Quantum to classical transition of free electrons interaction with light

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ABSTRACT

We present a theory for free electrons interaction with radiation in both classical and quantum regimes and delineate their transition, based on a model of quantum electron wavepacket (QEW). The theory has general validity for a wide range of free electron interaction and radiation sources, including Free Electron Lasers, Cerenkov radiation, and transition radiation. We exemplify our analysis with the schemes of Smith-Purcell radiation and dielectric laser acceleration (DLA) [1, 2]. These interactions, which were studied in terms of point particle physics, have a quantum nature in a phenomenon known as "photon-induced near-field electron microscopy" (PINEM) [3].

Our QEW model identifies three universal distinct interaction regimes: (i) near-point-particle acceleration/deceleration DLA regime, (ii) PINEM regime of multiphoton induced electron energy sidebands, and (iii) anomalous PINEM regime (APINEM) of a newly reported periodic spectral bunching. See the three regimes in Fig.2.

The formulation displays the transition of the FEL stimulated gain expression from the quantum to classical limit. Elsewhere we provided extension of the semiclassical model to quantum electrodynamics [4] to include spontaneous emission and spontaneous superradiance by modulated QEW similar to the classical prebunched-beam superradiant FEL in the classical point-particle picture [5, 6].

SETUP and MODELLING

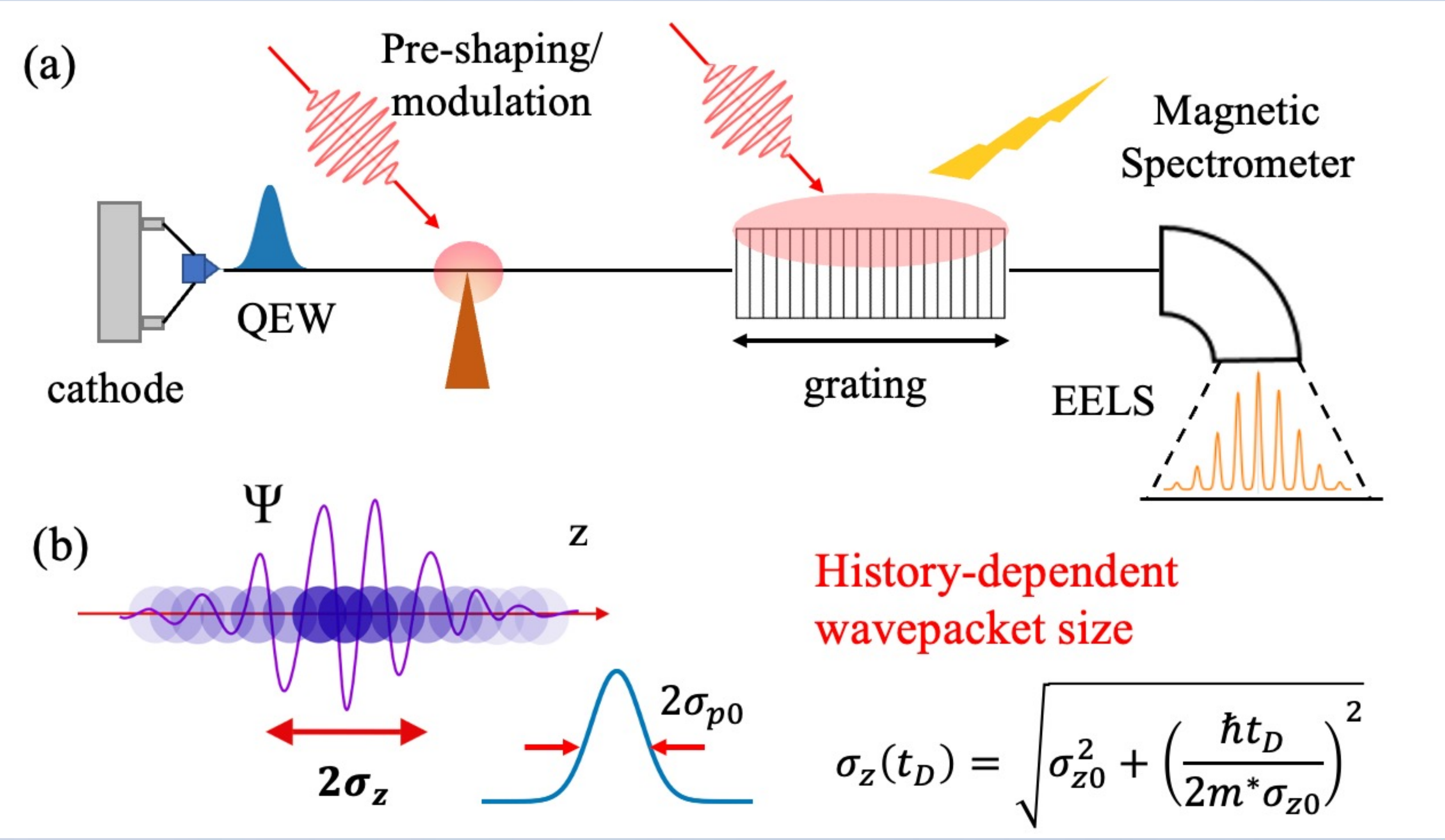

Figure 1. (a) stimulated Smith-Purcell setup for free electron interaction with the optical near-field on a grating. (b) the history-dependent electron wavepacket in space domain.

Our setup is shown in Fig. 1a, the electron is described as a quantum electron wavepacket (QEW) with finite size, which can be controlled by a THz or IR laser [7]. The initial QEW is modeled as a 1D Gaussian wavepacket [8, 9] (Fig. 1b)

$$\psi^{(0)}(z,t) = \int \frac{dp}{\sqrt{2\pi\hbar}}\, c_i(p) e^{-iE_p t/\hbar} e^{ipz/\hbar}$$

where $c_i(p) = \frac{1}{(2\pi\sigma_{p0}^2)^{1/4}} \exp\left[-\frac{(p-p_0)^2}{4\sigma_{p0}^2}\right]$ is the momentum wavefunction and $\sigma_p = \hbar/2\sigma_{z_0}$ is determined by the QEW waist size σ_{z_0}. The electron wavepacket size depends on both the waist size σ_{z_0} and the pre-drift time t_D, defined as $\sigma_z(t_D) = \sqrt{\sigma_{z_0}^2 + \left(\frac{\hbar t_D}{2m^*\sigma_{z0}}\right)^2}$.

The interaction of QEW with the near-field on the grating is described by Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(z,t) = [H_0 + H_I(t)]\,\psi(z,t)$$

where H_0 is the free electron kinetic energy [6] and the interaction Hamiltonian $H_I(t) = -\frac{e}{\gamma m}\boldsymbol{A}(t)\cdot\boldsymbol{p}$ is the general form of light-matter interaction. The coupling constant between the free electron and the near field can be defined as: $\Upsilon = \frac{eE_z L_I}{4\hbar\omega}$, which is proportional to the average photon number exchange on the grating.

RESULTS

Quantum to classical transition

The Schrödinger equation can be solved by first order perturbation, and the final electron state is

$$c_f(p) = \frac{1}{\sqrt{N}}\left(c_i(p) + c^{(1)}(p)\right)$$

where N is the normalization factor. Thus, the final momentum distribution can be expressed as

$$\rho^{(f)}(p) = \rho^{(0)}(p) + \rho^{(1)}(p) + \rho^{(2)}(p)$$

where $\rho^{(0)} = \frac{1}{N}|c_i(p)|^2$ and $\rho^{(2)}(p) = \frac{1}{N}\left|c^{(1)}(p)\right|^2 \propto \Upsilon^2$ both have a symmetric distribution around p_0 while the interference term $\rho^{(1)} = \frac{1}{N}2Re\left[c^{(1)}(p)c_i^*(p)\right] \propto \Upsilon$ is antisymmetric. Thus, the main momentum transfer due to the interaction is

$$\Delta p^{(1)} = \int dp\, p\, \rho^{(1)}(p) = \Delta p_{point}\, e^{-\Gamma^2/2}$$

where the parameter Γ is a function of both waist wavepacket size σ_{z_0} and pre-drift time t_D, defined by the ratio between wavepacket size and the optical wavelength

$$\Gamma(\sigma_{z_0}, t_D) = \frac{\omega}{v_0}\sigma_z(t_D) = 2\pi\frac{\sigma_t(t_D)}{T}$$

As shown in Fig. 2b, the acceleration occurs when $\sigma_t(t_D) < T$. In the limit of $e^{-\Gamma^2/2} \to 1$, Δp_{point} yields the classical acceleration of QEW (**DLA**).

The opposite effect is called **PINEM** (Fig. 2a) when the electron satisfies large recoil condition ($\sigma_E < \hbar\omega$ and $\sigma_t(t_D) > \sigma_{t_0} > T$), where $\Delta p^{(1)} \to 0$ and the 2nd order $\rho^{(2)}(p)$ is dominant and the electron exhibits energy spectrum with sidebands separated equally by the photon energy $\hbar\omega$.

The **APINEM** [9] happens in the presence of QEW energy chirp, where the $\sigma_t(t_D) > T > \sigma_{t_0}$. As shown in Fig. 2c, the sidebands are not related to the quantum recoil energy.

The 2nd order provides expression for stimulated energy transfer. By conservation of energy, it results in the FEL gain expression in the quantum regime:

$$\Delta\nu^{(2)} = -\frac{\Delta p^{(2)}}{\hbar\omega/v_0} = \frac{v_0}{\hbar\omega}\int dp\, p\, \rho^{(2)}(p) = \Upsilon^2\left[sinc^2\left(\frac{\bar\theta_e}{2}\right) - sinc^2\left(\frac{\bar\theta_a}{2}\right)\right]$$

$\bar\theta_{e,a} = \bar\theta \pm \epsilon/2$, where $\bar\theta = \left(\frac{\omega}{v_0} - q_z\right)L_I$ is the detuning parameter and $\epsilon = \frac{\hbar\omega^2 L_I}{2m^* v_0^3}$ is the quantum recoil parameter. When $\epsilon \to 0$, the formula is consistent with photon emission rate in the classical FEL regime:

$$\Delta\nu^{(2)} \to \Upsilon^2\epsilon\frac{\partial}{\partial\bar\theta}sinc^2\left(\frac{\bar\theta}{2}\right)$$

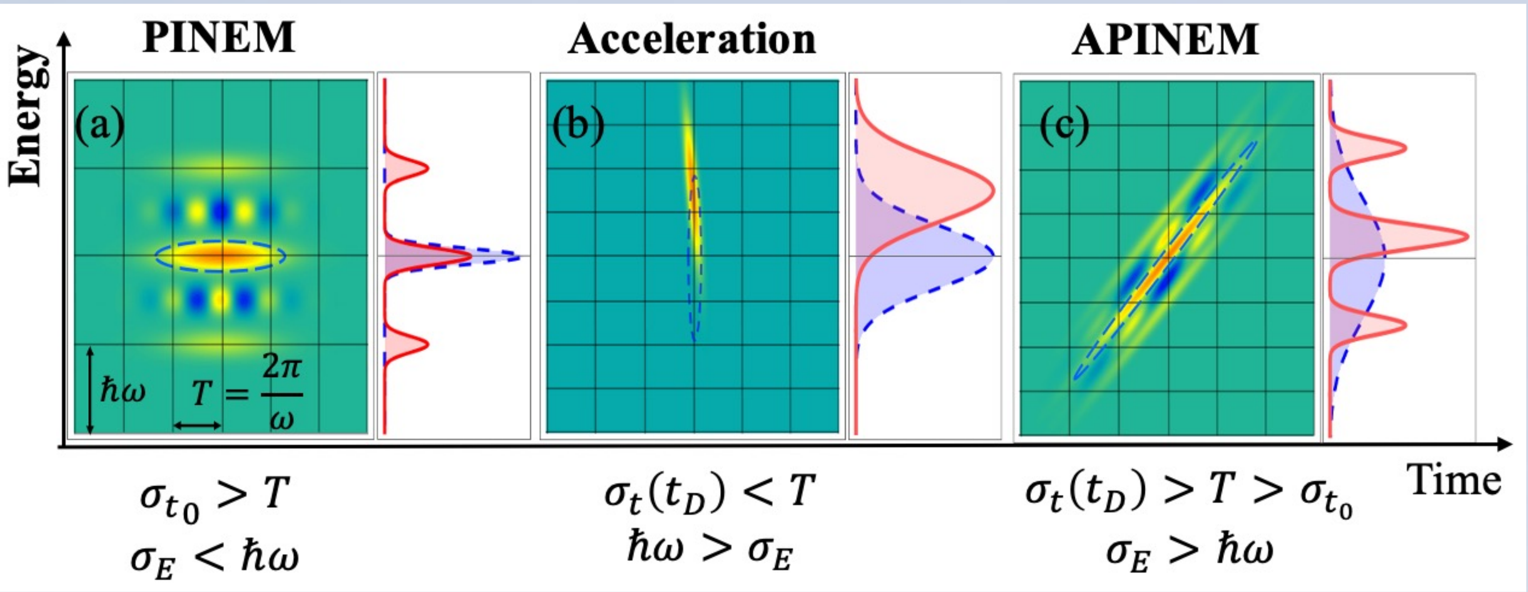

Figure 2. The numerically calculated Wigner distribution of initial QEW (dashed) and final QEW in phase space ($E - t$) for different interaction regimes $\Gamma(\sigma_{z_0}, t_D)$

CONCLUSIONS

The quantum to classical transition of the free electron interaction with radiation could be determined by the parameter $\Gamma(\sigma_{z_0}, t_D)$.

When $\Gamma(\sigma_{z_0}, t_D) > 1$, the quantum effect emerges:

1. For large σ_{z_0} (corresponding to the large recoil), we can get the PINEM effect with sidebands separated by the photon energy.
2. For small σ_{z_0} with large chirping (large pre-drift t_D), we can get the APINEM effect.

When $\Gamma(\sigma_{z_0}, t_D) < 1$, the electron exhibits the classical acceleration behaviour.

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