Comparative Study of Virtual Pepper-Pot and Differential Virtual Pepper-Pot Methods for 4D Transverse Phase Space Reconstruction of Small Beams

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4D Transverse Phase-Space Diagnostics for Small Beams

$x-y$ Coupled Beam

Projected second moments

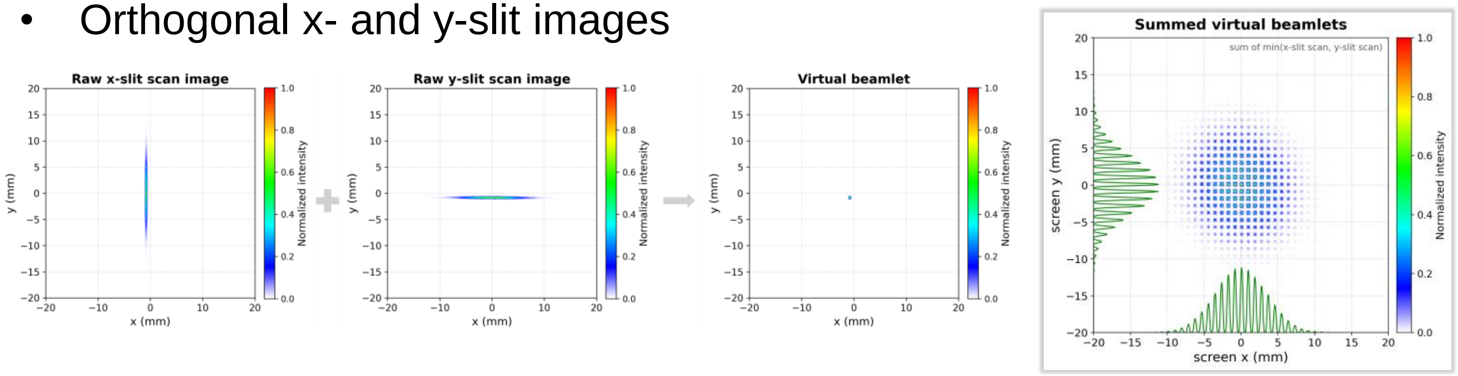
+

$x-y$ coupling terms

$$\Sigma_{4D} = \begin{pmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{pmatrix}$$

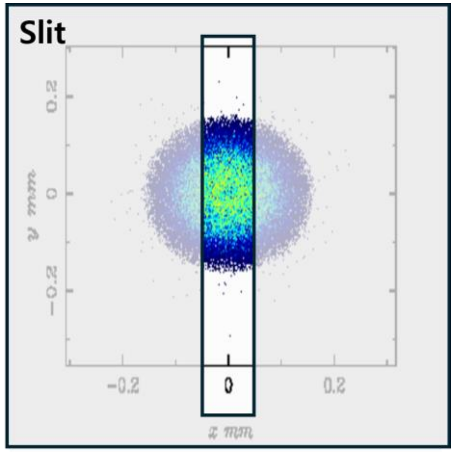
Virtual Pepper-Pot (VPP)

- Orthogonal x- and y-slit images



[1] G. Z. Georgiev and M. Krasilnikov, Proc. IBIC'19 (2019)

Finite-Slit-Width Limitation



- Limited spatial resolution for small beams
- Particle-position uncertainty within the slit
- Uncertainty in the reconstructed angle

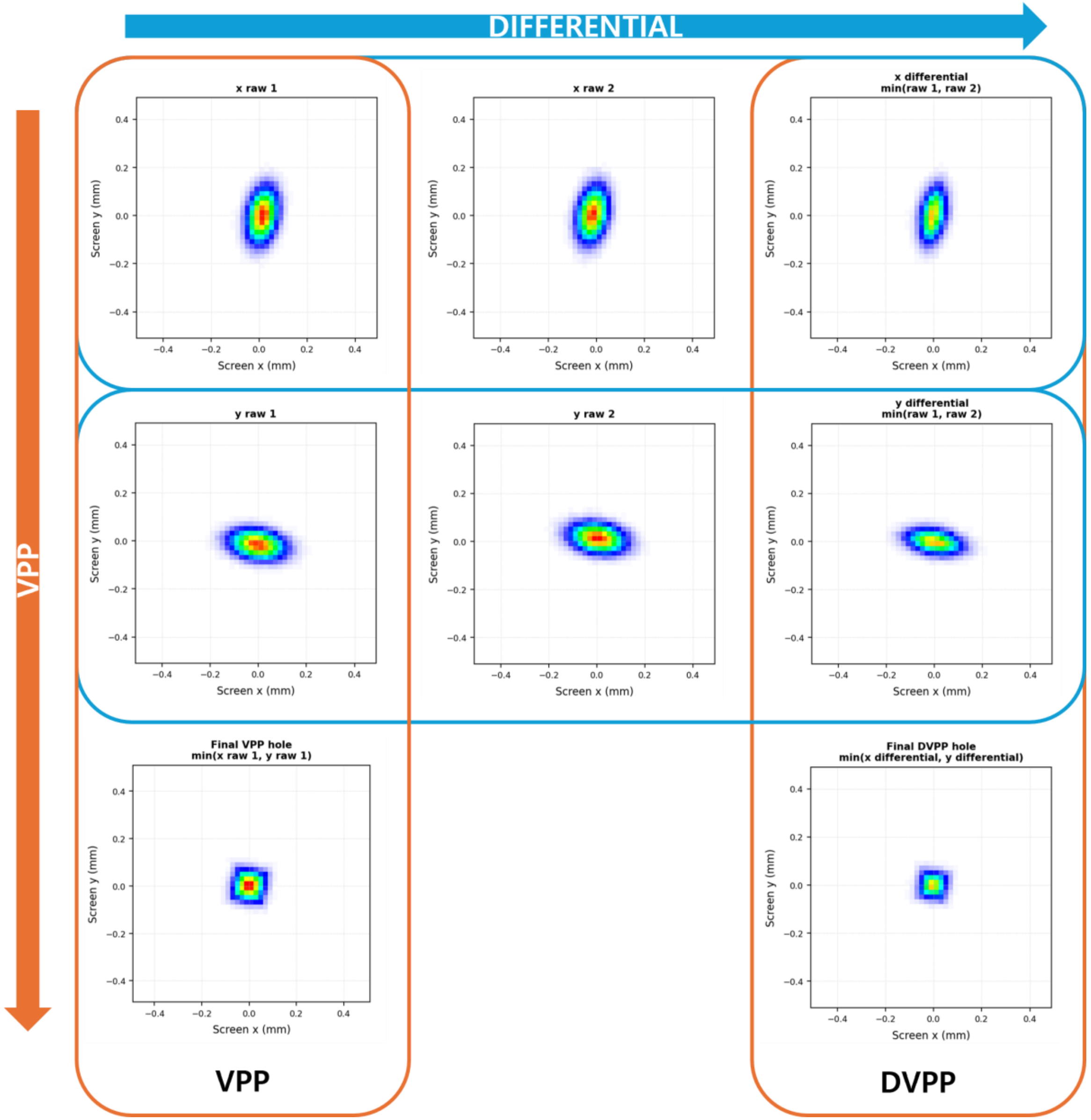
Differential Virtual Pepper-Pot (DVPP)

- Parallel slit images *separated by one slit width*

⇒ Virtually narrower slit images

[2] B. Shin et al, Proc. IBIC'23 (2023)

VPP and DVPP: Reconstruction Workflows



- Pixel-wise Minimum Operation:

$$Q_{min} = \min(Q_i, Q_j)$$

VPP

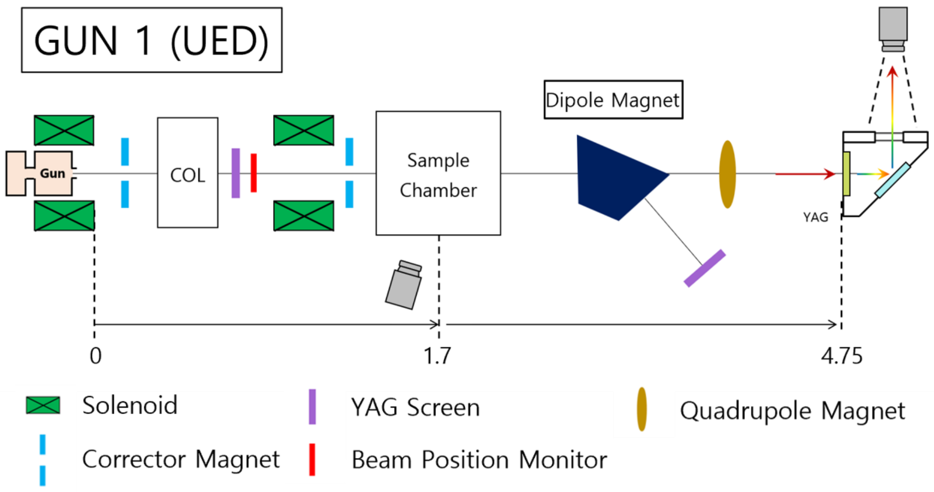
1. Orthogonal images (x- and y-slit images)
   Pixel-wise minimum operation
2. Virtual beamlets
   → 4D beam-matrix reconstruction

Differential VPP (DVPP)

1. Parallel slit-image pairs (one slit width apart)
   1^{st} Pixel-wise minimum operation
2. Orthogonal differential images
   2^{nd} Pixel-wise minimum operation
3. Virtual beamlets
   → 4D beam-matrix reconstruction

PAL-eLABs Gun1: Beamlet Space Charge with the L-Shaped Slit Scanner

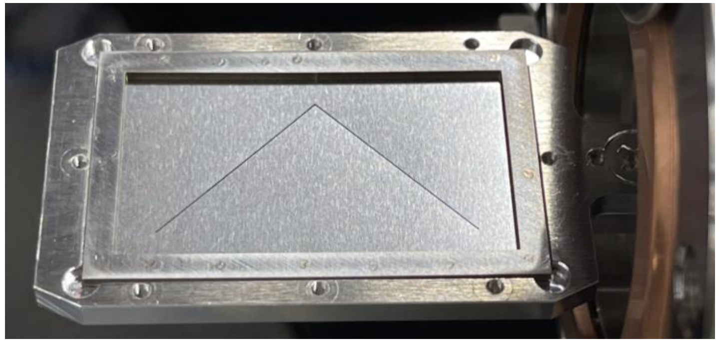
PAL-eLABs Gun1 Layout

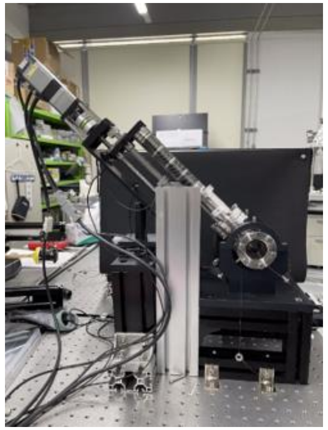


Gun1 Operating Range

Parameter	Value
Particle	Electron
Energy	$3 - 5\ \mathrm{MeV}$
Charge	$< 1\ \mathrm{pC}$

Existing L-shaped Slit Scanner





[2] B. Shin et al, Proc. IBIC'23 (2023)

Fabricated for Gun2 VPP and DVPP

- PAL-eLABs Gun2:

Beam Energy	Charge	Slit width
$70\ \mathrm{MeV}$	$< 250\ \mathrm{pC}$	$d = 140\ \mu m$

PAL-eLABs Gun1 Application Test

Beam Energy	Charge		
$3.768\ \mathrm{MeV}$	$75\ \mathrm{fC}$	$\Rightarrow$	Beamlet space charge is non-negligible

PAL-eLABs Gun1: Geometry Selection and Residual Space Charge

1. Slit-width Selection

- R_b: beamlet space-charge ratio
  $\Rightarrow R_b \ll 1$: emittance-dominated

Slit width	R_b	$Q_{beamlet}$
$d = 10\ \mu\mathrm{m}$	0.045	4.24 fC
$\boldsymbol{d = 50\ \mu\mathrm{m}}$	**1.13**	**21.21 fC**
$d = 100\ \mu\mathrm{m}$	4.52	40.58 fC

Space Charge (SC)↔ Signal **Trade-off**

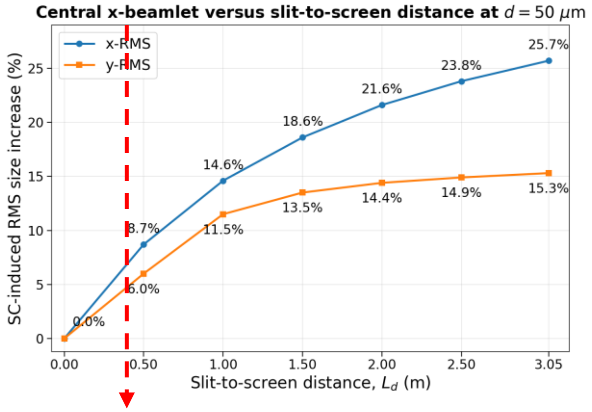
2. Reduce L_d at $d = 50\ \mu m$

- R_{sp}: angular-resolution parameter
  $\Rightarrow R_{sp} > 1$: lower-bound criterion for L_d

$$L_d > 0.180\ \mathrm{m}$$

Selected : $\boldsymbol{L_d = 0.4\ \mathrm{m}\ (R_{sp} = 2.22)}$

Beamlet RMS size Increases



Selected $L_d = 0.4\ \mathrm{m}$ $+7.14\%\ (x)$ $+4.36\%\ (y)$

Residual SC assessment

- Conventional slit-scan validation of ϵ_x $\Rightarrow$ SC-induced increase in relative reconstruction error

$\boldsymbol{d = 100\ \mu\mathrm{m},\ L_d = 3.05\ \mathrm{m}}$ → $\boldsymbol{d = 50\ \mu\mathrm{m},\ L_d = 0.4\ \mathrm{m}}$

33.31 pp → 6.57 pp

$\approx 80\ \%$ reduction, but residual SC remains

VPP and DVPP: Representative Reconstruction Results

* Note: SC excluded during slit-to-screen transport to isolate VPP/DVPP performance

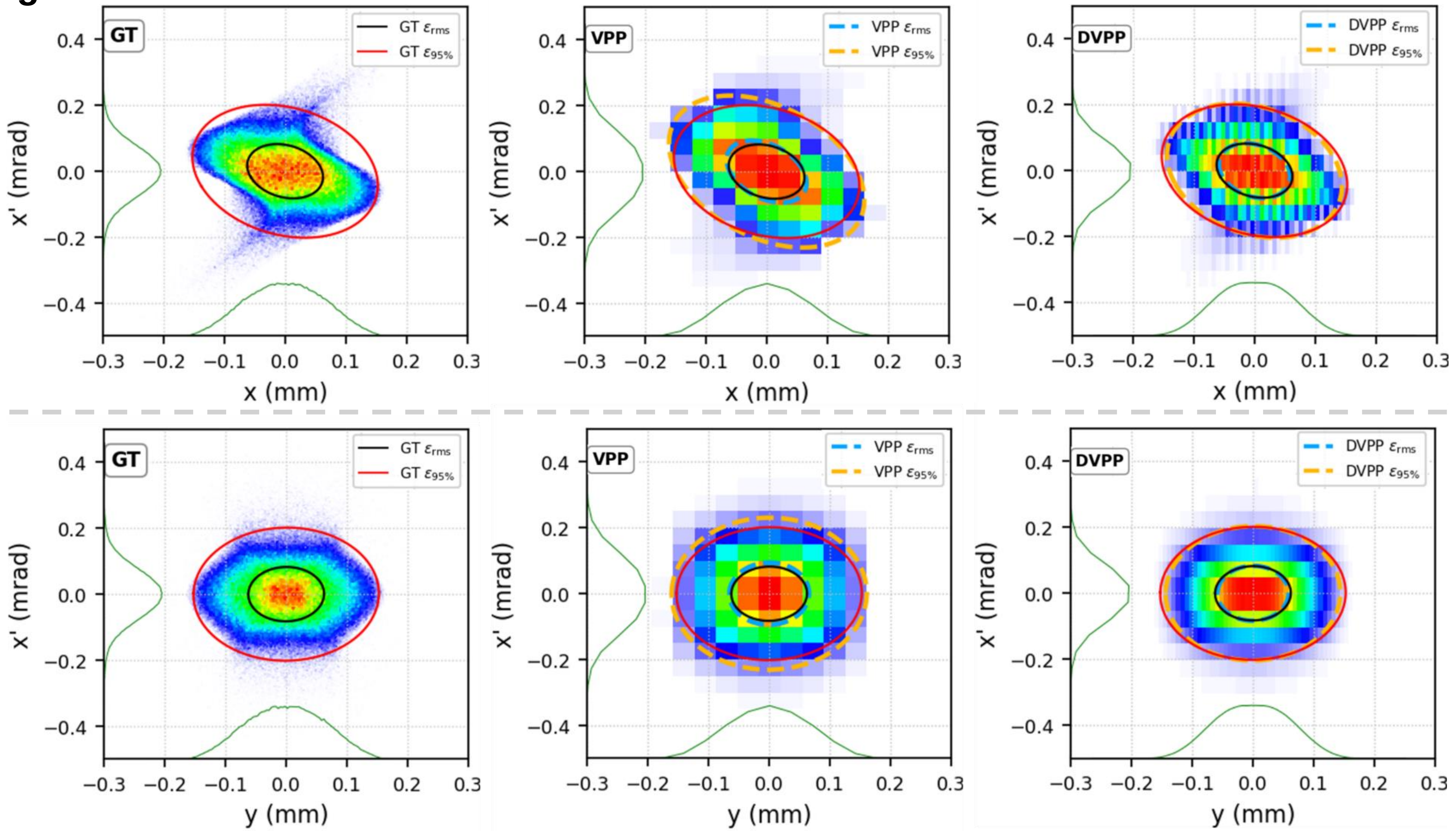
VPP
- Scan step size: $50\ \mu\mathrm{m}$

DVPP
- Scan step size: $10\ \mu\mathrm{m}$
- Differential slit image pair: $(i^{th}, i+5^{th})$

Weak-Coupling Case

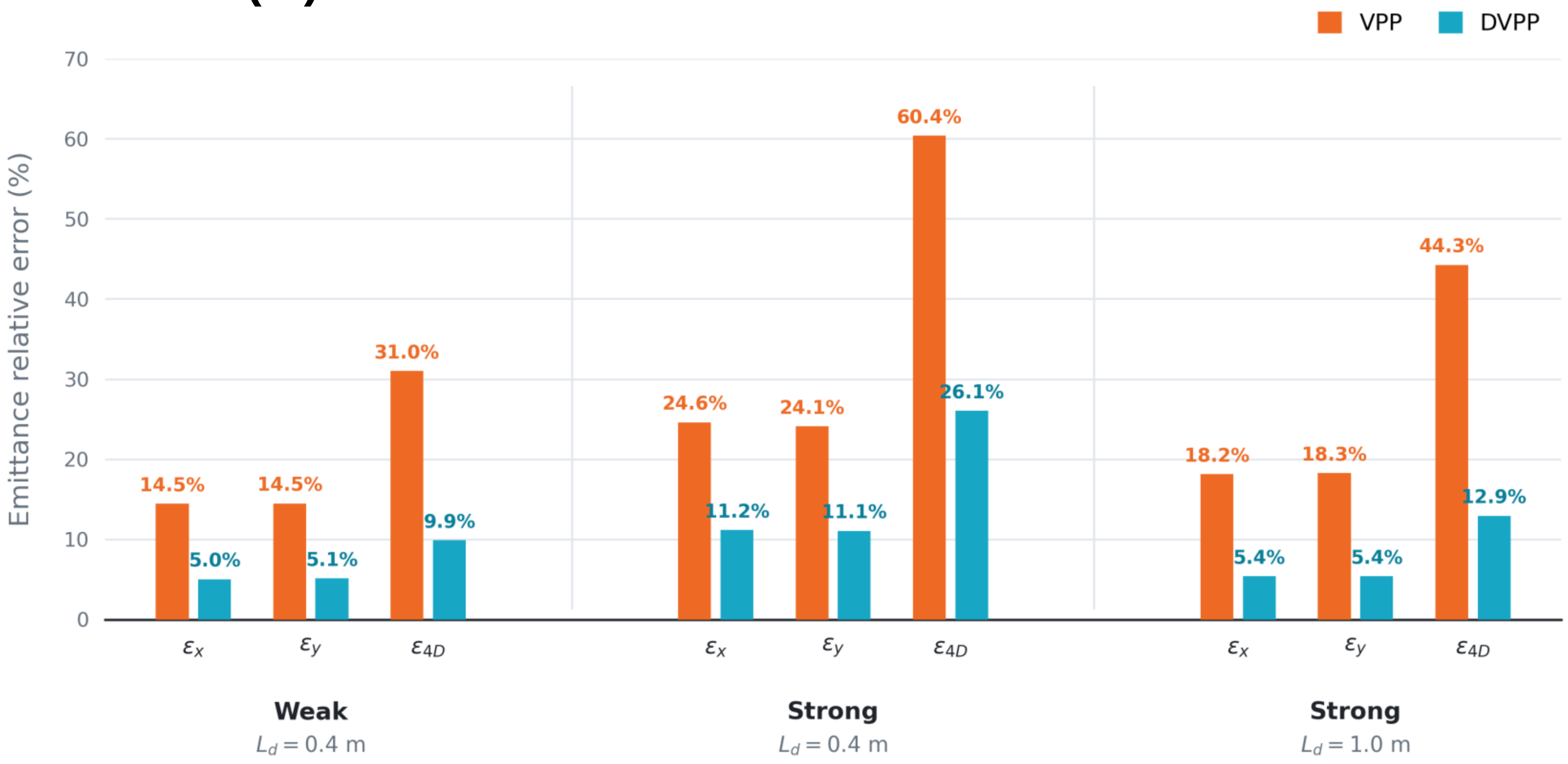
$x - x'$

$y - x'$



VPP and DVPP: Quantitative Comparison

- **Emittance Relative Error (%)**



- **Coupling-Factor Reconstruction**

Coupling	L_d (m)	Ground Truth t	VPP t	DVPP t
Weak	0.4	0.115×10^{-3}	0.072×10^{-3}	0.085×10^{-3}
Strong	0.4	0.101	0.062	0.078
Strong	1.0	0.101	0.066	0.083

VPP/DVPP Study for PAL-eLABs Gun1

1. PAL-eLABs Gun1 Space-Charge Study

- Existing $140\ \mu\mathrm{m}$ Gun2 slit: non-negligible beamlet SC in Gun1
- Selected: $d = 50\ \mu\mathrm{m}, L_d = 0.4\ \mathrm{m}$
- SC-induced increase in ϵ_x error: $33.31 \rightarrow 6.57\ \mathrm{pp}$

  ⇒ $\approx 80\%$ reduction, but residual SC remains

2. VPP/DVPP Reconstruction Comparison

- *SC excluded during slit-to-screen transport for intrinsic comparison*
- DVPP

  ⇒ Lower relative errors in $\epsilon_x, \epsilon_y,\ \mathrm{and}\ \epsilon_{4D}$

  ⇒ Coupling factor closer to GT

3. [Outlook] Options for Practical Gun1 Application

① Narrower slit with further L_d optimization

② SC-corrected reconstruction algorithm

Thank you · Poster TUPO011