 0)}{N(D_{(s)}^+, X > 0) + N(D_{(s)}^+, X < 0)}$$

$$A_{\bar{X}}(D_{(s)}^-) \equiv \frac{N(D_{(s)}^-, \bar{X} > 0) - N(D_{(s)}^-, \bar{X} < 0)}{N(D_{(s)}^-, \bar{X} > 0) + N(D_{(s)}^-, \bar{X} < 0)}$$

$$\mathcal{A}_{CP}^X \equiv \frac{A_X(D_{(s)}^+) - A_{\bar{X}}(D_{(s)}^-)}{2}$$

$X$	$\times 10^{-3}$	$\mathcal{A}_{CP}^X$ Belle	$\mathcal{A}_{CP}^X$ Belle II	Combined $\mathcal{A}_{CP}^X$	Significance
$D^+$	$C_{\text{TP}}$	$-4.0 \pm 5.9 \pm 3.0$	$-0.2 \pm 7.0 \pm 1.8$	$-2.3 \pm 4.5 \pm 1.5$	$0.5\sigma$
	$C_{\text{QP}}$	$-1.0 \pm 5.9 \pm 2.5$	$-0.4 \pm 7.0 \pm 2.4$	$-0.7 \pm 4.5 \pm 1.7$	$0.2\sigma$
	$C_{\text{TP}} C_{\text{QP}}$	$+6.4 \pm 5.9 \pm 2.2$	$+0.6 \pm 7.0 \pm 1.3$	$+3.9 \pm 4.5 \pm 1.1$	$0.8\sigma$
	$\cos \theta_{K_S^0} \cos \theta_{K^-}$	$-4.7 \pm 5.9 \pm 3.0$	$-0.6 \pm 6.9 \pm 3.0$	$-2.9 \pm 4.5 \pm 2.1$	$0.6\sigma$
	$C_{\text{TP}} \cos \theta_{K_S^0} \cos \theta_{K^-}$	$+1.9 \pm 5.9 \pm 2.0$	$-0.2 \pm 7.0 \pm 1.9$	$+1.0 \pm 4.5 \pm 1.4$	$0.2\sigma$
	$C_{\text{QP}} \cos \theta_{K_S^0} \cos \theta_{K^-}$	$+14.9 \pm 5.9 \pm 1.4$	$+7.0 \pm 7.0 \pm 1.6$	$+11.6 \pm 4.5 \pm 1.1$	$2.5\sigma$
$D_s^+$	$C_{\text{TP}}$	$-0.3 \pm 3.1 \pm 1.3$	$+1.0 \pm 3.9 \pm 1.1$	$+0.2 \pm 2.4 \pm 0.8$	$0.1\sigma$
	$C_{\text{QP}}$	$+0.6 \pm 3.1 \pm 1.2$	$+2.0 \pm 3.9 \pm 1.4$	$+1.1 \pm 2.4 \pm 0.9$	$0.4\sigma$
	$C_{\text{TP}} C_{\text{QP}}$	$+1.5 \pm 3.2 \pm 1.4$	$-2.7 \pm 3.9 \pm 1.7$	$-0.2 \pm 2.5 \pm 1.1$	$0.1\sigma$
	$\cos \theta_{K_S^0} \cos \theta_{K^-}$	$-3.7 \pm 3.1 \pm 1.1$	$-6.3 \pm 3.9 \pm 1.2$	$-4.7 \pm 2.4 \pm 0.8$	$1.8\sigma$
	$C_{\text{TP}} \cos \theta_{K_S^0} \cos \theta_{K^-}$	$-4.4 \pm 3.2 \pm 1.4$	$+0.8 \pm 3.9 \pm 1.4$	$-2.2 \pm 2.5 \pm 1.0$	$0.8\sigma$
	$C_{\text{QP}} \cos \theta_{K_S^0} \cos \theta_{K^-}$	$-1.6 \pm 3.1 \pm 1.3$	$-0.0 \pm 3.9 \pm 1.7$	$-1.0 \pm 2.4 \pm 1.0$	$0.4\sigma$

# Precision measurement of $|V_{cd(s)}|$

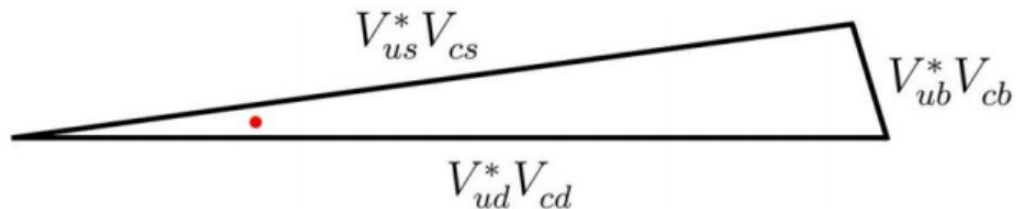
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

BESIII+B  
factories+LQCD

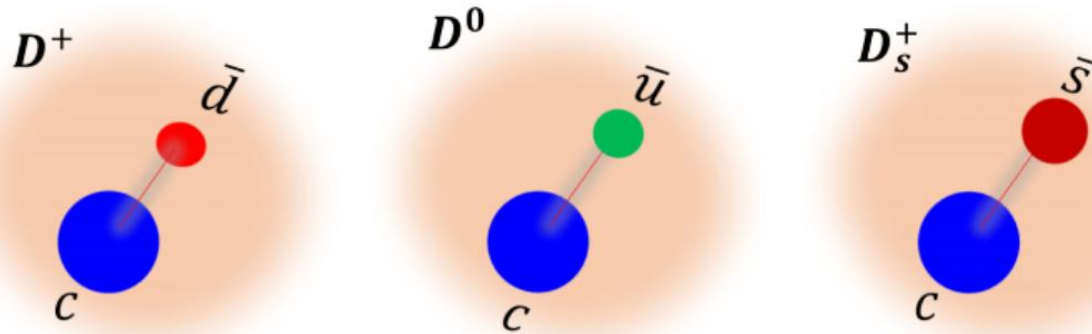
$$\begin{aligned} |V_{ud}| &= 0.97367(32), & |V_{us}| &= 0.22431(85), & |V_{ub}| &= 0.00382(20) \\ |V_{cd}| &= \mathbf{0.221(4)}, & |V_{cs}| &= \mathbf{0.975(6)}, & |V_{cb}| &= 0.0411(12) \\ |V_{td}| &= 0.0086(2), & |V_{ts}| &= 0.0415(9), & |V_{tb}| &= 1.010(27) \end{aligned}$$

BESIII precision < 2%?

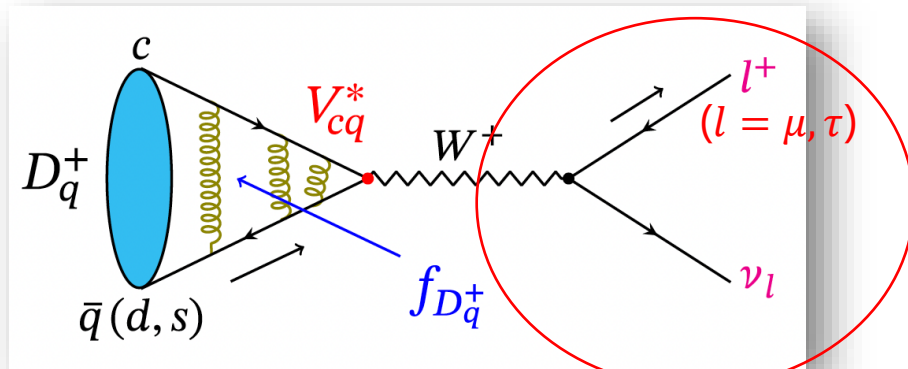
- Precision test of the CKM unitarity is an important approach to search for new physics
- Besides the CP-violating phases in the triangle  $V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$ ,  $|V_{cd(s)}|$  are also key parameters



# Precision test of lepton flavor universality in charm sector



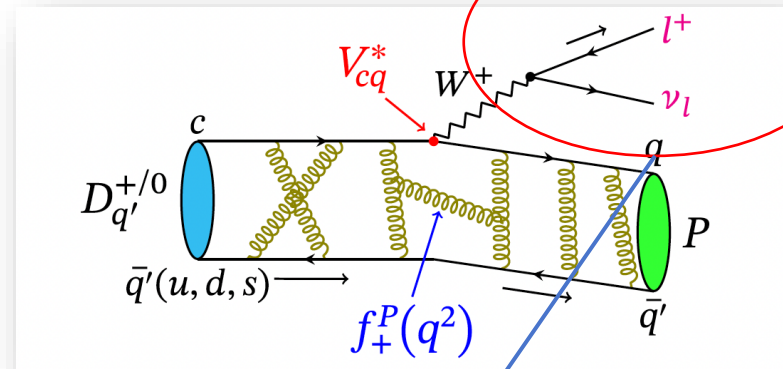
Purely leptonic decays



$$\Gamma(D^+ \rightarrow l^+ \nu_l) = \frac{G_F^2 f_{D^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D^+} \left(1 - \frac{m_l^2}{m_{D^+}^2}\right)^2$$

$$\mathcal{R}_{\tau/\mu} = \frac{\mathcal{B}(D^+ \rightarrow \tau^+ \nu_\tau)}{\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu)}$$

Semi-leptonic decays ( $l = e, \mu$ )

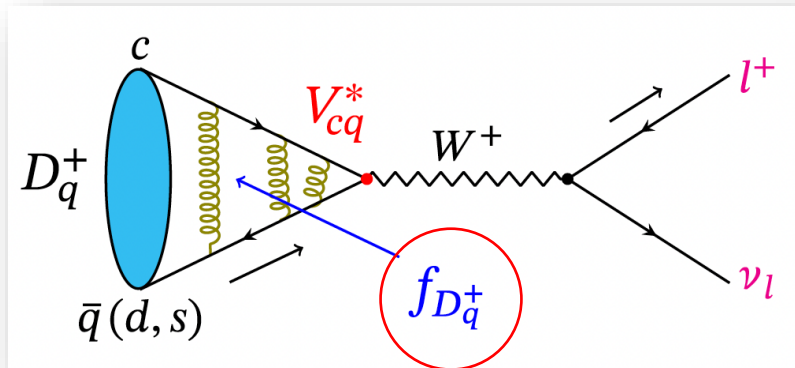


$$\frac{d\Gamma}{dq^2} = x \frac{G_F^2 p^3}{24\pi^3} |f_+^P(q^2)|^2 |V_{cd(s)}|^2$$

$$\mathcal{R}_{\mu/e} = \frac{\mathcal{B}(D \rightarrow X \mu^+ \nu_\mu)}{\mathcal{B}(D \rightarrow X e^+ \nu_e)}$$

# Precision test of LQCD calculations

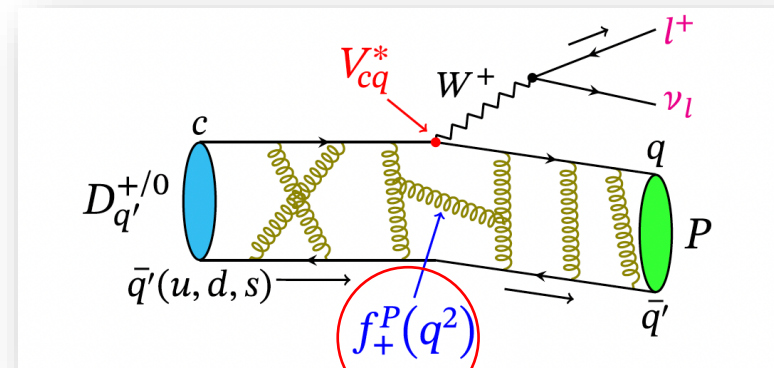
## Purely leptonic decays



$$\Gamma(D^+ \rightarrow l^+ \nu_l) = \frac{G_F^2 f_{D^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D^+} \left(1 - \frac{m_l^2}{m_{D^+}^2}\right)^2$$

**Decay constant**

## Semi-leptonic decays



$$\frac{d\Gamma}{dq^2} = x \frac{G_F^2 p^3}{24\pi^3} |f_+^P(q^2)|^2 |V_{cd(s)}|^2$$

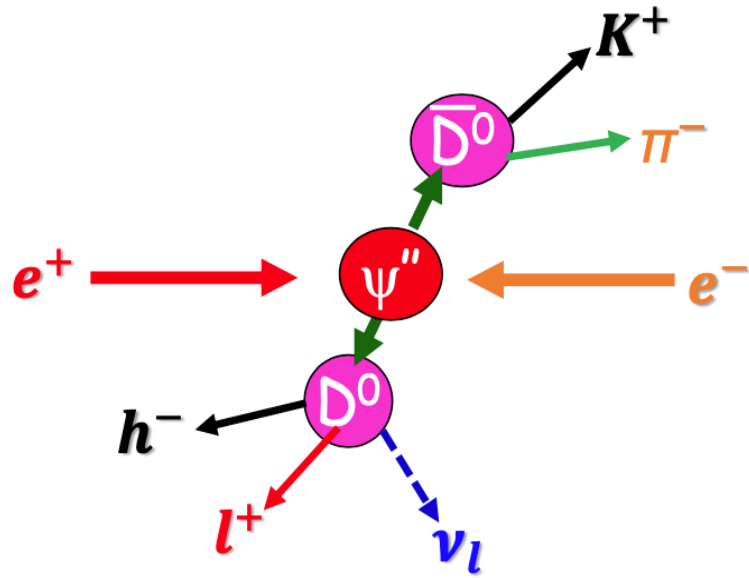
**Form factors**

**LQCD calculations**

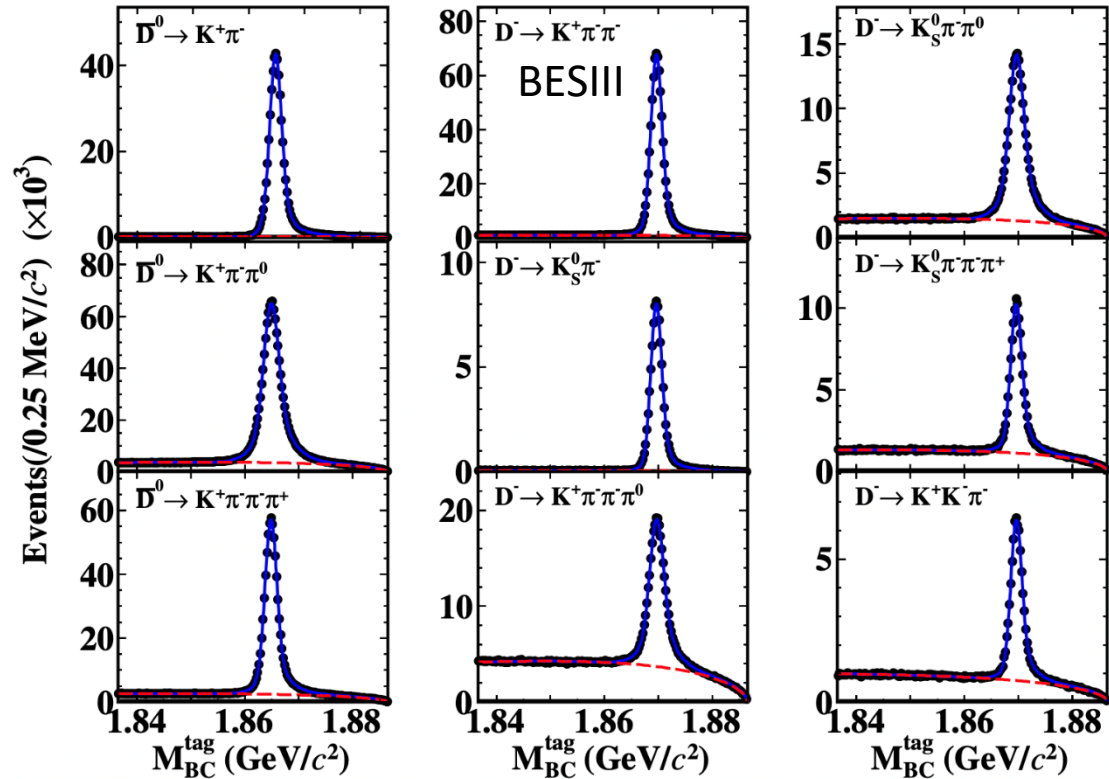
Probe the nature of light mesons like  $f_0(500)$  /  $a_0(980)$  /  $K_1(1270)$ ...

- $D \rightarrow \ell^+ \nu_\ell$
- $D \rightarrow \text{Pseudoscalar} \ell^+ \nu_\ell$
- $D \rightarrow \text{Vector} \ell^+ \nu_\ell$
- $D \rightarrow \text{Scalar} \ell^+ \nu_\ell$
- $D \rightarrow \text{Axialvector} \ell^+ \nu_\ell$

# Double tagged $D$ mesons



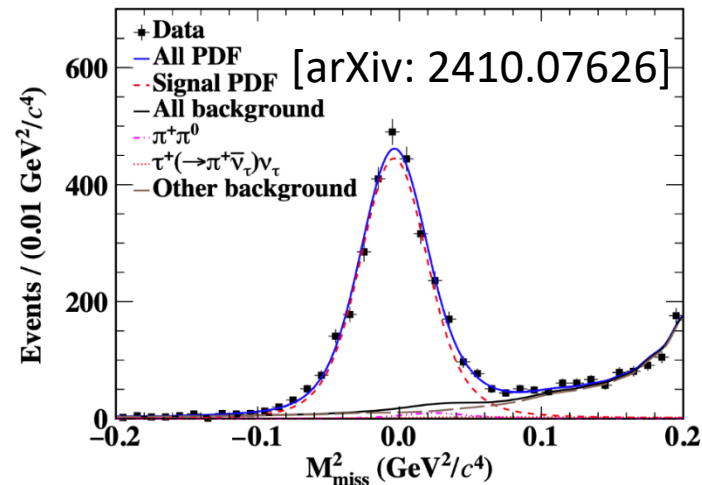
- $20.3 \text{ fb}^{-1}$  @  $3.773 \text{ GeV}$
- $7.33 \text{ fb}^{-1}$  @  $4.128\text{-}4.226 \text{ GeV}$



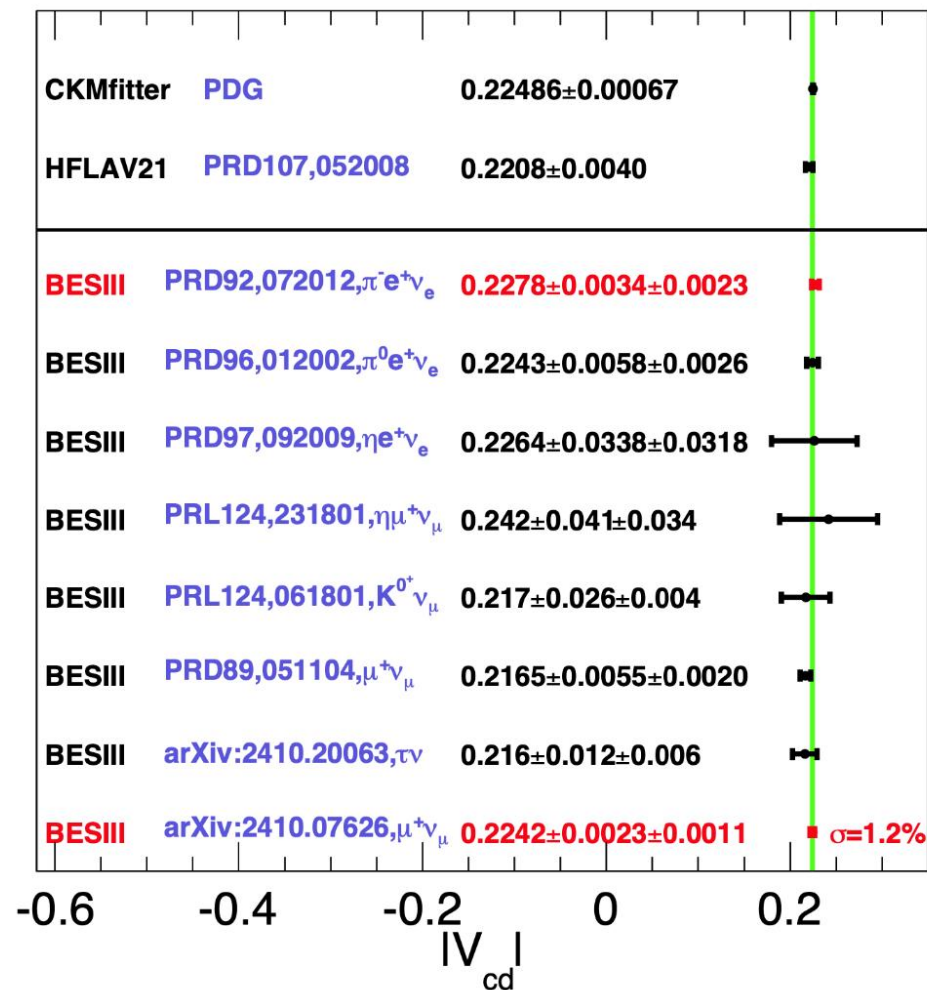
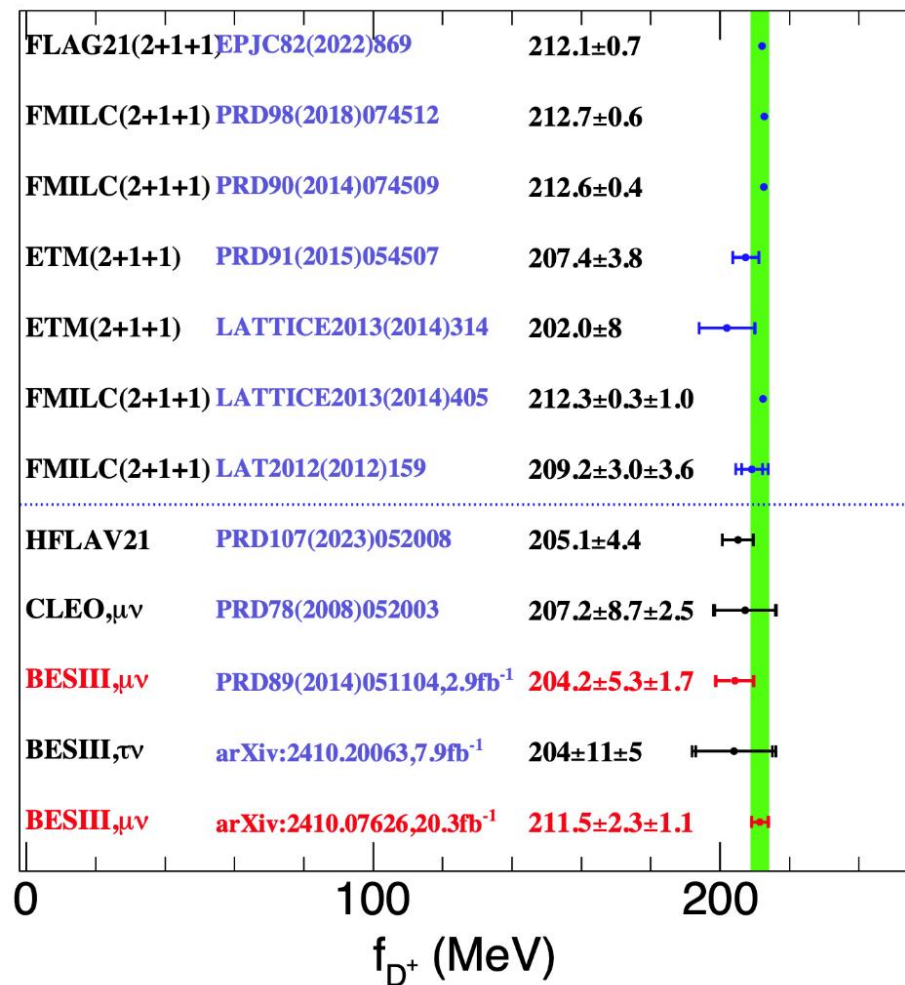
Reconstruct the missing neutrino

$$U_{\text{miss}} = E_{\text{miss}} - |\vec{p}_{\text{miss}}|$$

$$M_{\text{miss}}^2 = E_{\text{miss}}^2 - |\vec{p}_{\text{miss}}|^2$$

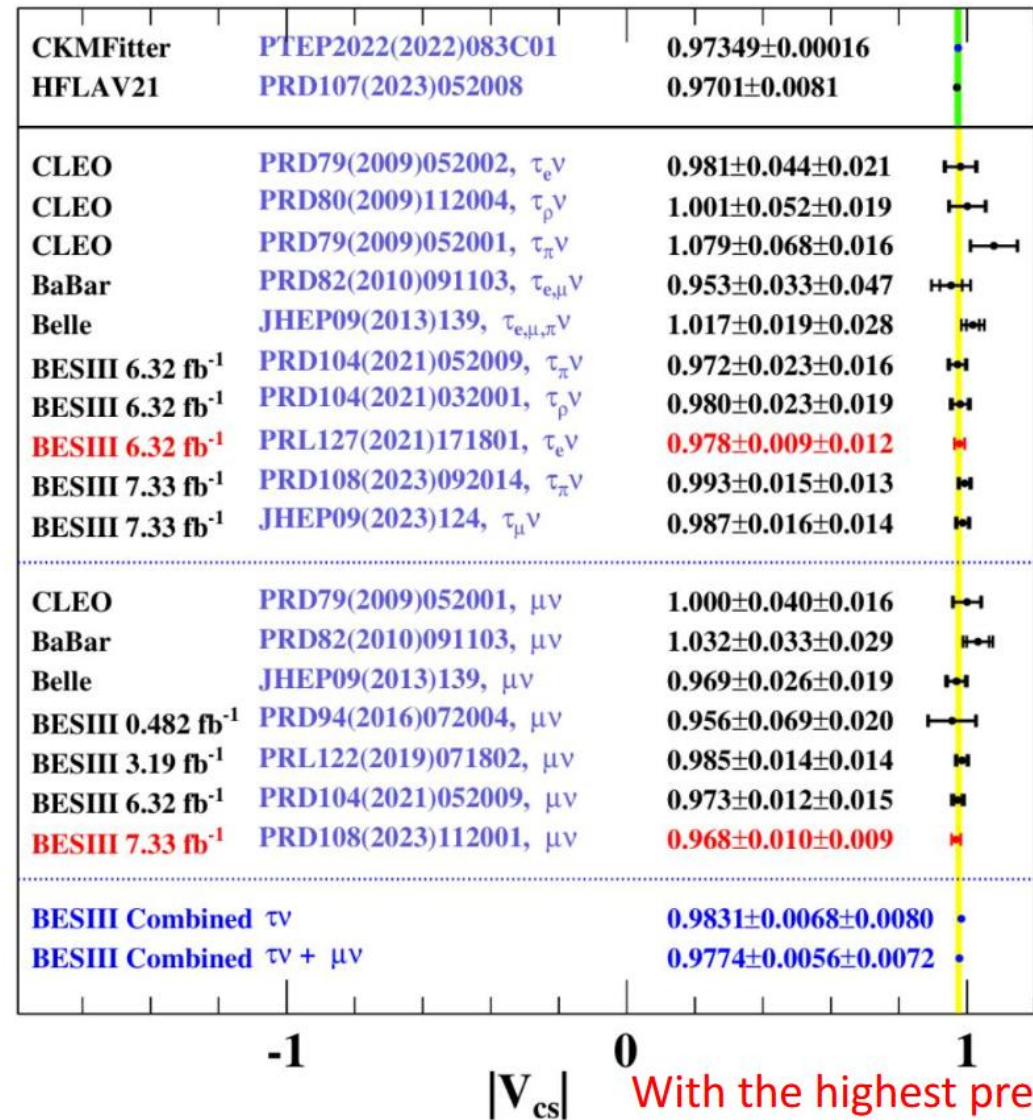
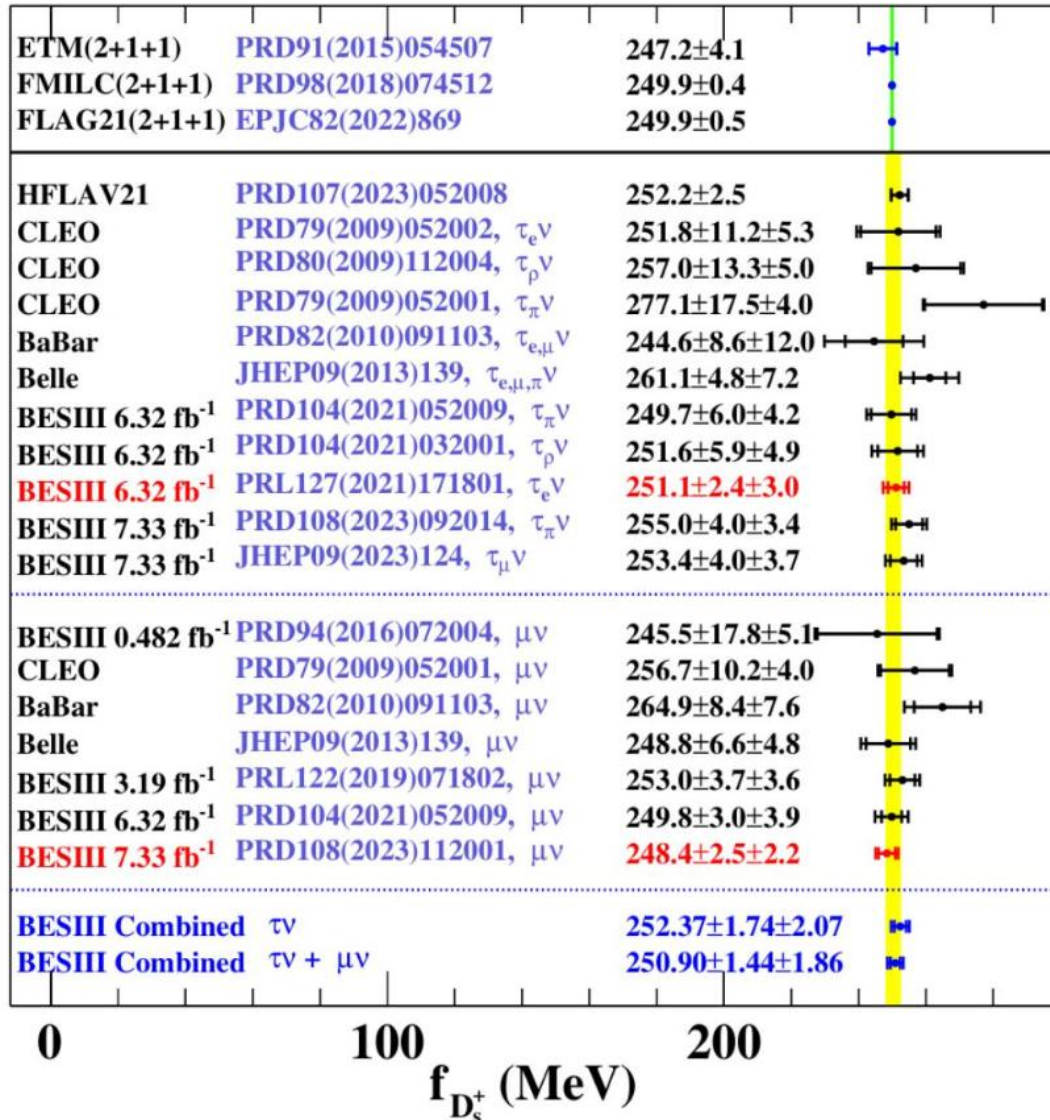


# Decay constant $f_{D^+}$ and $|V_{cd}|$ with $D^+ \rightarrow l^+ \nu_l$



Precision of decay constant and  $|V_{cd}|$  reach 1.2%

# Decay constant $f_{D_s^+}$ and $|V_{cs}|$ with $D_s^+ \rightarrow l^+ \nu_l$



With the highest precision: 0.9%

# Study of the $D \rightarrow K \ell^+ \nu_\ell$ decay

➤ 7.93 fb<sup>-1</sup> data @ 3.773 GeV

➤  $\mathcal{B}(D^0 \rightarrow K^- e^+ \nu_e) = (3.521 \pm 0.009 \pm 0.016)\%$

$\mathcal{B}(D^0 \rightarrow K^- \mu^+ \nu_\mu) = (3.419 \pm 0.011 \pm 0.016)\%$

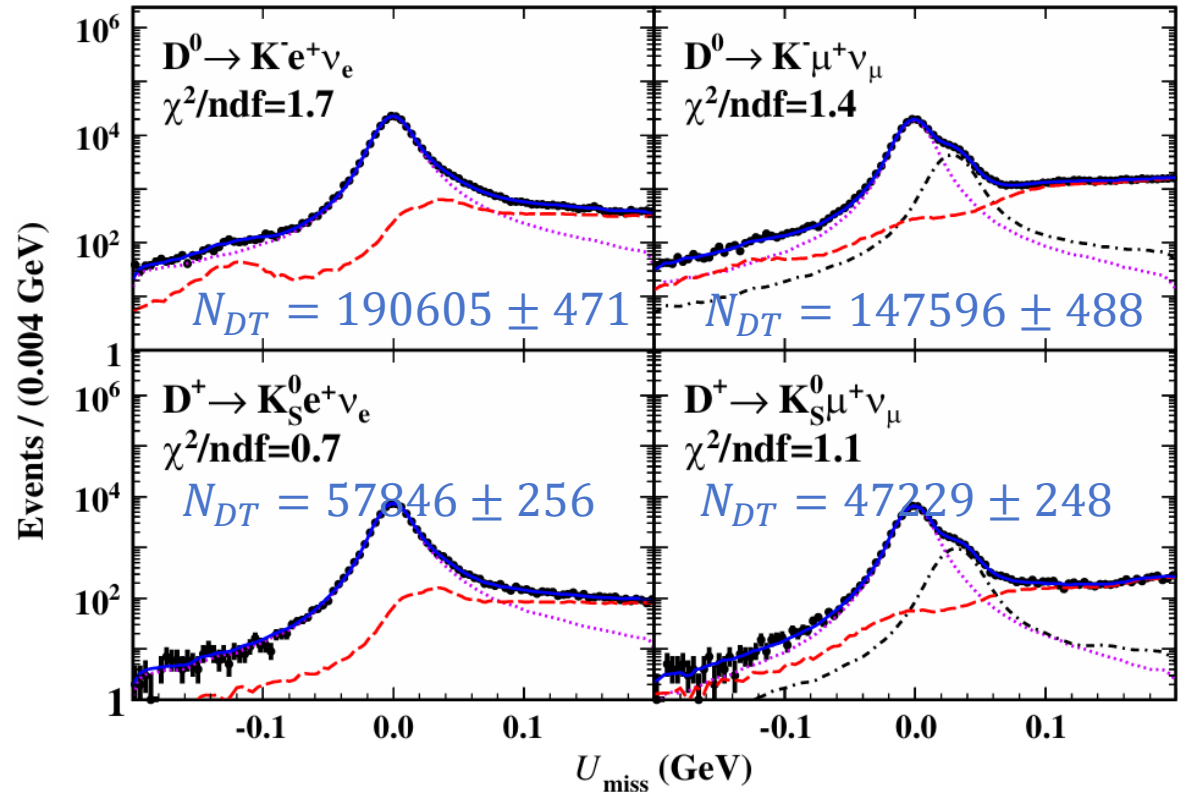
$\mathcal{B}(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = (8.864 \pm 0.039 \pm 0.082)\%$

$\mathcal{B}(D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu) = (8.665 \pm 0.046 \pm 0.084)\%$

➤ LFU test (SM:  $0.975 \pm 0.001$ )

$$\frac{\mathcal{B}(D^0 \rightarrow K^- \mu^+ \nu_\mu)}{\mathcal{B}(D^0 \rightarrow K^- e^+ \nu_e)} = 0.971 \pm 0.004 \pm 0.006 (\sim 0.7\%)$$

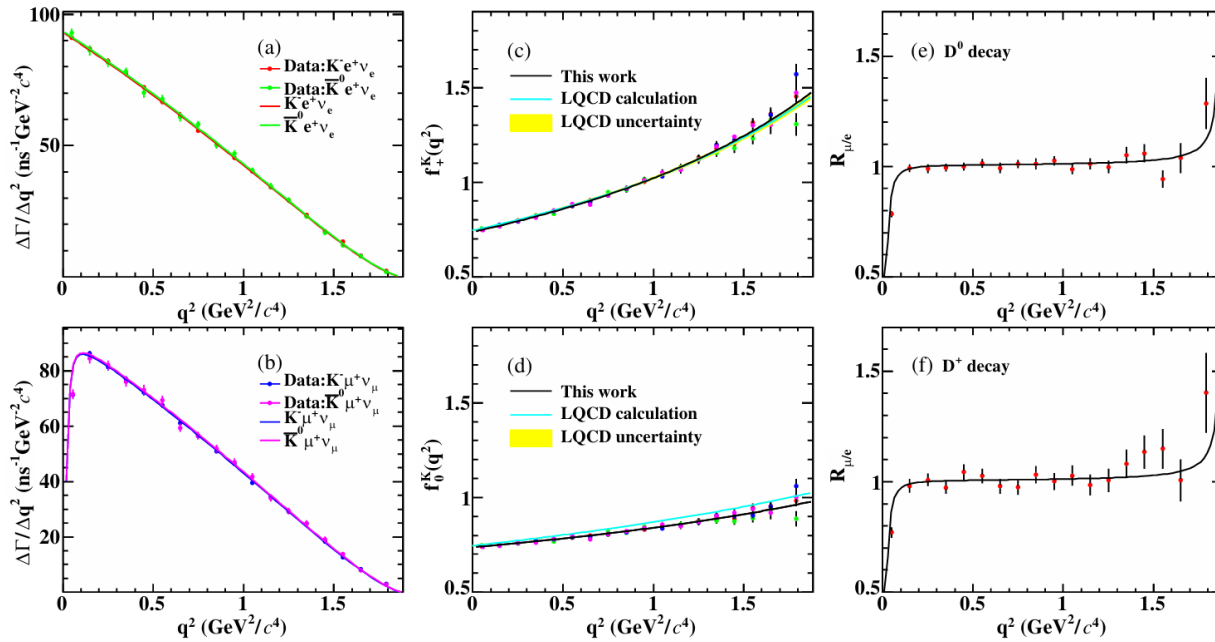
$$\frac{\mathcal{B}(D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu)}{\mathcal{B}(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)} = 0.978 \pm 0.007 \pm 0.013 (\sim 1.5\%)$$



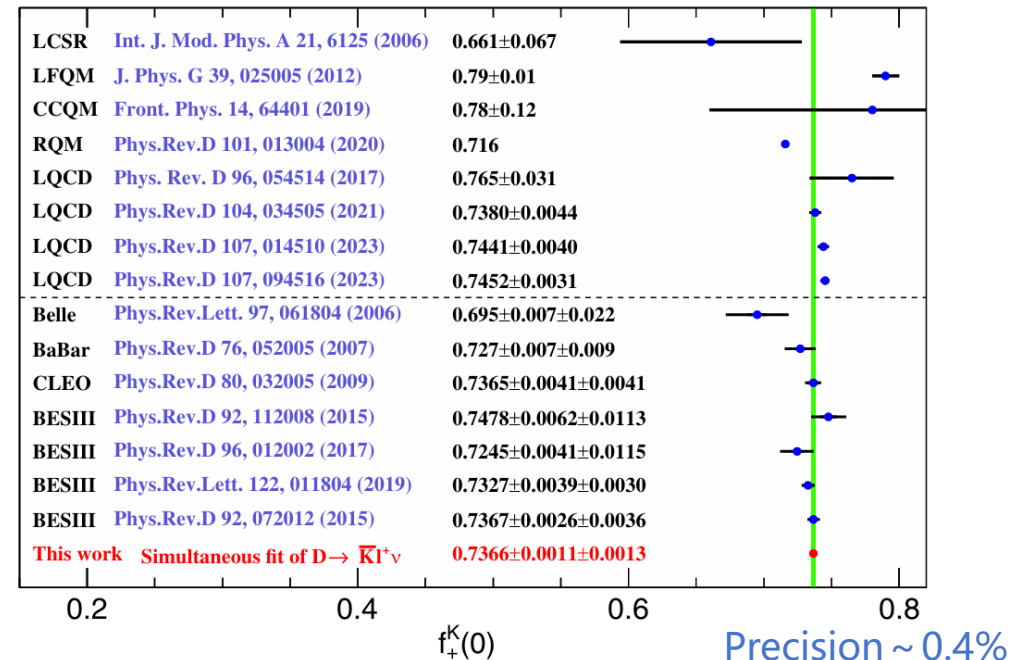
[PRD 110 (2024) 112006]



# Study of the $D \rightarrow K \ell^+ \nu_\ell$ decay



[PRD 110 (2024) 112006]



- $f_+^K(0) = 0.7366 \pm 0.0011 \pm 0.0013$ , consistent with the LQCD calculation
- $|V_{cs}| = (0.9623 \pm 0.0015 \pm 0.0017 \pm 0.0040) \text{ MeV}$

# Summary

- Precision study of charm mixing and CP violation would be one of the most important goals of heavy flavor physics in coming years
- More precise determination of the CKM matrix elements in the charm sector is also essential to test SM
- Charm physics studies at Belle (II) and BESIII (or STCF in the future) are complementary to each other
- There are a lot of other progresses, eg. charmed baryons, rare charm decays etc...
  - See [Belle](#) , [Belle II](#), [BESIII](#)

Thanks!