Coupled Dynamics and Transverse Matching of Heavy Ion Beams

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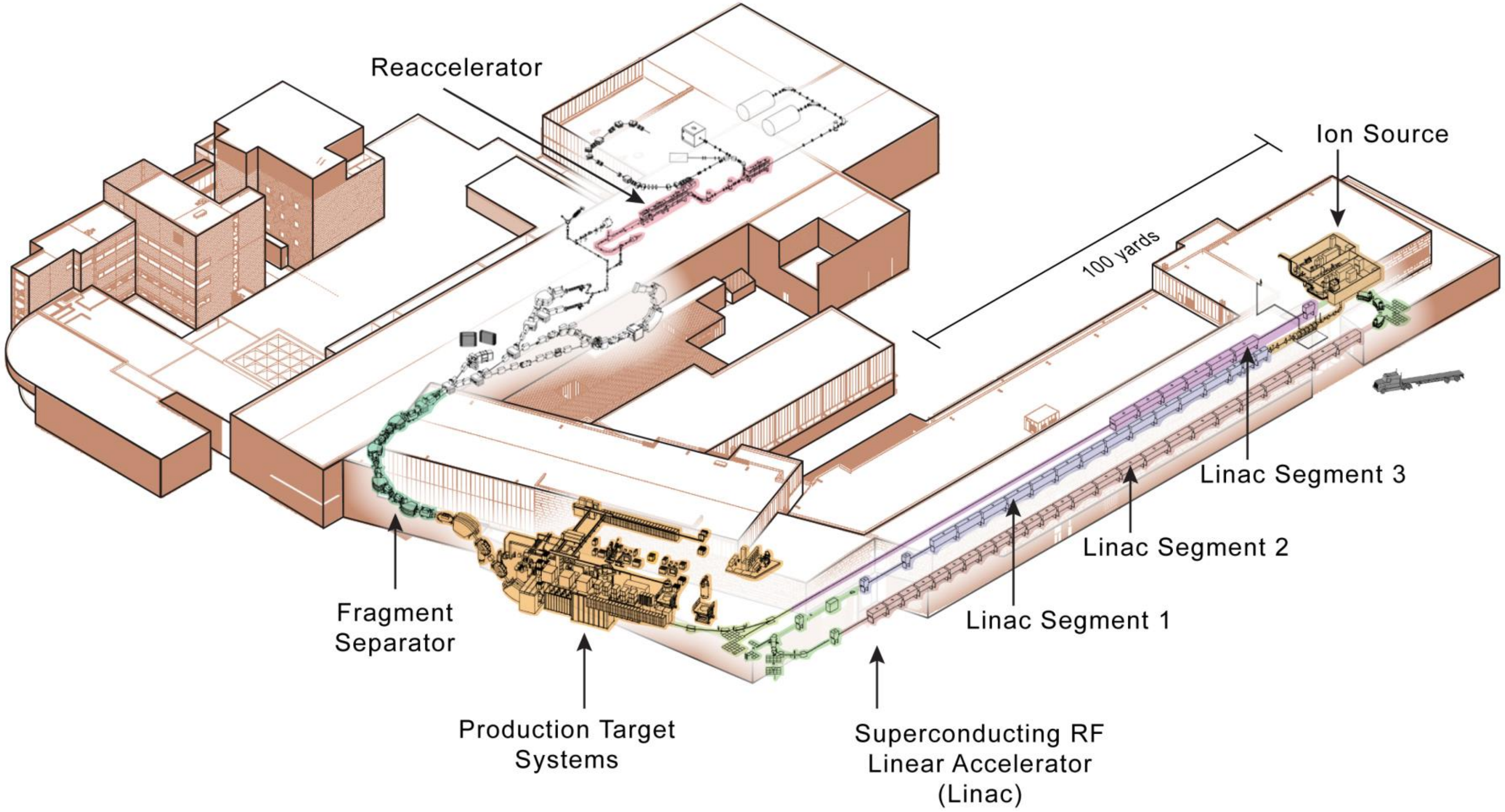
17 August 2026

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics and used resources of the Facility for Rare Isotope Beams (FRIB) Operations, which is a DOE Office of Science User Facility under Award Number DE-SC0023633.

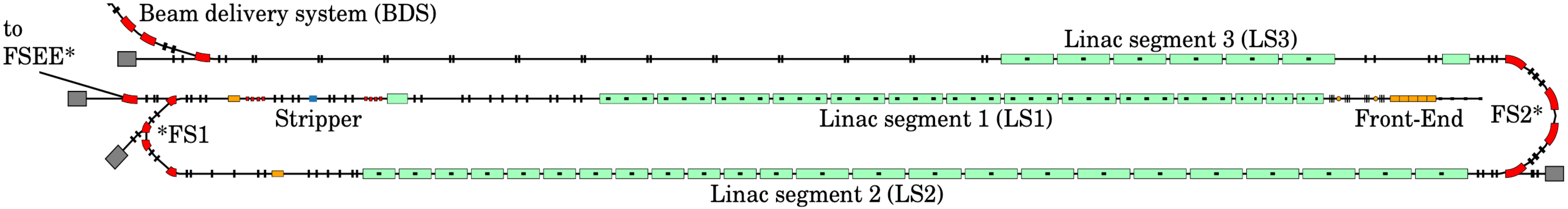
Outline

- Introduction
- Reconstruction + Matching
- 2D Emittance Evolution
- 4D Emittance Evolution
- Future Improvements



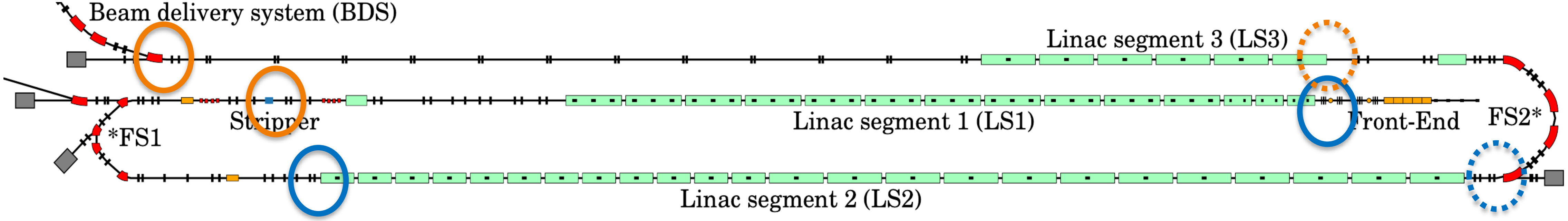
FRIB Linac

- Reliably delivers 20 kW heavy-ion beams to the target [Oxygen (A=16) to uranium (A=238)]
  - Final beam energies from 177-300 MeV/u
  - 30 kW uranium beam demonstrated in October 2025
- Can simultaneously accelerate multiple charge states after unique liquid lithium charge stripper
- Low-loss tune (beam losses < 10^{-4})
  - Instant Phase Setting (IPS) code used for longitudinal motion
    - Setting phase/amplitude of 324 SRF accelerating cavities
  - Requires transverse re-matching in ~5 locations
    - Profile monitors used for transverse measurements



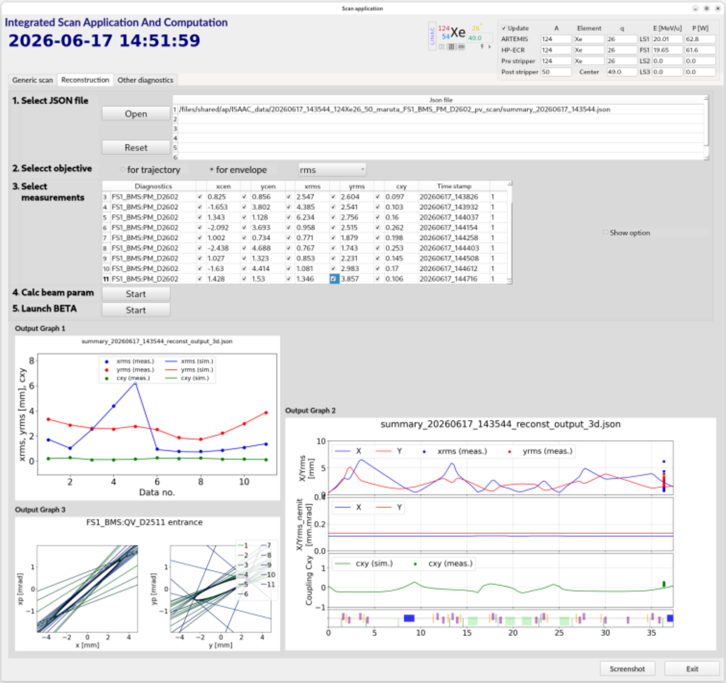
Transverse Reconstruction/Matching at FRIB

- Typically measure and match Courant-Snyder (CS) parameters at (solid circles):
  - LS1 entrance
  - Stripper
  - LS2 entrance
  - BDS
- If unexpected losses are observed, sometimes measure at (dashed circles):
  - FS2 bending section
  - LS3 entrance
- Measurement + reconstruction time = ~20 minutes
  - Time-consuming to measure multi-charge-state beams
- Currently, 4D emittance reconstructions are reliable at blue circled locations

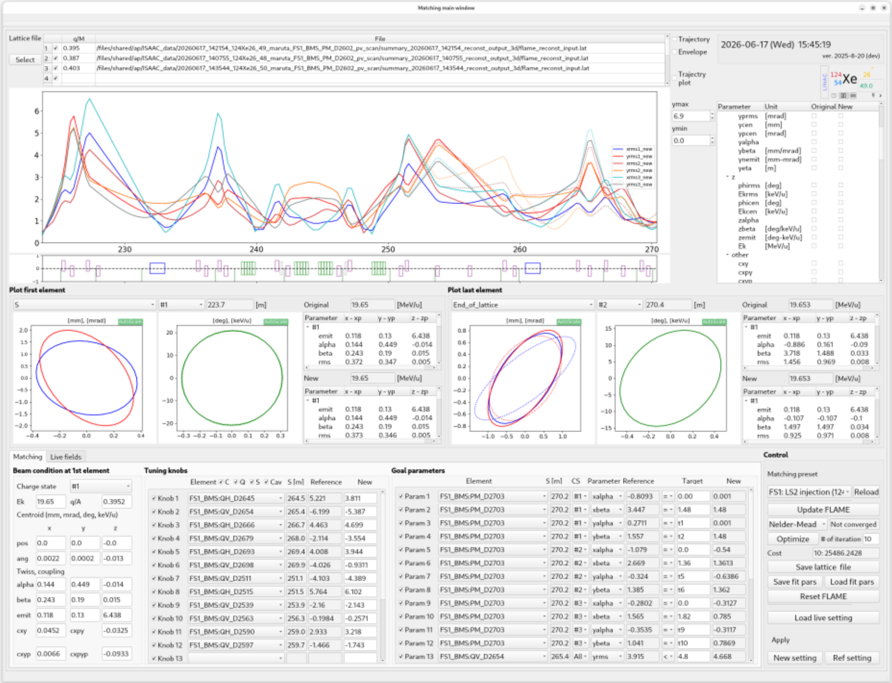


Matching Procedure

- Integrated Scan Application And Computation (ISAAC) application performs quadrupole scan measurements and reconstruction



- BETA application optimizes quadrupole settings to match all charge states to design CS parameters



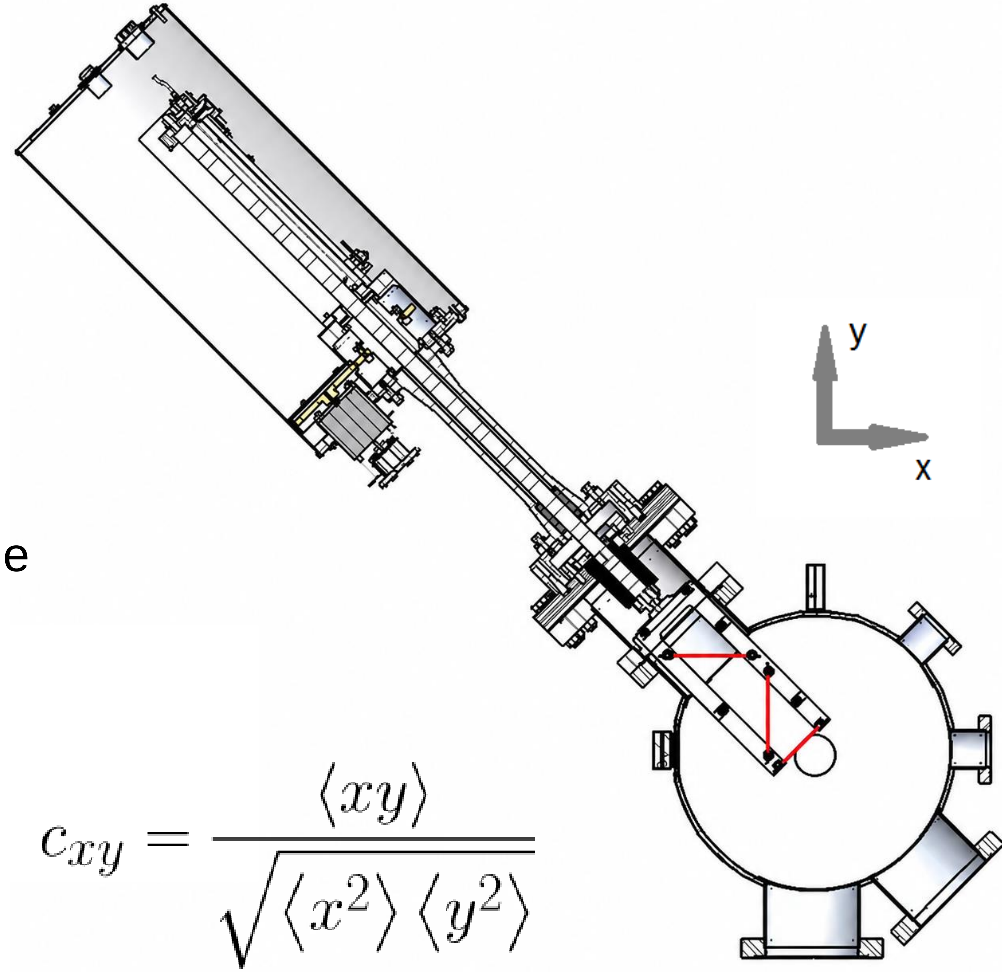
- Both applications based on FLAME (linear envelope code)

4D Emittance Reconstruction

- **Measurements**
  - Quadrupole scan (6-10 measurements)
  - 3-wire profile monitors (x, y, 45 deg)
    - » Measure x_{rms}, y_{rms}, c_{xy}
  - Optimize sigma matrix parameters to best fit measurements with FLAME simulations
  - 2D emittance reconstructions reliable, 4D needs improvement

- **Motivations** to improve 4D emittance measurements:
  - Limit emittance growth for high-power operation
    - » Solenoids used in FRIB cryomodules cause 2D emittance exchange
  - Accurate input coupling necessary for digital twin
    - » Beam comes from ECR ion source with initial angular momentum

Profile Monitor



$$\Sigma = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle \end{bmatrix}$$

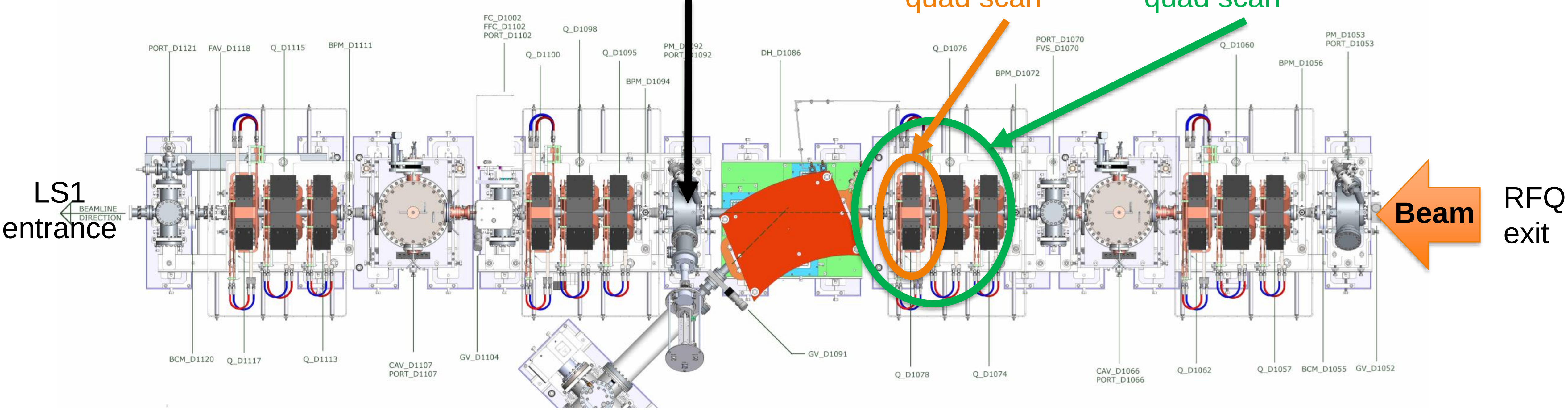
$$\epsilon_{4\mathrm{Dn}} = \beta^2\gamma^2\sqrt{\det(\Sigma)}$$

$$\Sigma_1 = R\Sigma_0 R^T$$

$$c_{xy} = \frac{\langle xy \rangle}{\sqrt{\langle x^2 \rangle \langle y^2 \rangle}}$$

Medium Energy Beam Transport (MEBT)

- Beam energy = 0.5 MeV/u

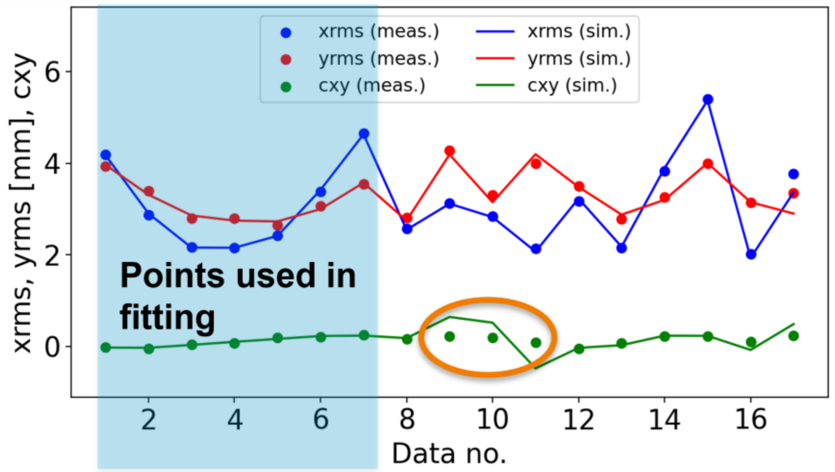


- Old quad scan:
  - 1 quadrupole scanned
  - Good 2D emittance reconstructions
  - Unreliable 4D emittance reconstructions

- New quad scan:
  - Use 3 quadrupoles
  - Improved 4D reconstructions
  - Same 2D results

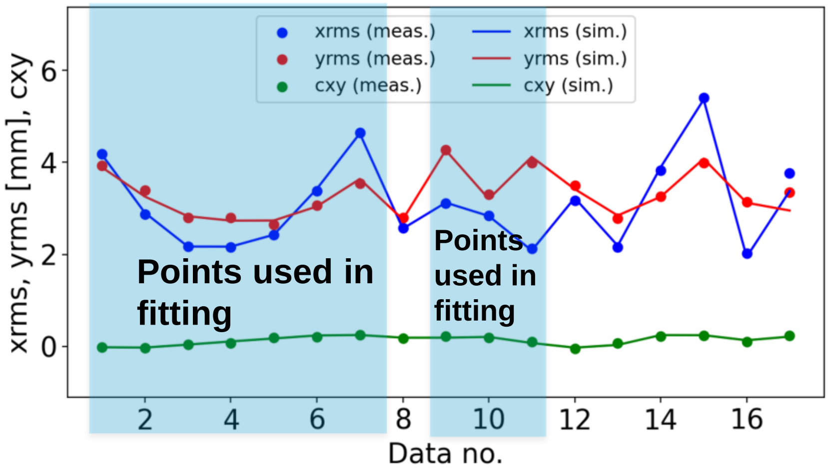
$^{40}Ar^{9+}$ Study (12/16/2025)

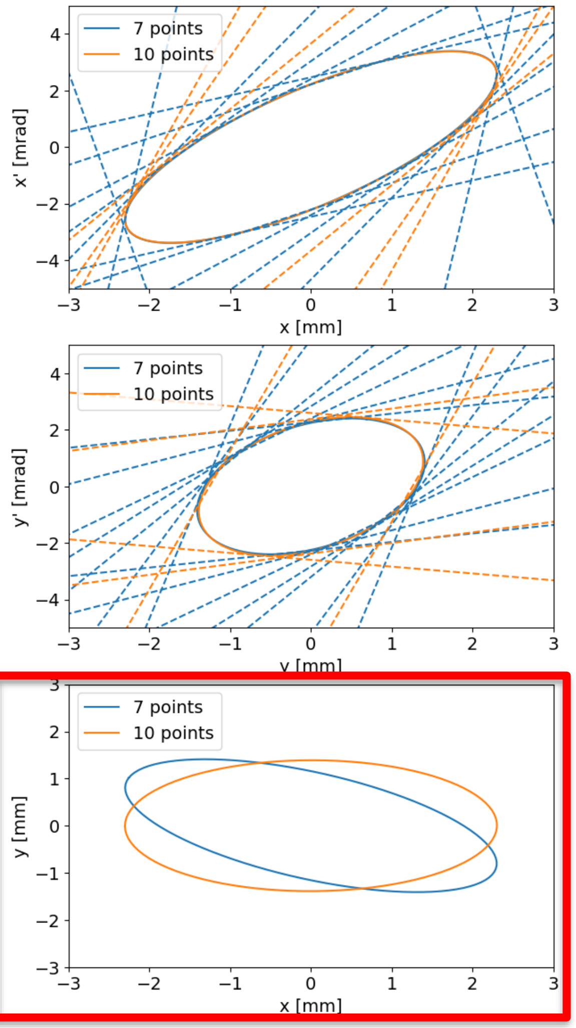
7-point fitting (1 quad)



Add circled points to fitting

10-point fitting (3 quads)





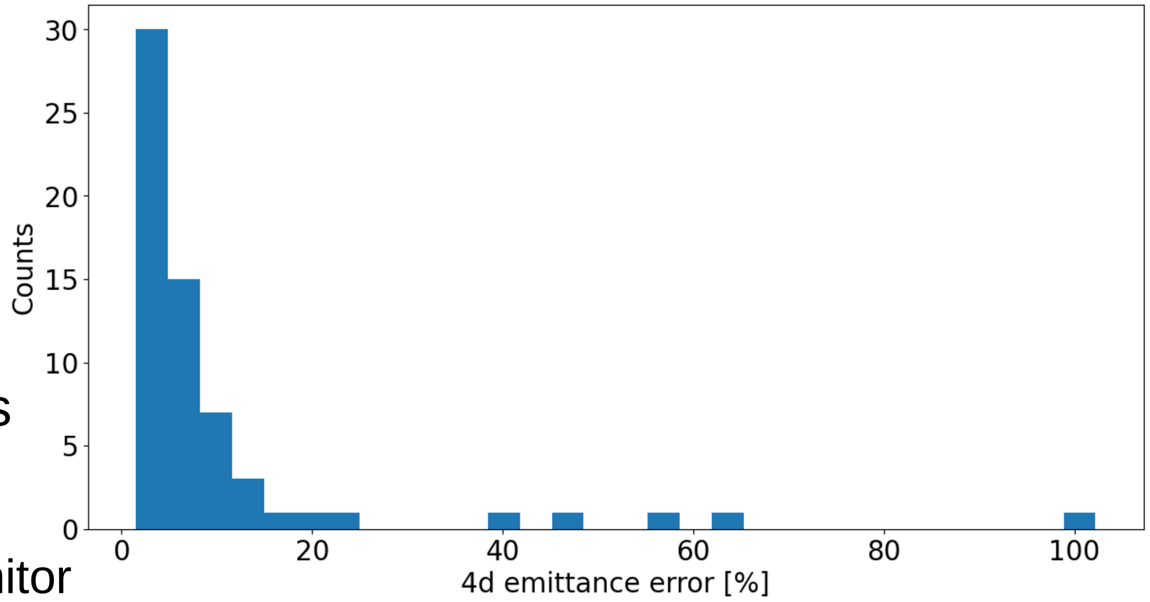
# points fitted	E4dn [(π*mm*mrad)²]	Exn [π*mm*mrad]	Eyn [π*mm*mrad]	Cxy
7	1.5e-3	0.168	0.104	**-0.828**
10	1.7e-2	0.169	0.104	**-0.133**

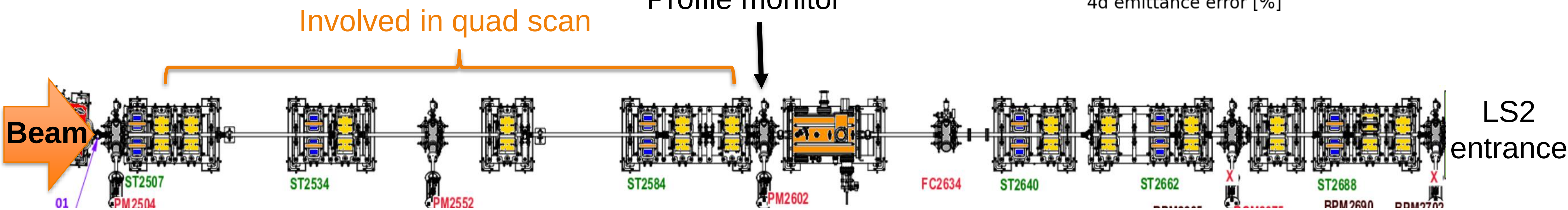
- Results replicated with $^{208}Pb^{33+}$ beam
- New points added to standard MEBT scan preset

LS2 Entrance

- Beam energy = 16.5-20 MeV/u
- 12 quadrupoles in segment
  - 6 used in quad scan
  - Installing 2 more to help with matching
- Very reliable, low error results for 2D and 4D emittances, no changes required
  - Error > 100% at other locations before improvements

69 measurements from Mar. 2025 to Jun. 2026



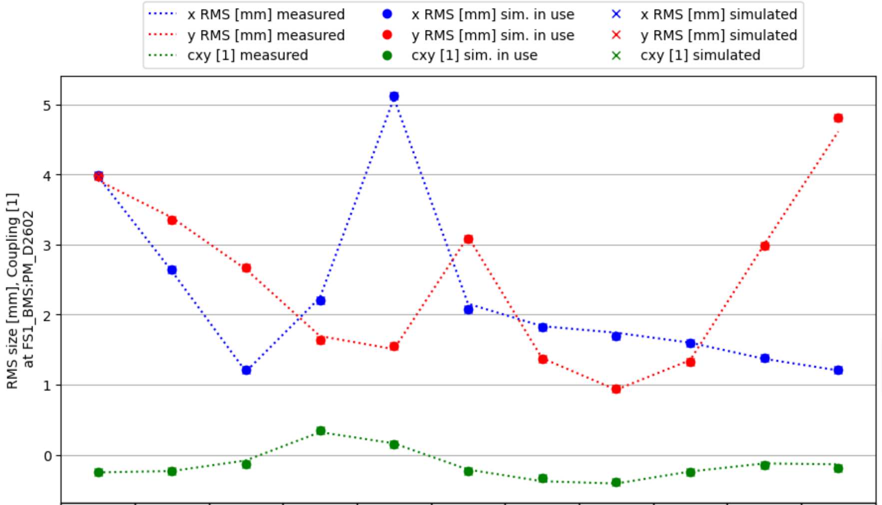


Error Evaluation of Q-Scan Result

▪ Propagate measurement error to the phase space error

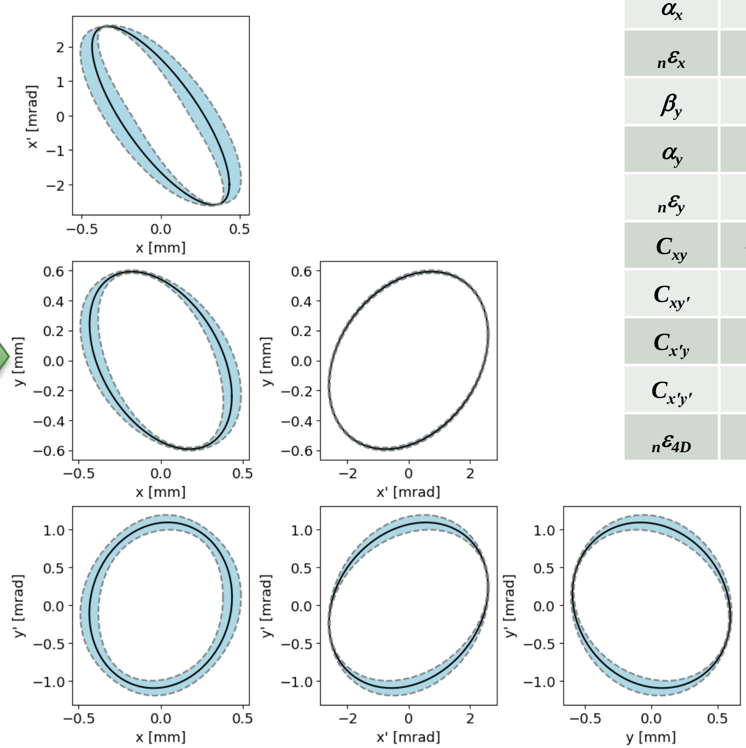
- Error propagation equation: $\Sigma_f = J_x \Sigma_x J_x^T + J_y \Sigma_y J_y^T + \ldots$
  - » $\Sigma_{f,x,y}$: variance-covariance matrix
  - » $J_{x,y}$: Jacobian matrix of f with respect to x, y
- E.g., assume f is an emittance, x is the measured beam size, and y is the quadrupole field
- $\Sigma_{x,y}$ represents the error, $J_{x,y}$ are calculated by the auto-differentiation FLAME model

Q-Scan Result ($^{82}Se^{32+}$)



	No.0	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8	No.9	No.10
FS1_BMS:QV_D2511	-6.37	-6.37	-6.37	-6.37	-6.37	-5.84	-5.84	-5.84	-5.84	-5.84	-5.84
FS1_BMS:QH_D2515	6.76	6.76	6.76	6.76	6.76	7.05	7.05	7.05	7.05	7.05	7.05
FS1_BMS:QV_D2539	-0.00	-0.24	-0.51	-1.02	-1.53	-1.66	-1.66	-1.66	-1.66	-1.66	-1.66
FS1_BMS:QV_D2563	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.77	-1.17	-1.53	-2.30	-3.06
FS1_BMS:QH_D2590	0.00	0.00	0.00	0.00	0.01	5.31	5.31	5.31	5.31	5.31	5.31
FS1_BMS:QV_D2597	-0.00	-0.00	-0.00	-0.00	-0.01	-4.91	-4.91	-4.91	-4.91	-4.91	-4.91

Reconstructed rms phase space ellipses (black) including calculated uncertainty (blue)



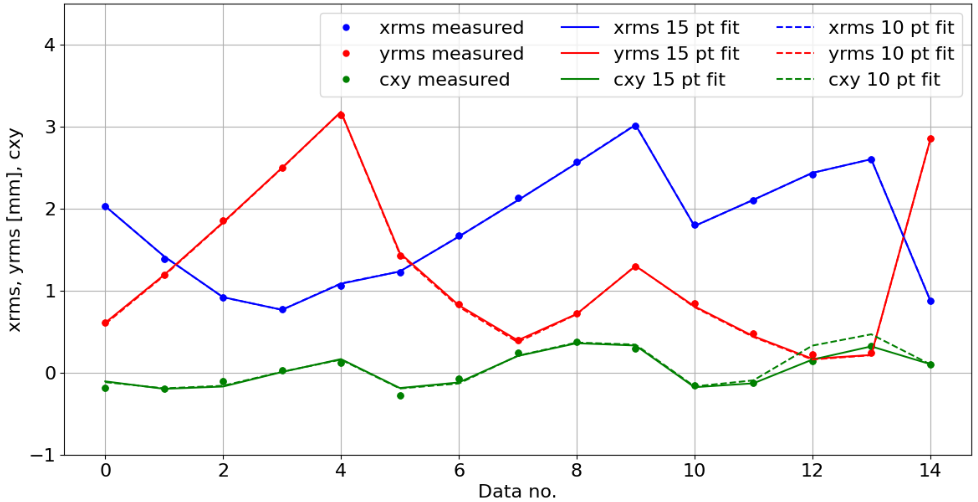
Phase Space Errors Assuming 5% Measurement Error and 0.1 A Quadrupole Error

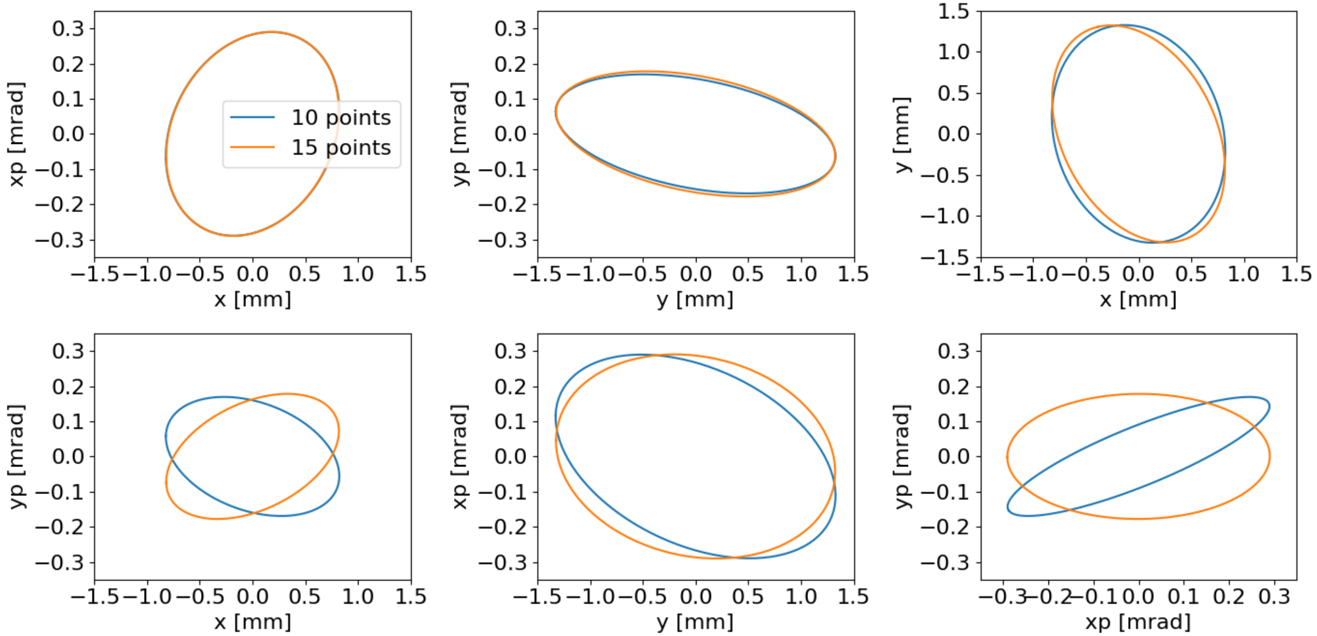
	Mean ± Error	% Error	Unit
x_{RMS}	4.33e-01 ± 5.48e-02	**12.7 %**	[mm]
x'_{RMS}	2.59e+00 ± 2.67e-02	**1.0 %**	[mrad]
y_{RMS}	5.92e-01 ± 1.06e-02	**1.8 %**	[mm]
y'_{RMS}	1.10e+00 ± 9.85e-02	**9.0 %**	[mrad]
β_x	2.63e-01 ± 3.54e-02	**13.5 %**	[m/rad]
α_x	1.22e+00 ± 4.82e-01	**39.7 %**	[1]
$_n\varepsilon_x$	1.48e-01 ± 5.32e-02	**35.9 %**	[mm-mrad]
β_y	5.46e-01 ± 4.94e-02	**9.0 %**	[m/rad]
α_y	1.39e-01 ± 8.11e-02	**58.4 %**	[1]
$_n\varepsilon_y$	1.34e-01 ± 1.50e-02	**11.2 %**	[mm-mrad]
C_{xy}	-4.05e-01 ± 7.94e-02	**19.6 %**	[1]
$C_{xy'}$	9.90e-02 ± 1.25e-01	**125.8 %**	[1]
$C_{x'y}$	2.87e-01 ± 2.98e-02	**10.4 %**	[1]
$C_{x'y'}$	2.17e-01 ± 7.46e-02	**34.4 %**	[1]
$_n\varepsilon_{4D}$	1.60e-02 ± 7.84e-03	**48.9 %**	[mm^2-mrad^2]

LS2 Exit

- Beam energy > 130 MeV/u
- 4 quadrupoles between exit of LS2 and profile monitor
  - Standard scan uses 2 quads
  - Adding 1 more quad significantly reduced 4D emittance error

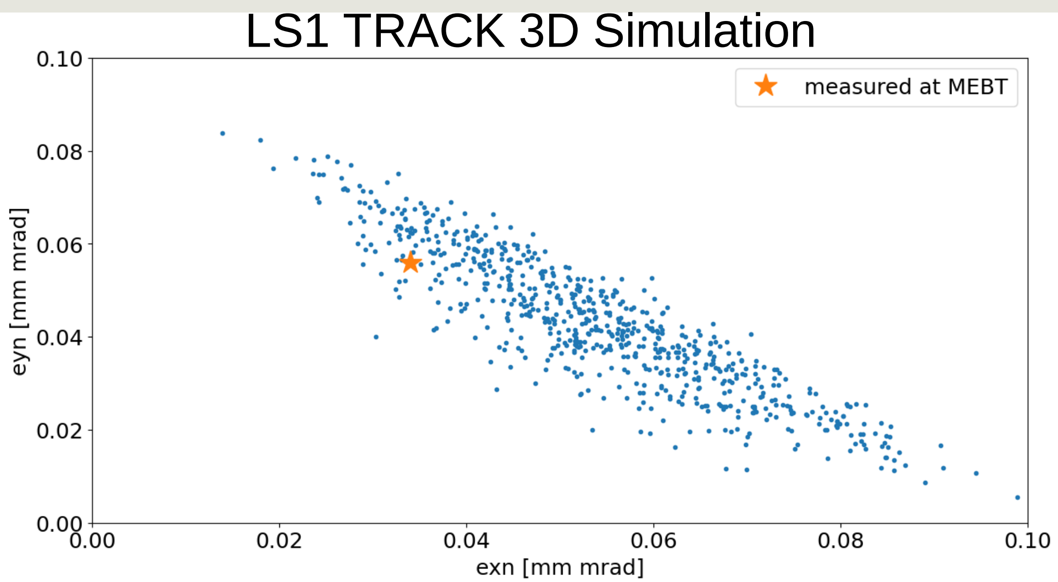
# Quads scanned	# Points	e4Dn [(π*mm*mrad)^2]	Error [%]
2	10	2.6e-6	6e8
3	15	1.1e-2	20





2D Emittance Evolution

- TRACK simulation along LS1
  - 3D fields for cavities and solenoids
  - Input emittances from measurement and matched input CS parameters
  - Randomly generated input coupling
- 2D emittances at exit of accelerating segments highly dependent on input coupling



- Better knowledge of coupling makes emittances more predictable along the linac
- Better for transport segments if 2D emittances are equal

Measured Normalized 2D Emittances [π mm mrad]

Beam	MEBT (x)	MEBT (y)	LS2 ent (x)	LS2 ent (y)	BDS (x)	BDS (y)
$^{16}O^{6+}$	0.106	0.086	0.114	0.105	0.111	0.116
$^{208}Pb^{33+(68+)}$	0.060	0.044	0.079	0.038	0.062	0.117
$^{20}Ne^{8+(10+)}$	0.129	0.100	0.122	0.137	0.235	0.119
$^{195}Pt^{32+(66+)}$	0.050	0.056	0.078	0.038	0.072	0.094

4D Emittance Evolution

Measured Normalized 4D Emittance [(π mm mrad)^2]

Beam	MEBT	LS2 entrance	BDS
$^{16}O^{6+}$	0.009	0.010	0.011
$^{208}Pb^{33+(68+)}$	0.002	0.003	0.004
$^{20}Ne^{8+(10+)}$	0.012	0.014	0.014
$^{195}Pt^{32+(66+)}$	0.002	0.003	0.004

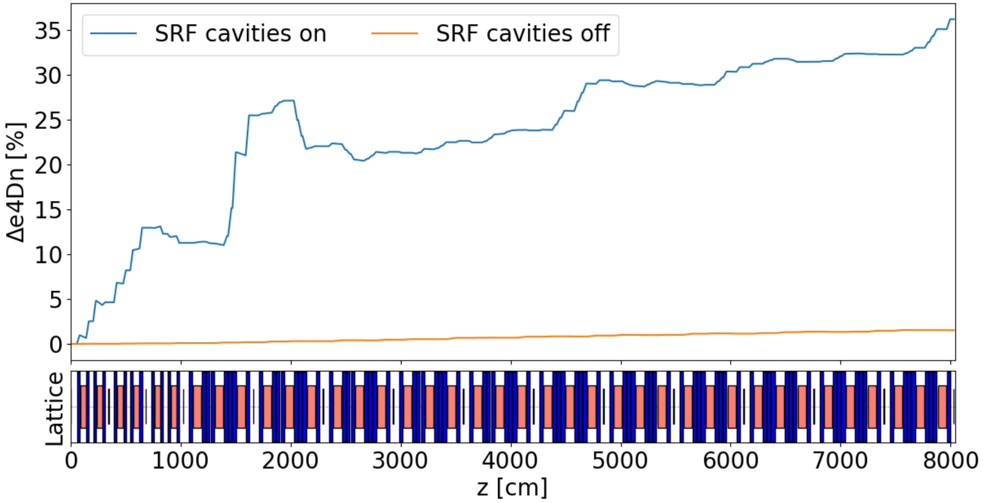
4 β = 0.041: 4 QWRs x 2 Solenoids

11 β = 0.085: 8 QWRs x 3 Solenoids

12 β = 0.29: 6 HWRs x 1 Solenoids

18 β = 0.53: 8 HWRs x 1 Solenoids

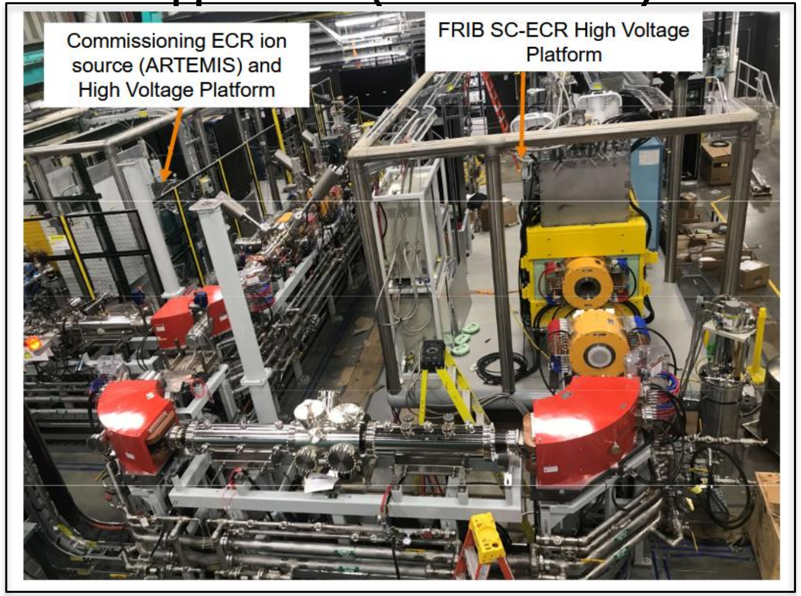
LS1 TRACK 3D Simulation



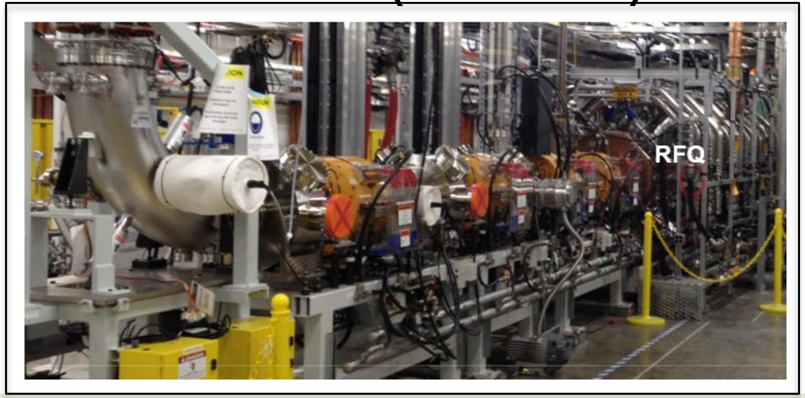
- TRACK simulations with 3D fields
  - Input distributions based on real reconstructions
  - LS1 (QWR cavities): 20-50% 4D emittance growth with cavities on
    - » LS1 with cavities off: < 5% 4D emittance growth
  - LS2 and LS3 (HWR cavities): < 5% growth
- Growth in LS1 caused by phase-dependent dipole fields in QWR cavities
- Growth in other segments caused by non-linear solenoid fringe fields

Future Improvements

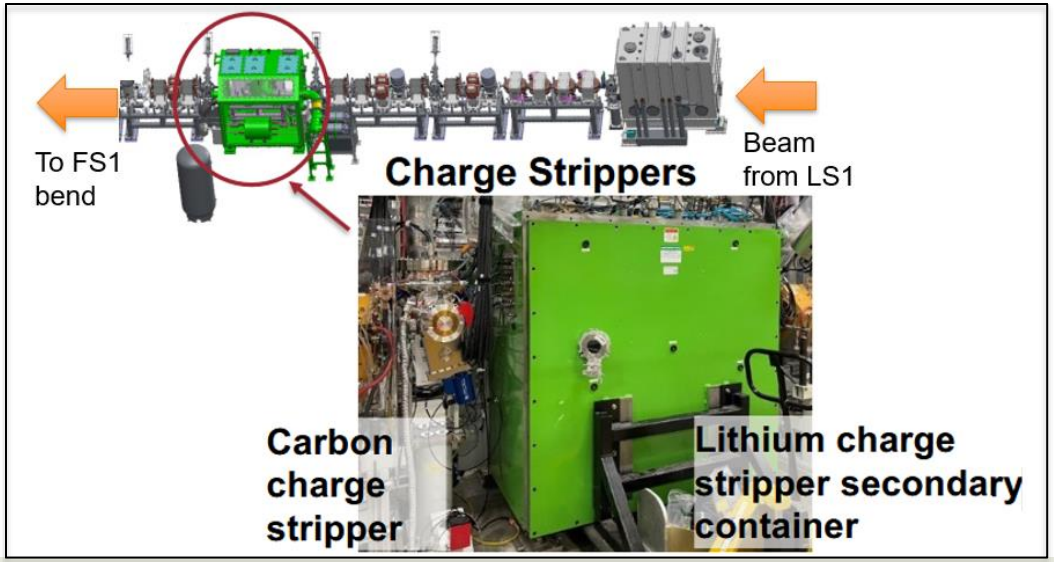
Upper LEBT (Above Ground)



Lower LEBT (Linac Tunnel)

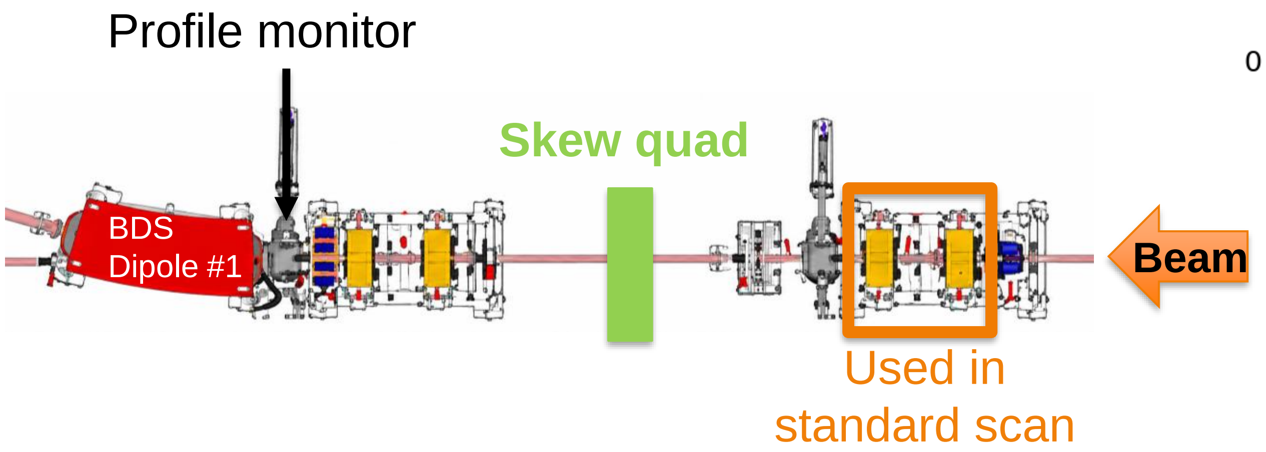


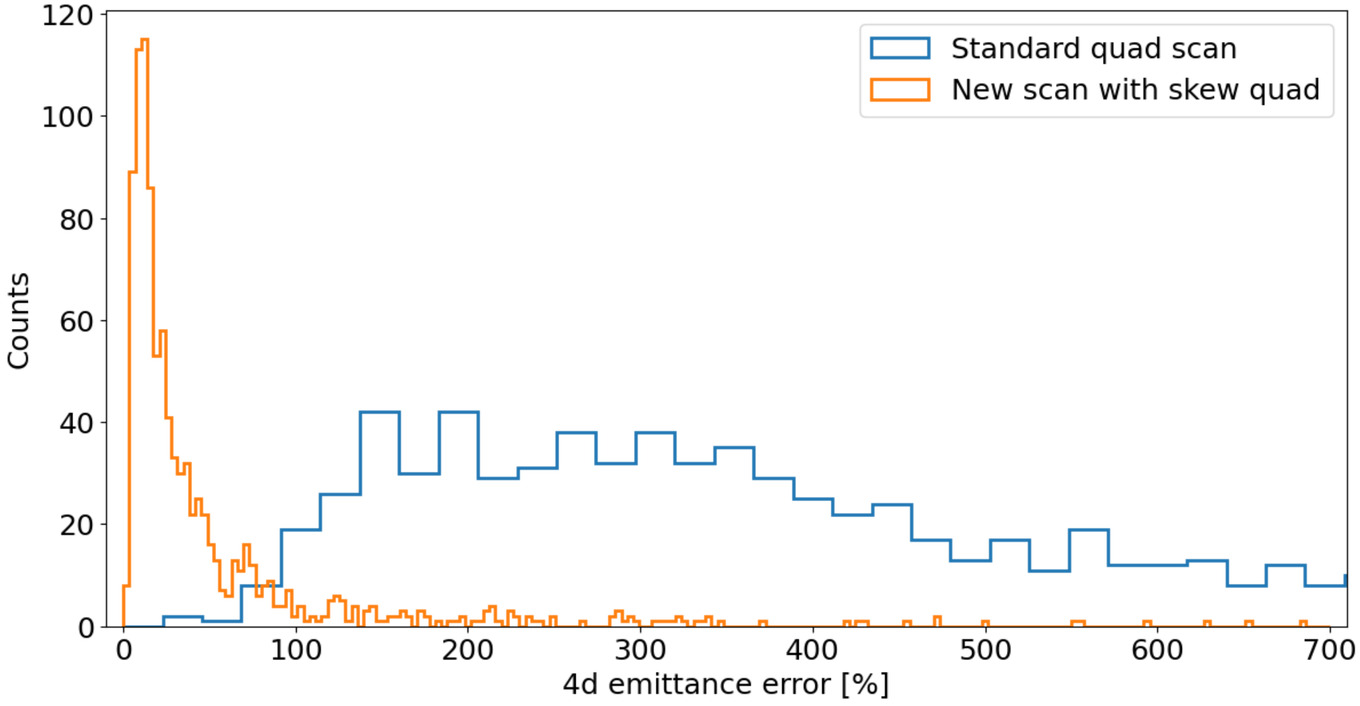
- Improve 4D emittance measurements in Low Energy Beam Transport (LEBT)
  - Solenoids can correct coupling before RFQ injection
- Improve reconstructions at the stripper and BDS
- Install skew quadrupoles
  - Manipulate coupling terms for matching to LS1, LS2, LS3
  - Improve quad scan reconstruction of coupling terms



Skew Quadrupole Scan Simulation Study

- Add skew quadrupole in BDS
- Optimize quad scan settings to minimize 4D emittance error
  - 3 quadrupoles (skew + 2 quads used in standard scan)
  - 4 points
- Simulate 1000 quad scans with randomly generated input parameters

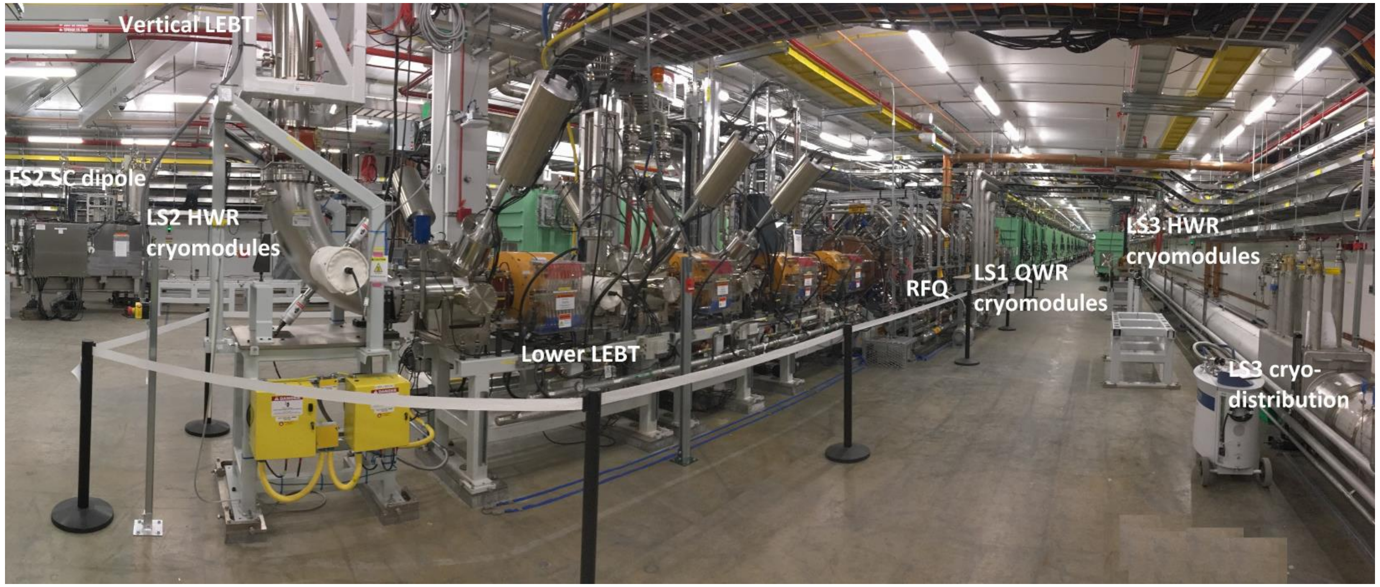




	# points	Median error [%]
Standard scan	8	450
New scan	4	23

Summary

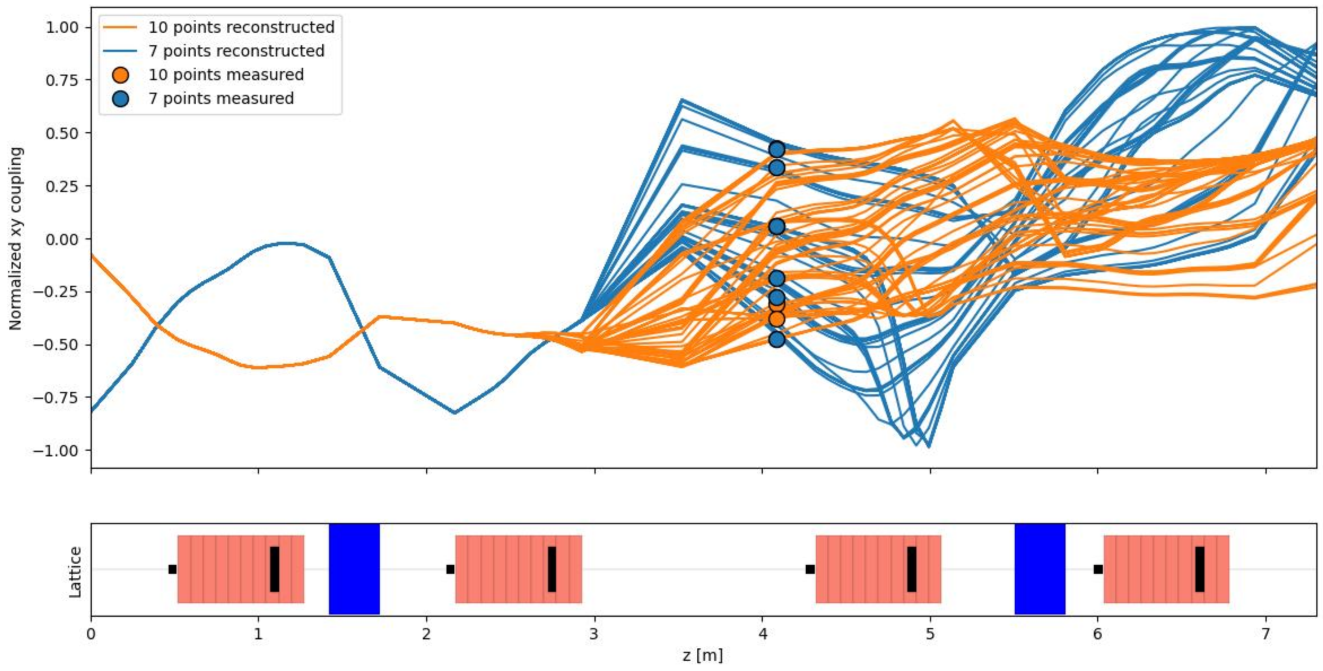
- Reliable 4D emittance measurements developed at:
  - MEBT, LS2 entrance, LS2 exit
- Improvements needed in:
  - LEBT, Charge stripper area, BDS
- Simulations with 3D fields help identify causes of 4D emittance growth in linac segments
- Installation of skew quads can improve 4D reconstruction accuracy and matching



Backups

208Pb33+ Study (04/01/2026)

- Same study with 208Pb beam
- Again, cxy much higher in 7-point scan
  - Unrealistic 4D emittance
- 10-point scan gives realistic 4D emittance
- Standard MEBT quad scan preset updated to include more points

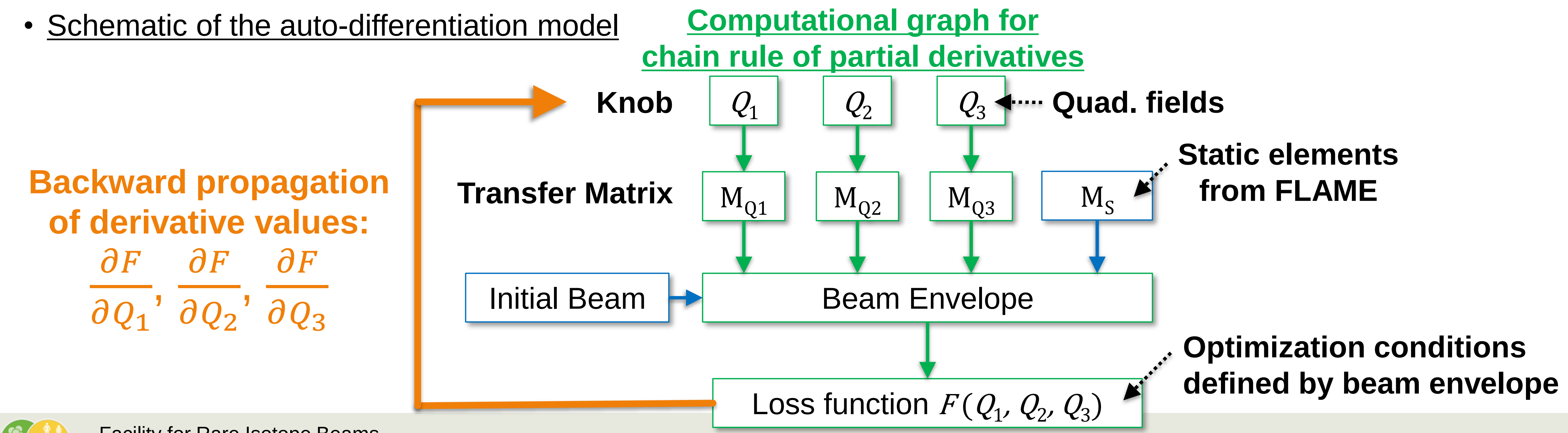


	E4dn [(pi*mm*mrad)^2]	Exn [pi*mm*mrad]	Eyn [pi*mm*mrad]	Cxy
7 points	4.4e-5	0.063	0.045	**-0.82**
10 points	2.0e-3	0.060	0.044	**-0.08**

Auto-Differentiation (AD) Model with FLAME

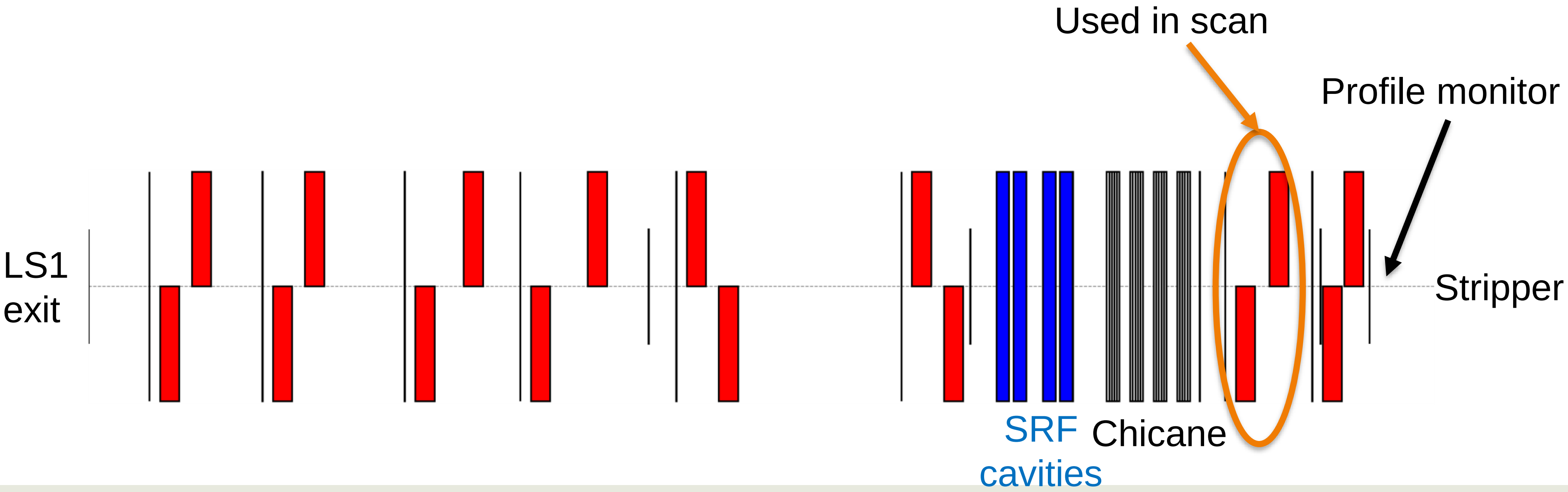
- AD bridging physics-based model with modern machine learning optimization techniques
- Inherit FLAME package to keep compatibility with all archived operation datasets
- Overwrite parametric values (Quad. field or CS parameter) with auto-differentiable variables
- Support backward differentiation for the optimization, the error propagation, and so fourth

  - Schematic of the auto-differentiation model



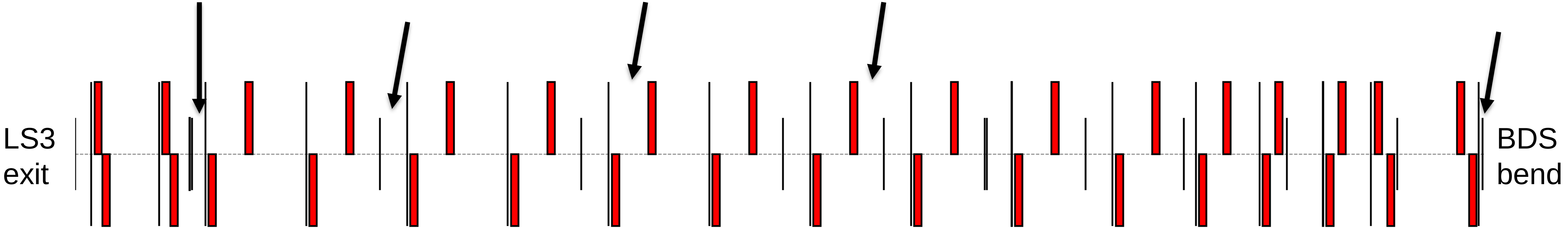
Matching to Stripper

- Beam energy = 17-20 MeV/u
- Need to focus beam on stripper to limit emittance growth
  - $\alpha = 0$, rms beam size = 0.3 mm
- Measured 2D emittances almost always greater than emittances measured downstream
- 4D emittance results very unreliable



Matching to BDS

- Beam energy = 177-300 MeV/u
- Long FODO transport section
- Use 5 PMs to reconstruct without scanning quadrupoles
  - Faster than previous 8-10 point quad scan
  - 60 deg phase advance
  - ~75% good 4D emittances
- Installing new fast PM for quad scan



4D Emittance

- FRIB cryomodules use solenoids for transverse focusing
  - 2D emittance exchange
  - 4D emittance should be conserved
- 4D emittance important to evaluate beam quality
- As we ramp up beam power, accurate coupling measurements become important to prevent losses
- Always had reliable 2D emittance measurements
- Working to improve reliability and consistency of 4D emittance measurements
- Normalized coupling terms: [-1, 1]

$$\epsilon_{4\mathrm{Dn}} = \beta^2\gamma^2\sqrt{\det(\Sigma)}$$

$$c_{xy} = \frac{\langle xy\rangle}{\sqrt{\langle x^2\rangle\langle y^2\rangle}}$$

Transfer matrix

$$\Sigma_1 = R\Sigma_0 R^T$$

Measurement location

Reconstruction location

Measured by profile monitor

$$\left\langle x^2 \right\rangle_1 = R_{11}^2 \left\langle x^2 \right\rangle_0 + R_{12}^2 \left\langle x'^2 \right\rangle_0 + 2R_{11}R_{12}^2 \left\langle xx' \right\rangle_0$$

$$\left\langle y^2 \right\rangle_1 = R_{33}^2 \left\langle y^2 \right\rangle_0 + R_{34}^2 \left\langle y'^2 \right\rangle_0 + 2R_{33}R_{34}^2 \left\langle yy' \right\rangle_0$$

$$\left\langle xy \right\rangle_1 = R_{11}R_{33} \left\langle xy \right\rangle_0 + R_{12}R_{33} \left\langle x'y \right\rangle_0 + R_{11}R_{34} \left\langle xy' \right\rangle_0 + R_{12}R_{34} \left\langle x'y' \right\rangle_0$$