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4D phase space tomography from 1D measurements

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U.S. DEPARTMENT of ENERGY

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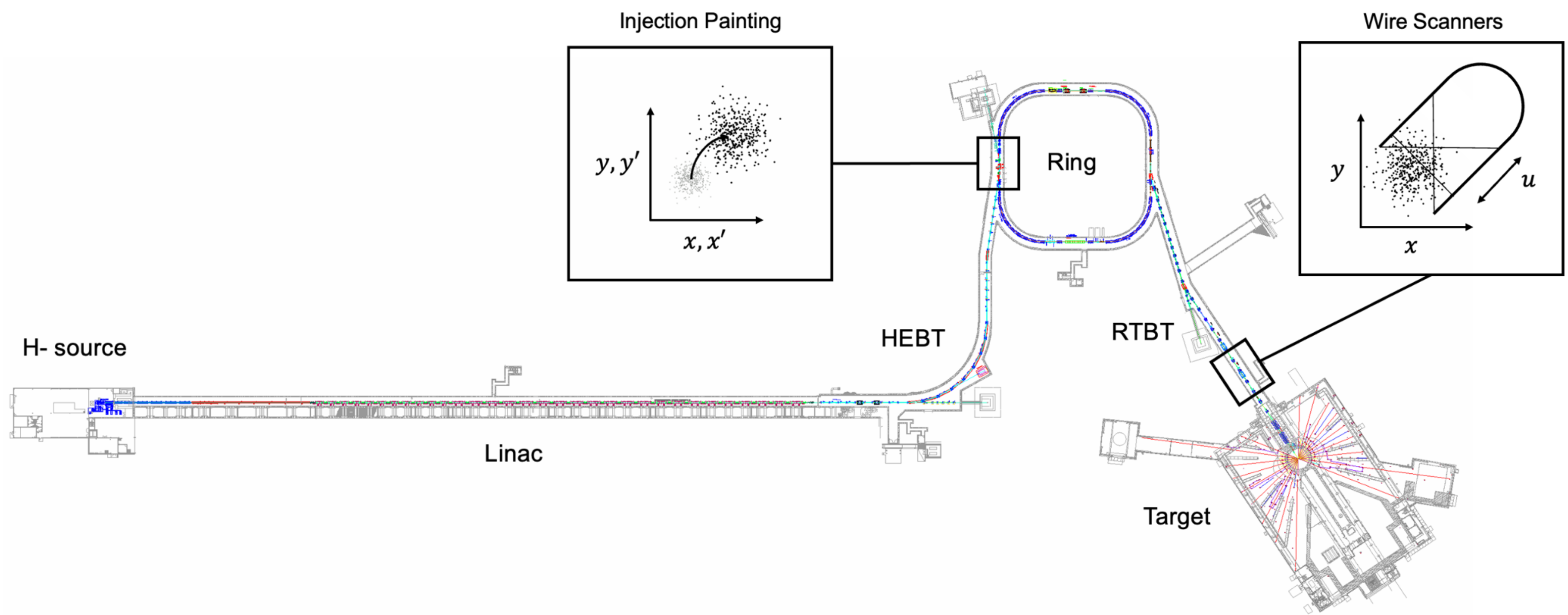
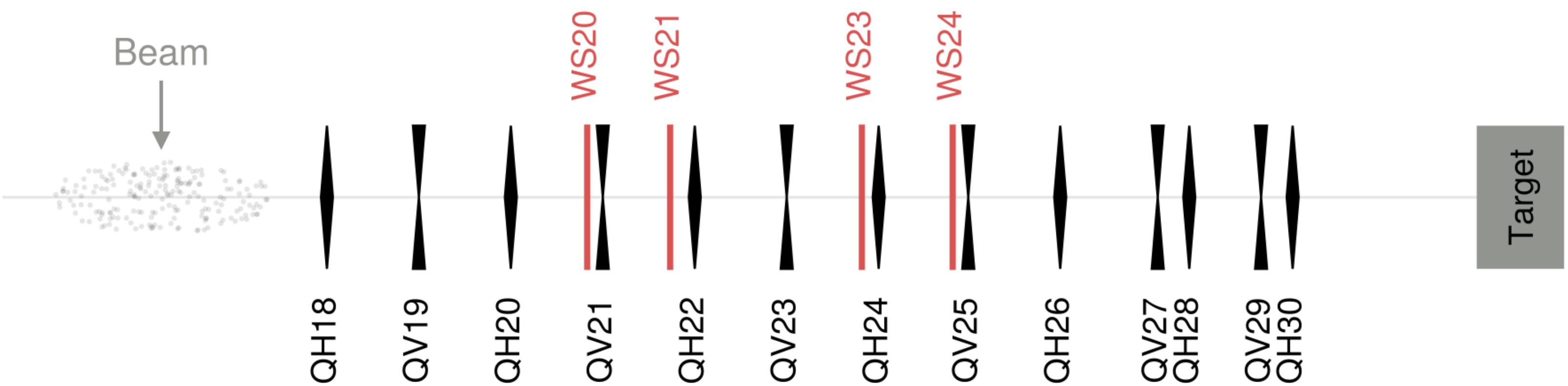
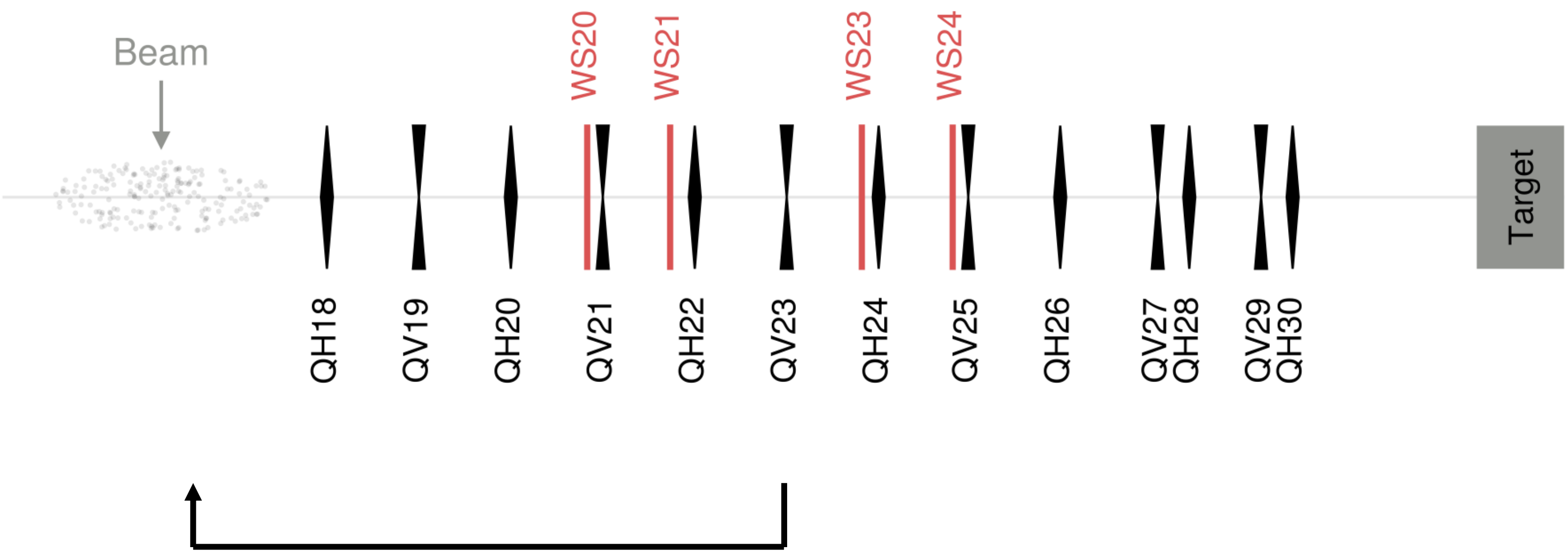
4D-from-1D tomography is motivated by phase space painting experiments at SNS



Wire scanners measure 1D profiles of extracted beam (horizontal, vertical, diagonal)



Can we reconstruct the 4D phase space distribution from this data?



$$f(x, x', y, y') = ?$$

First 4D reconstruction at LANL in 1981 (!)

IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

FOUR-DIMENSIONAL BEAM TOMOGRAPHY*

G. N. Minerbo, O. R. Sander, and R. A. Jameson
Los Alamos National Laboratory, Los Alamos, NM 87545

Summary

A computer code has been developed to reconstruct the 4-D transverse phase-space distribution of an accelerator beam from a set of linear profiles measured at different angles at three or more stations along the beam line. The code was applied to wire-scan data obtained on the low-intensity H^- beam of the LAMPF accelerator. A 4-D reconstruction was obtained from 10 wire-scan profiles; 2-D projections of the reconstruction agree fairly well with slit-and-collector measurements of the horizontal and vertical emittance distributions.

We assume, for simplicity that f has been normalized and centered as follows:

$$\int d^4v\, f(v) = 1, \int d^4v\, v_n\, f(v) = 0, \quad n = 1, \ldots, 4.$$

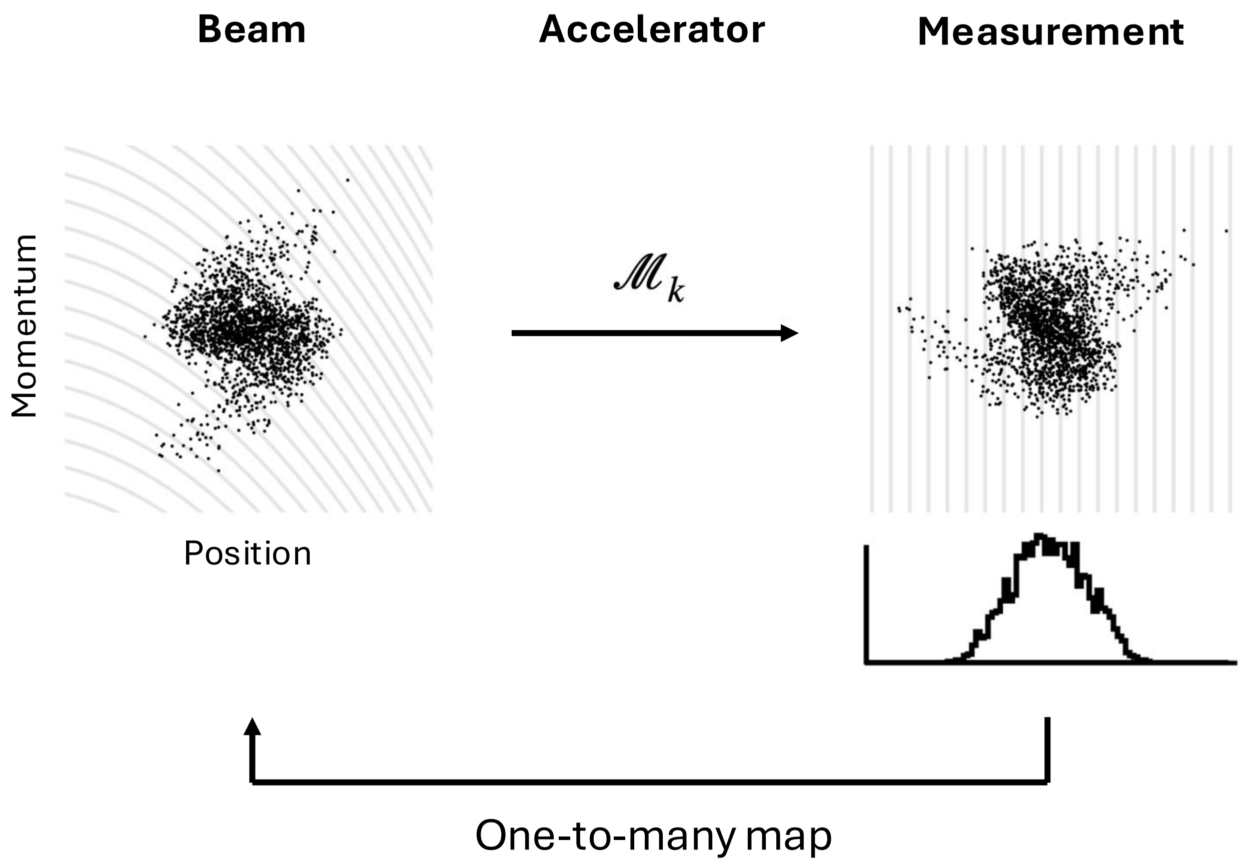
The transfer matrices have det $T_j = 1$ under very general conditions, hence the g_j will also be normalized and centered. We define b_j, the RMS width of g_j, by

$$b_j^2 = \int du\, u^2\, g_j(u), \; j = 1, \ldots, J \; . \qquad (3)$$

The covariance matrix σ of f is defined as

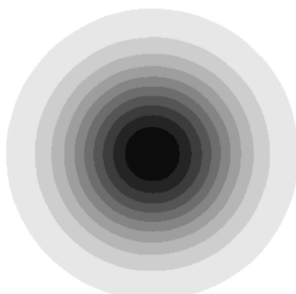
Maximum-entropy reconstruction method

Goal: infer initial distribution from projections of transformed distribution



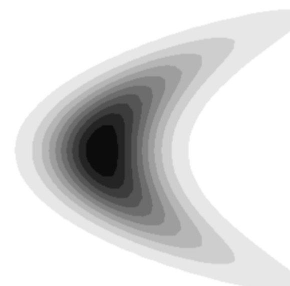
Entropy = relative simplicity

$p_*(x)$ $p(x)$



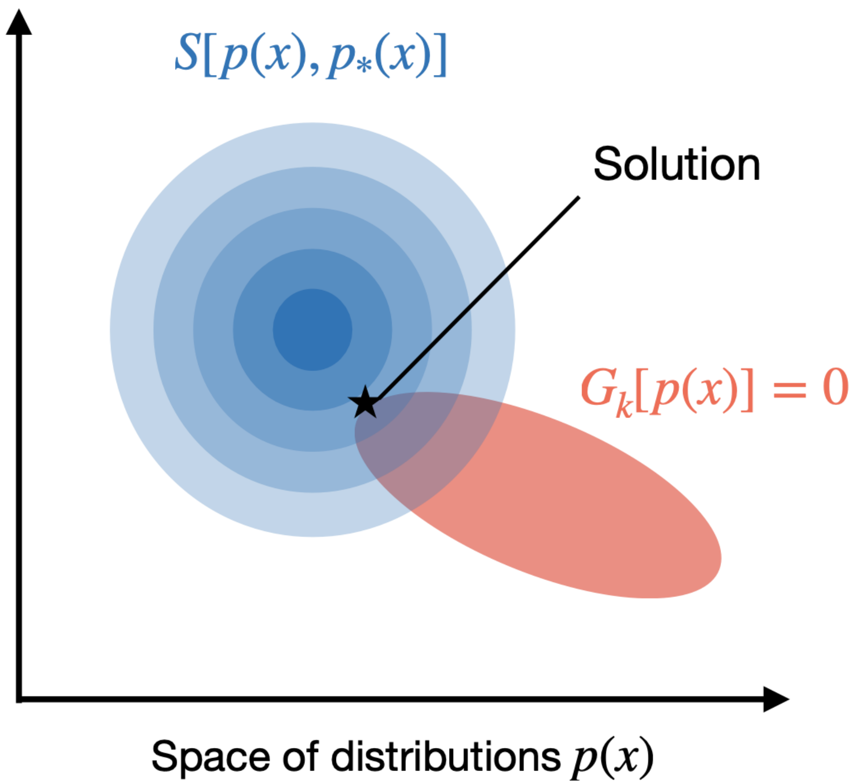
New information

$\longrightarrow$



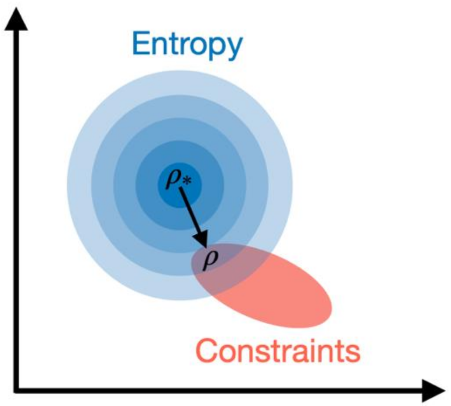
$$\boxed{S[p(x), p_*(x)] = -\int p(x) \log\left(\frac{p(x)}{p_*(x)}\right) dx}$$

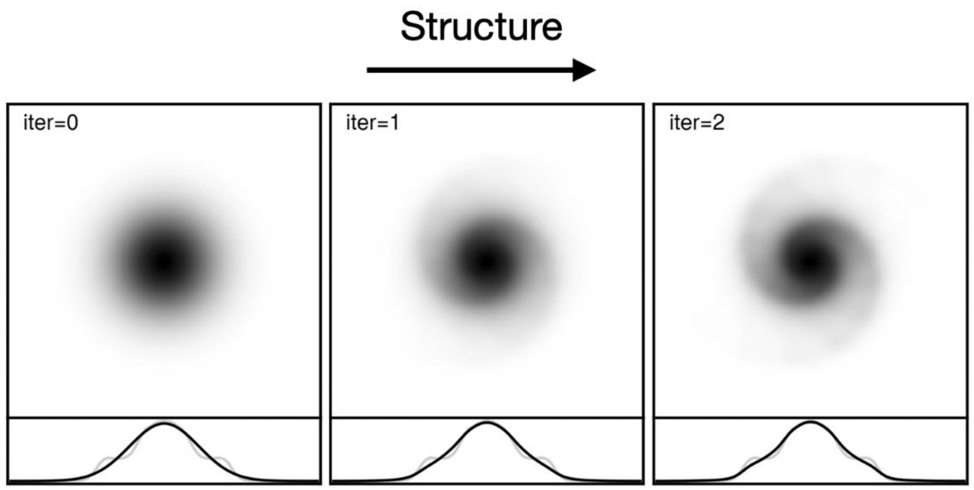
Strategy: rank solutions by their relative entropy

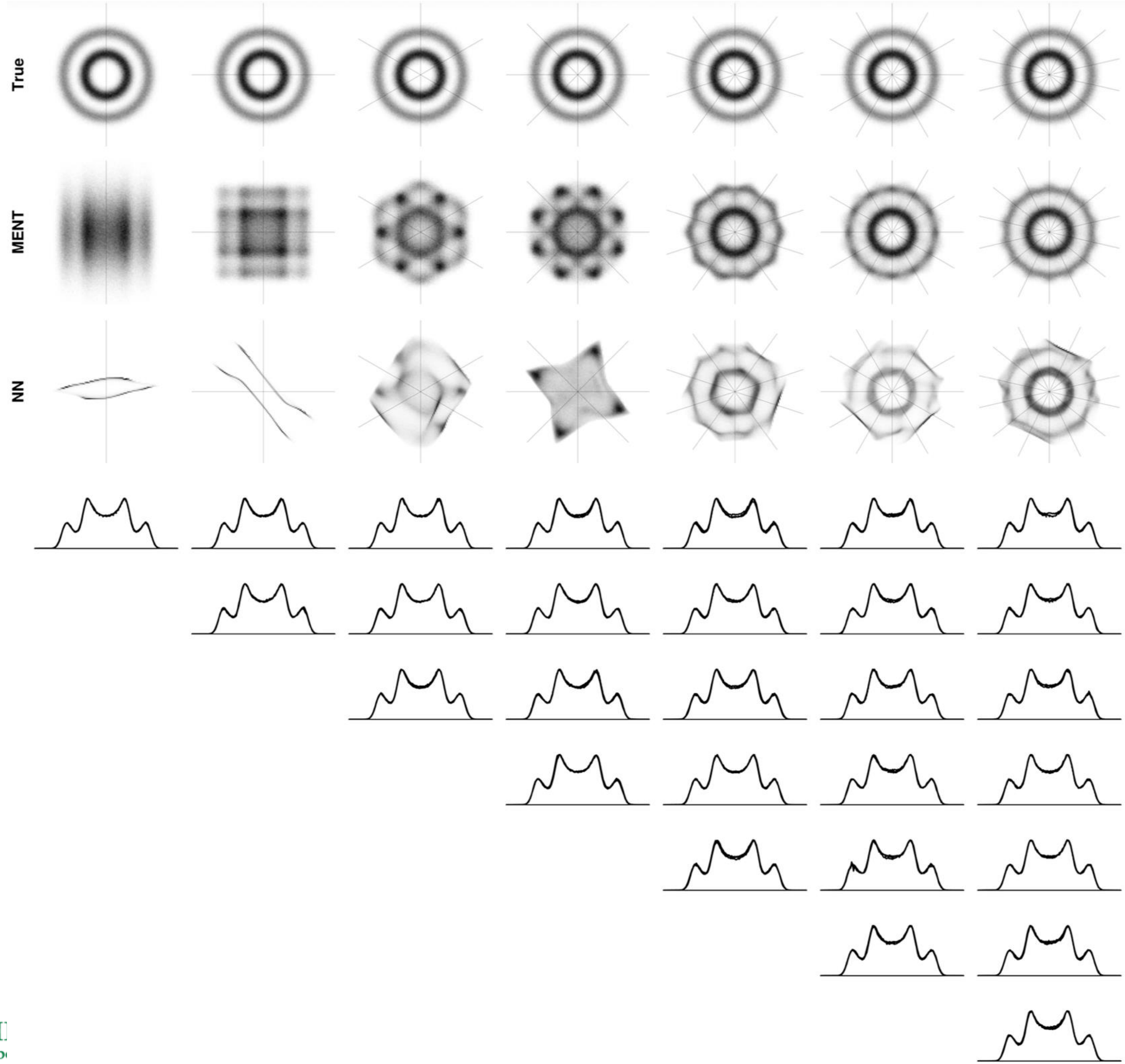


MENT iterations follow maximum-entropy trajectory until constraints satisfied

$$\boxed{h_k^{(i+1)} = h_k^{(i)} \left[(1-\omega) + \omega \left(g_k / \tilde{g}_k \right) \right]}$$






True
MENT
NN

Experimental results

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First solve linear system for unique elements of covariance matrix

$$\boxed{A\theta = \psi}$$

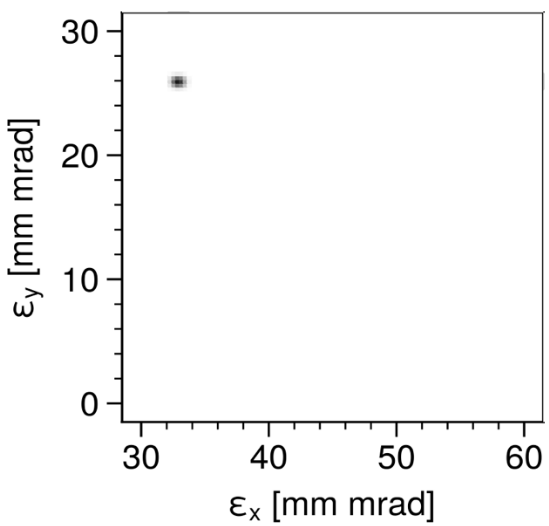
$\psi = \left[\psi^{(1)}, \dots, \psi^{(K)}\right]$ (measured beam moments)

$A = \left[A^{(1)}, \dots, A^{(K)}\right]$ (linear transformations)

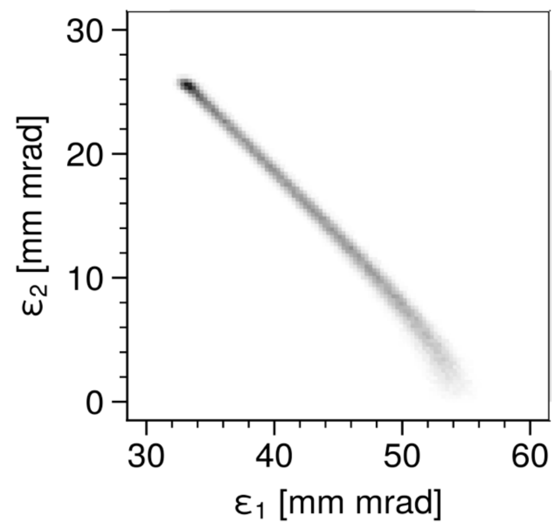
$\theta =$ unique elements of Σ

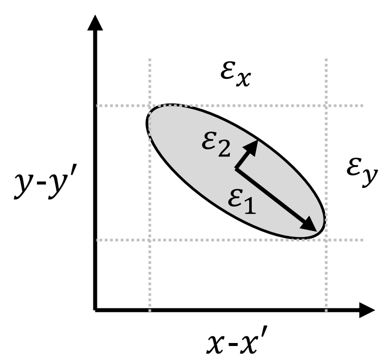
Nominal optics lead to high sensitivity to measurement errors

Projected emittances ($\varepsilon_x, \varepsilon_y$)

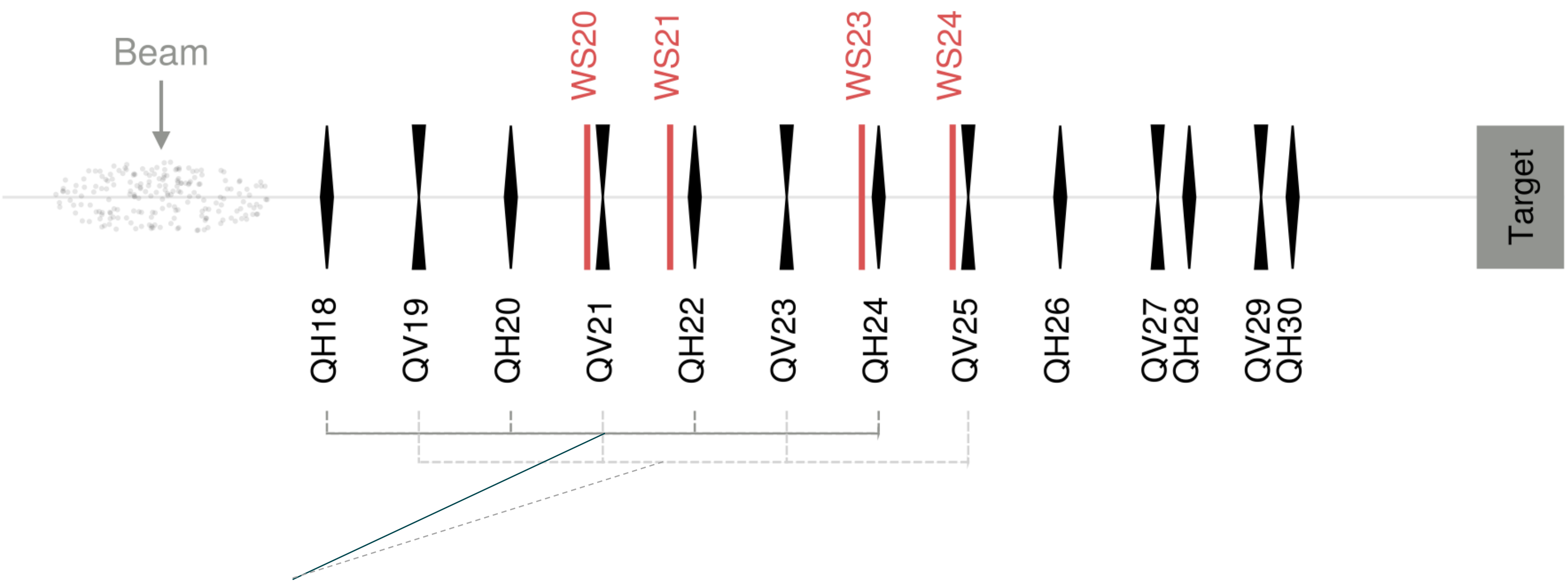


Intrinsic emittances ($\varepsilon_1, \varepsilon_2$)





We have some freedom to change quadrupoles upstream of measurement

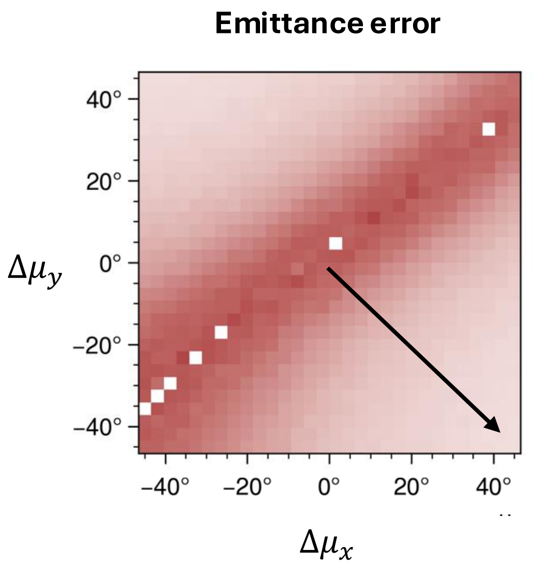


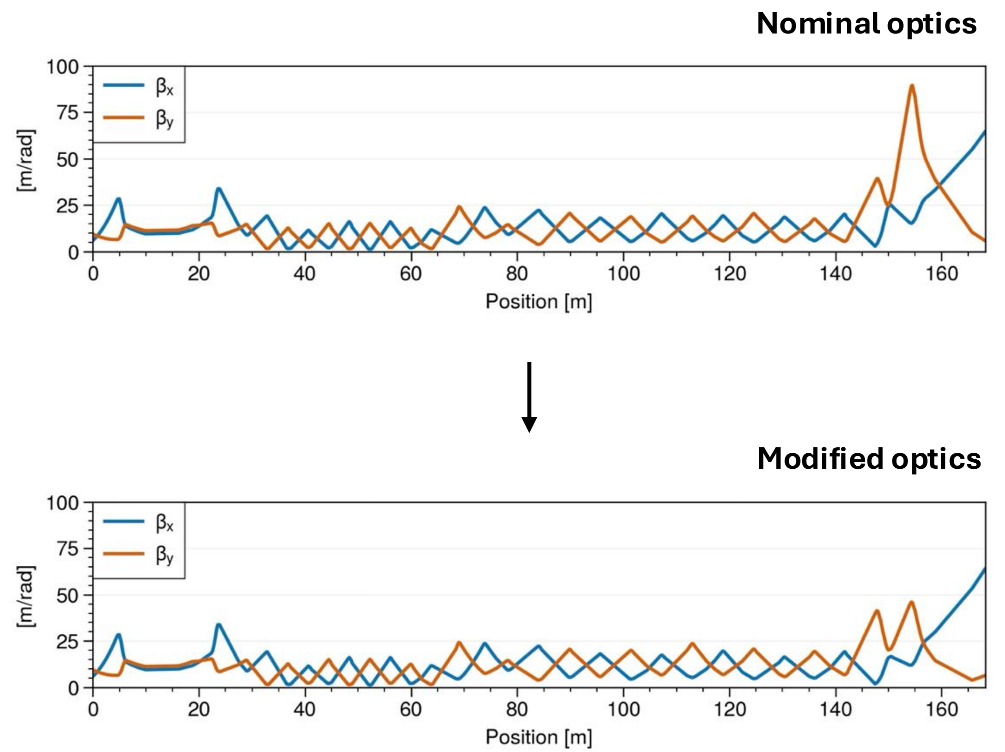
Parameters = two power supplies

Constraint #1: beam size in transport line

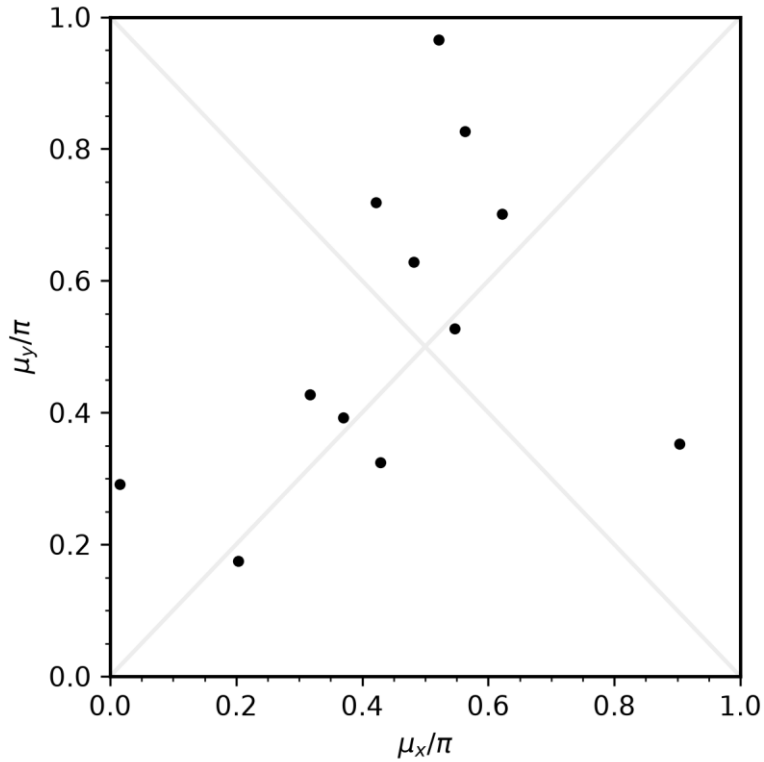
Constraint #2: beam size on target

Splitting phase advances leads to better-conditioned linear system

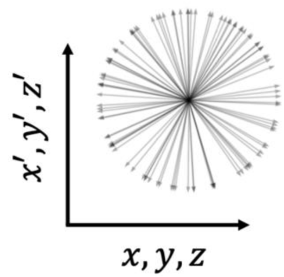




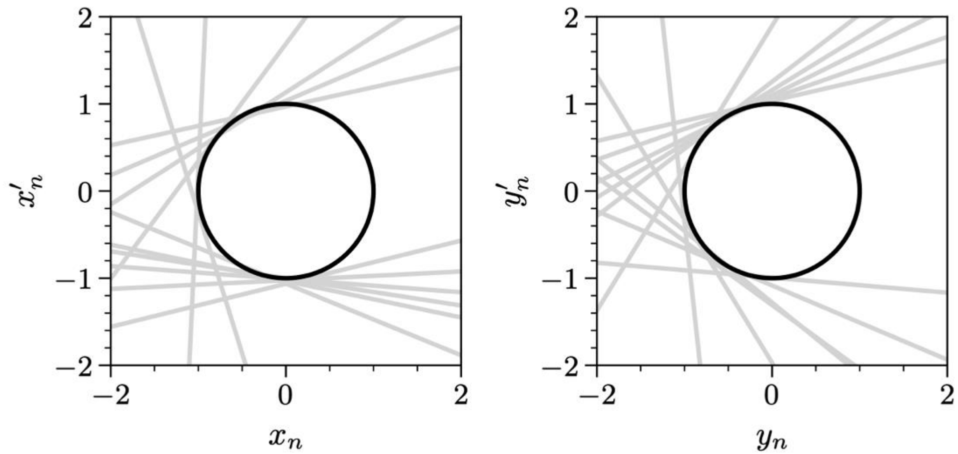
We used three sets of optics = 36 profiles

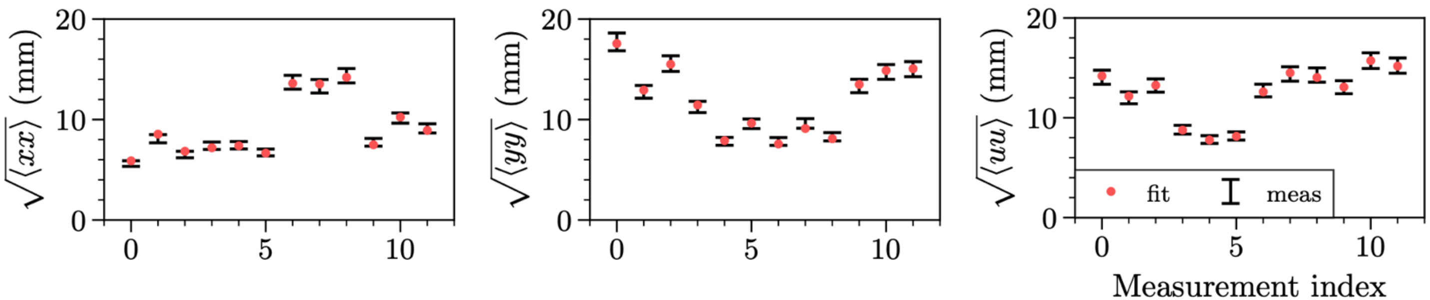


- Result for 2D projections from Hock and Wolski: evenly distribute points in μ_x-μ_y.
- Better/more general: dot products?

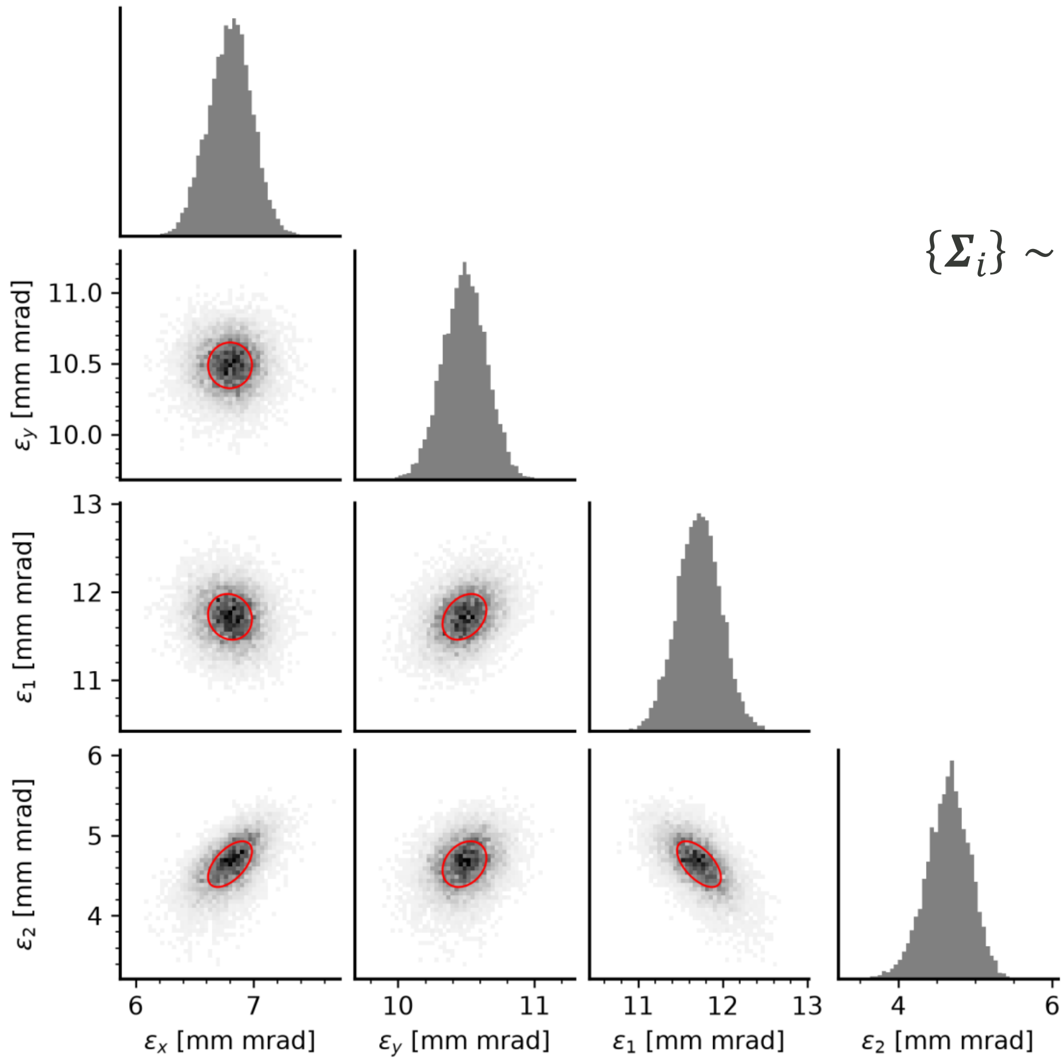


Projected emittances measurements are reliable



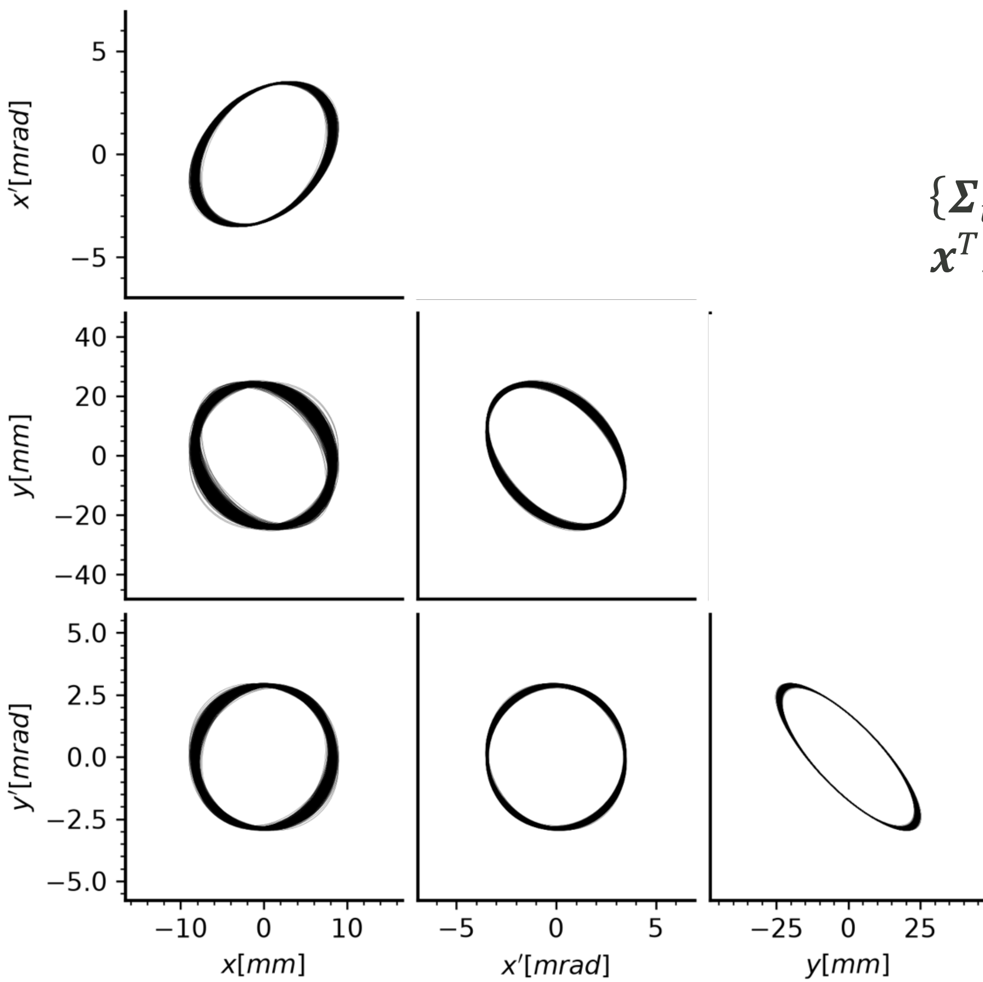


Intrinsic uncertainty estimates are larger but still only around 5%



$$\{\boldsymbol{\Sigma}_i\} \sim P(\boldsymbol{\Sigma}|\boldsymbol{\psi})$$

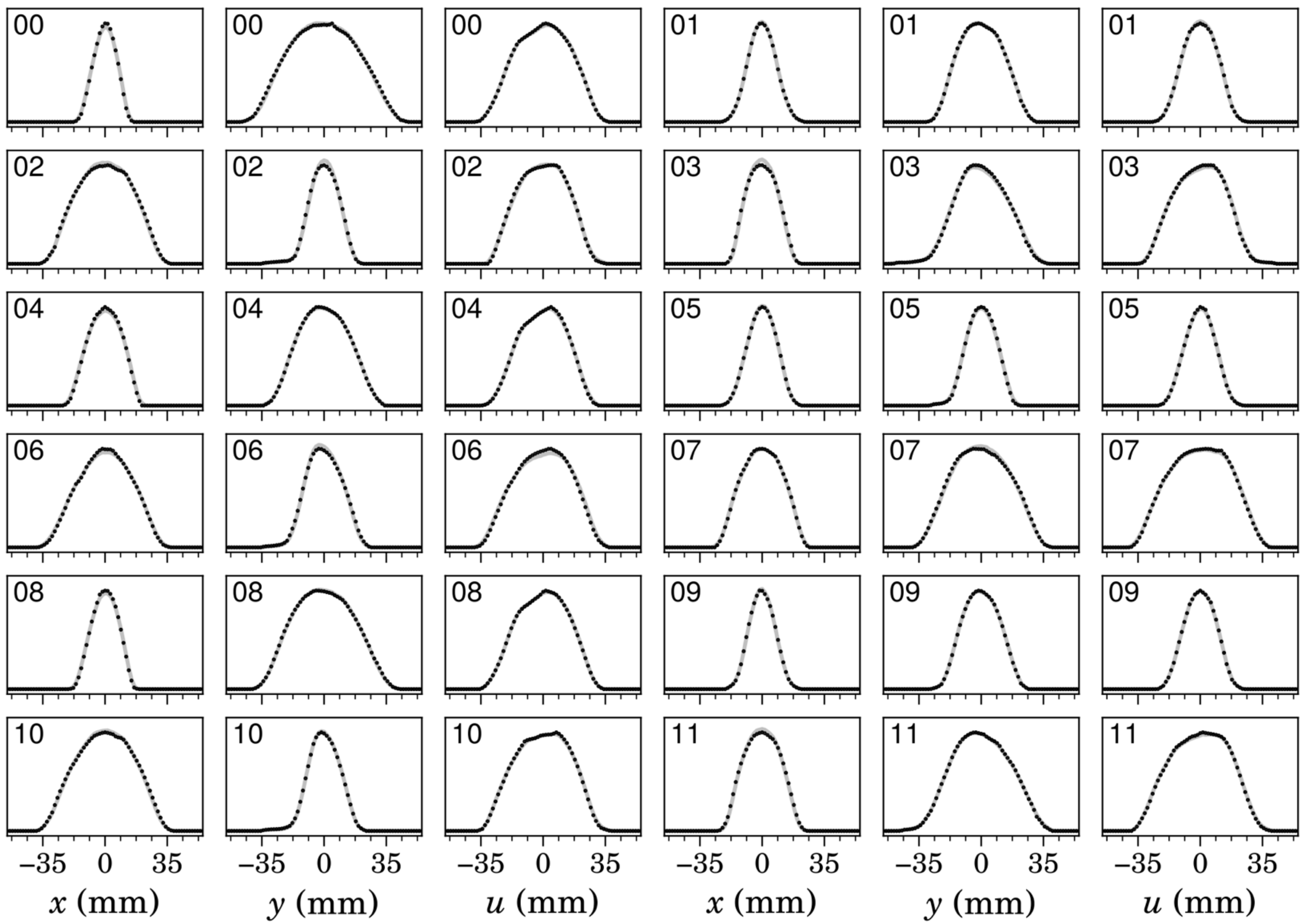
Visualize uncertainty by projecting RMS ellipsoid onto each plane



$\{\boldsymbol{\Sigma}_i\} \sim P(\boldsymbol{\Sigma}|\boldsymbol{\psi})$

$\boldsymbol{x}^T \boldsymbol{\Sigma}_i^{-1} \boldsymbol{x} = 1$

MENT profiles converge to measurements



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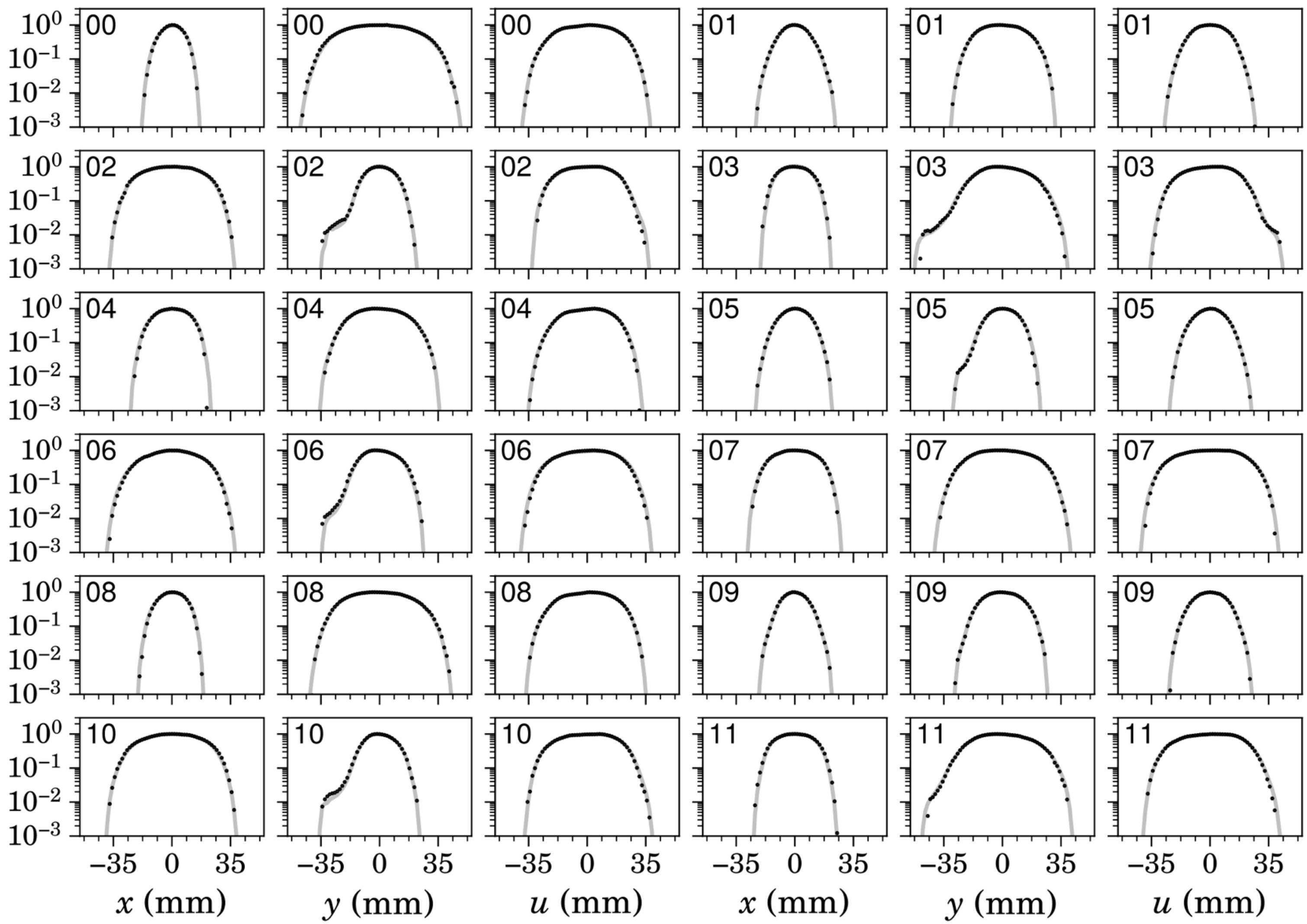
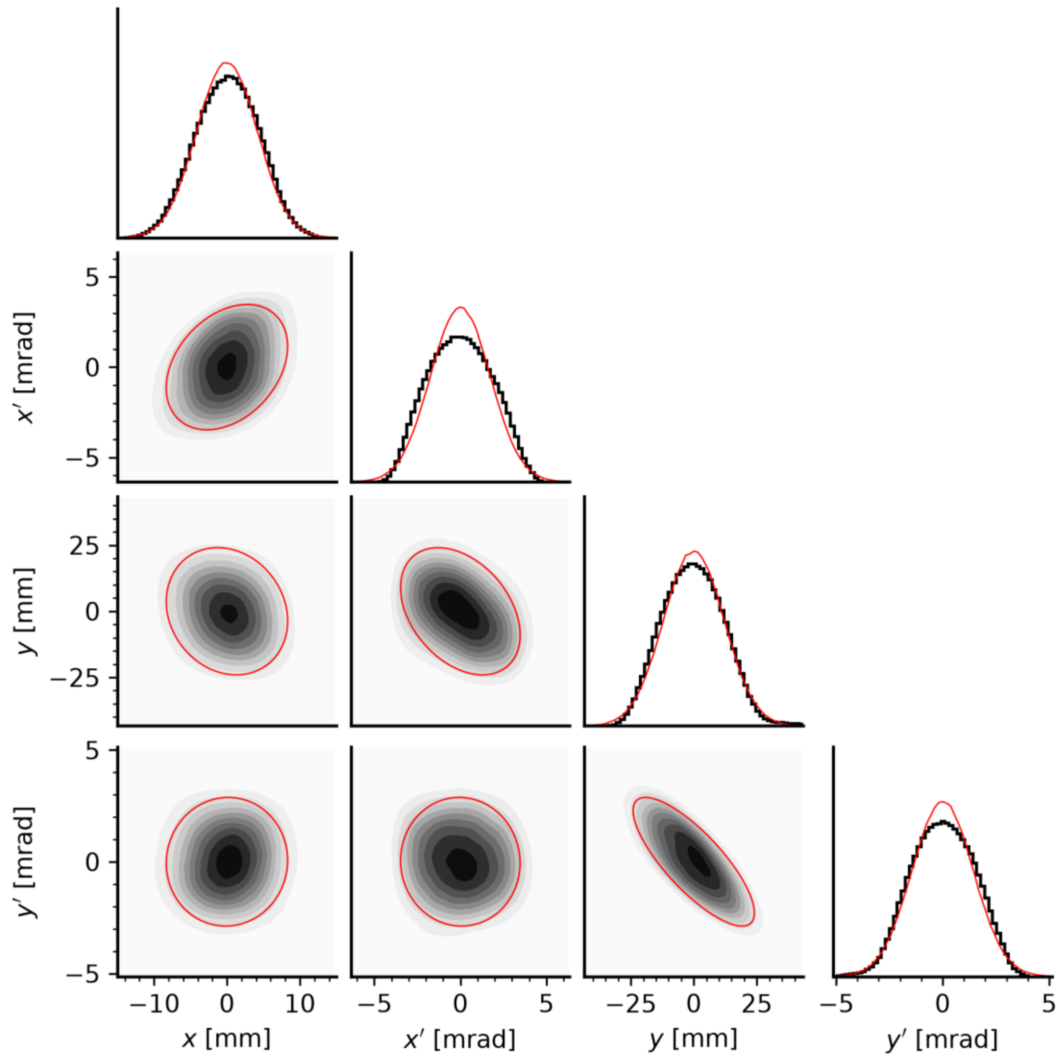
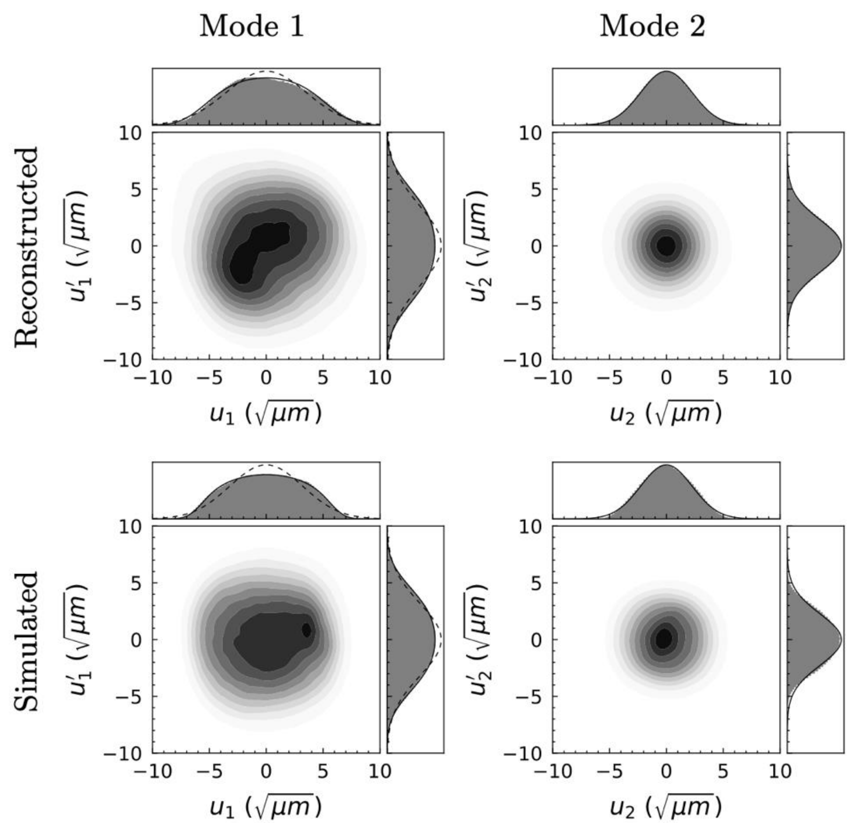
Low-density features reproduced (real or not)



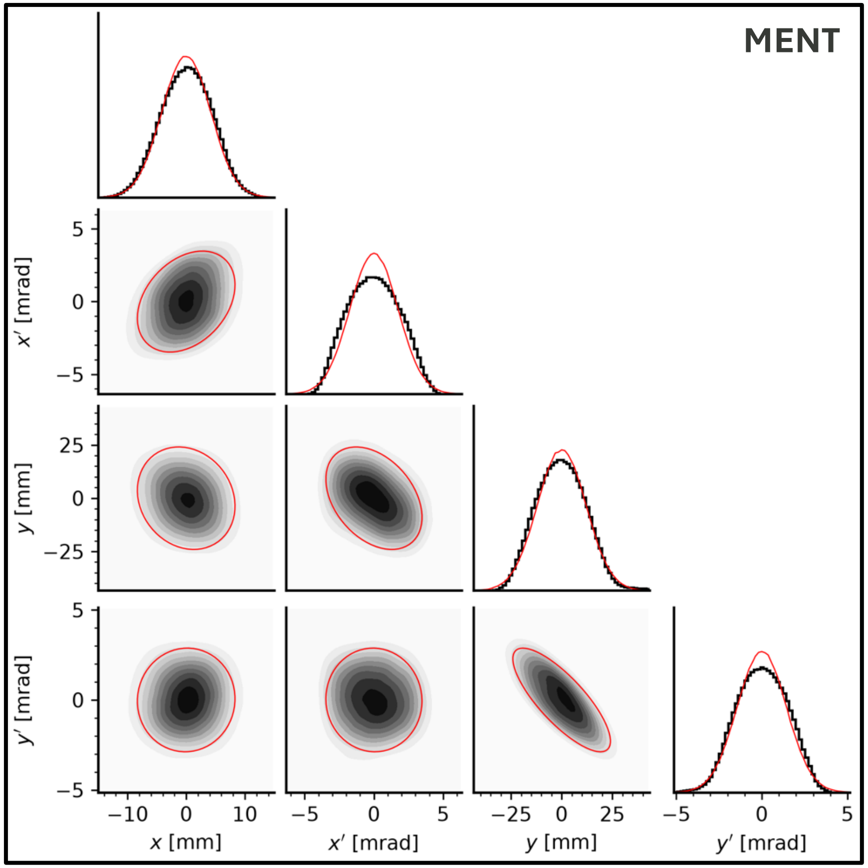
Reconstructed distribution deviates from Gaussian

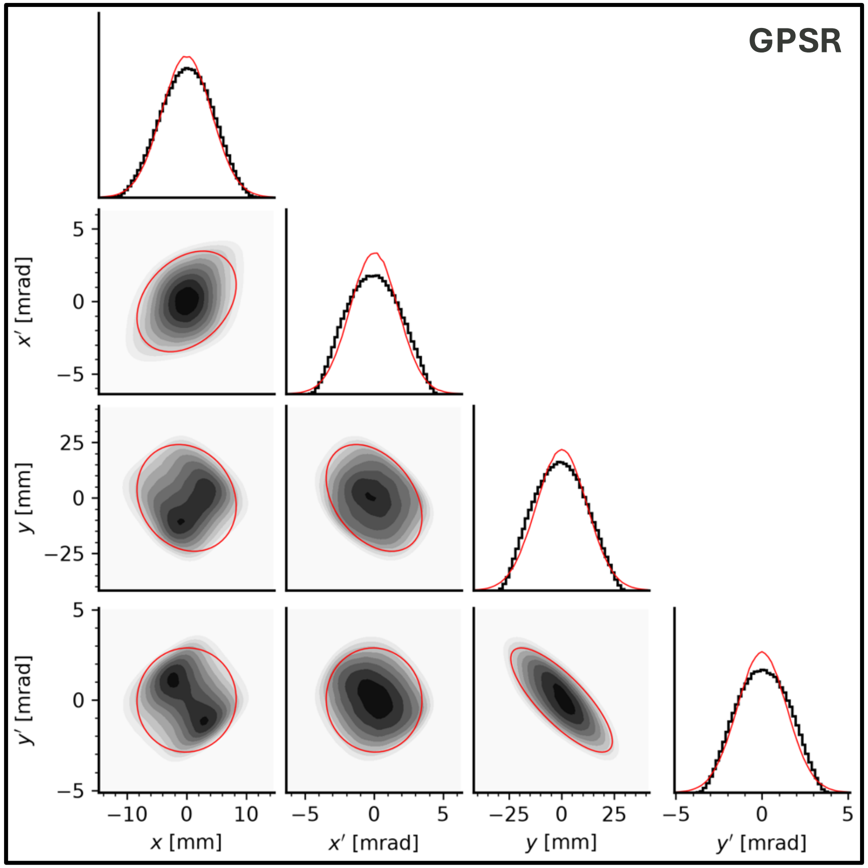


4D reconstruction enables comparison with simulation in <u>normalized</u> phase space

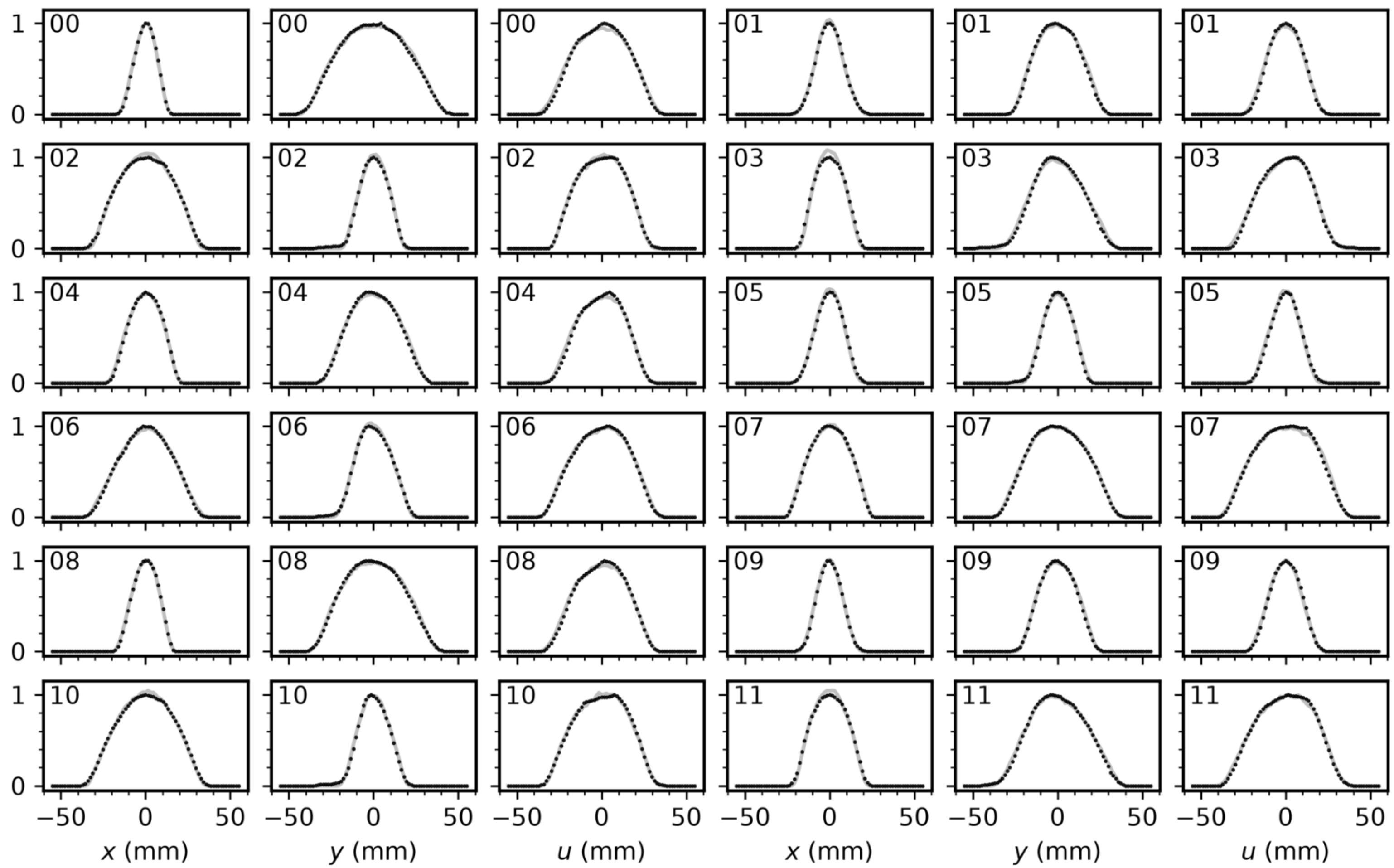


Careful!


MENT
x′ [mrad]
y [mm]
y′ [mrad]
x [mm]
x′ [mrad]
y [mm]
y′ [mrad]


GPSR
x′ [mrad]
y [mm]
y′ [mrad]
x [mm]
x′ [mrad]
y [mm]
y′ [mrad]

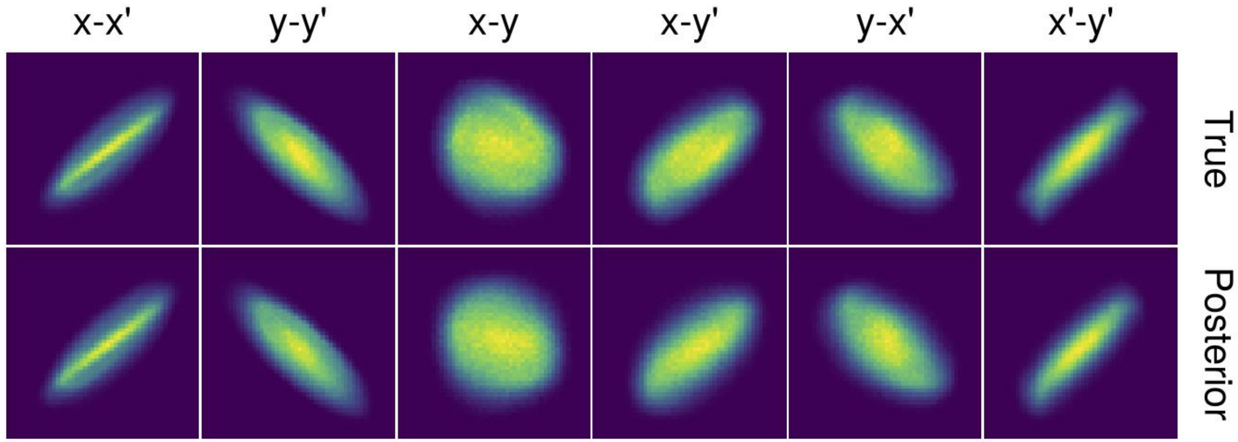
Careful!

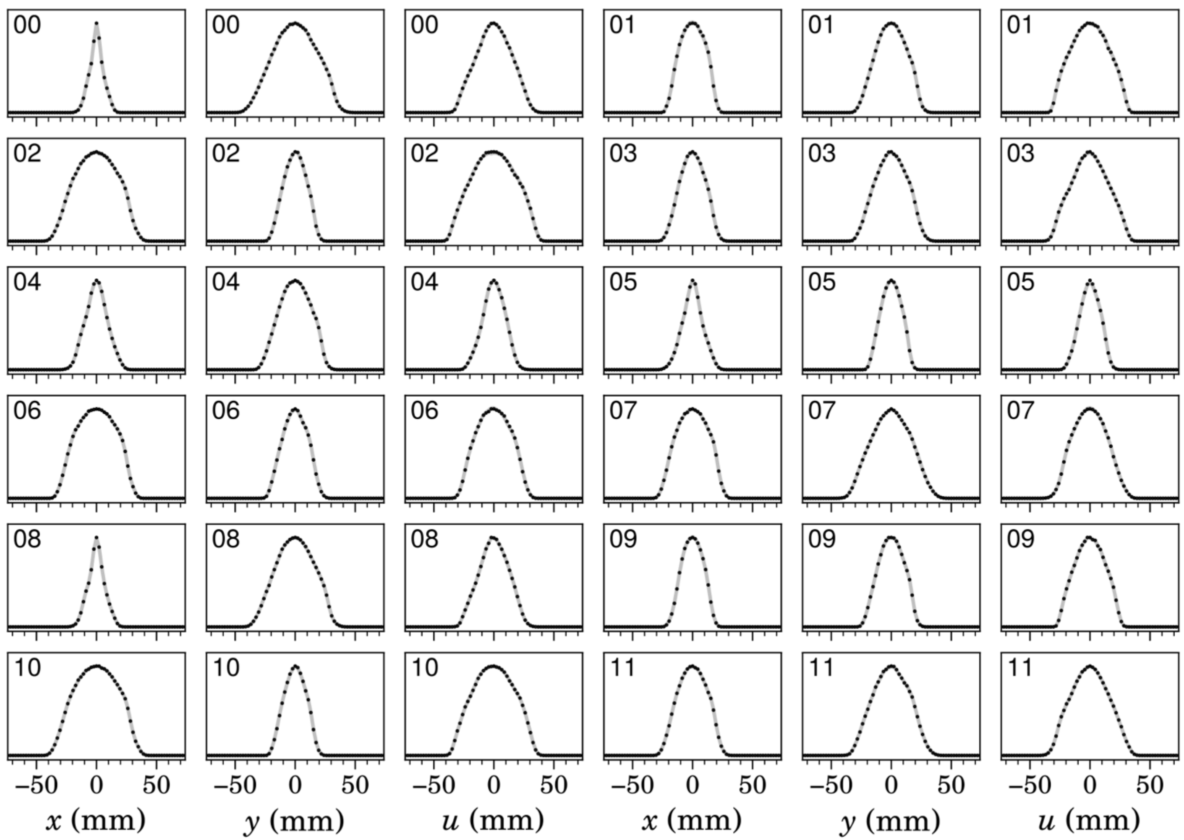


Uncertainty quantification

How accurate is the reconstructed distribution?

- Ignore model and measurement uncertainty, leaving intrinsic reconstruction uncertainty.
- Uncertainty depends on accelerator optics <u>and</u> unknown distribution.
- Robust uncertainty quantification (UQ) has not been demonstrated in the literature. (Thoughts later.)
- Adopt fake-data simulation approach.


x-x'
y-y'
x-y
x-y'
y-x'
x'-y'
True
Posterior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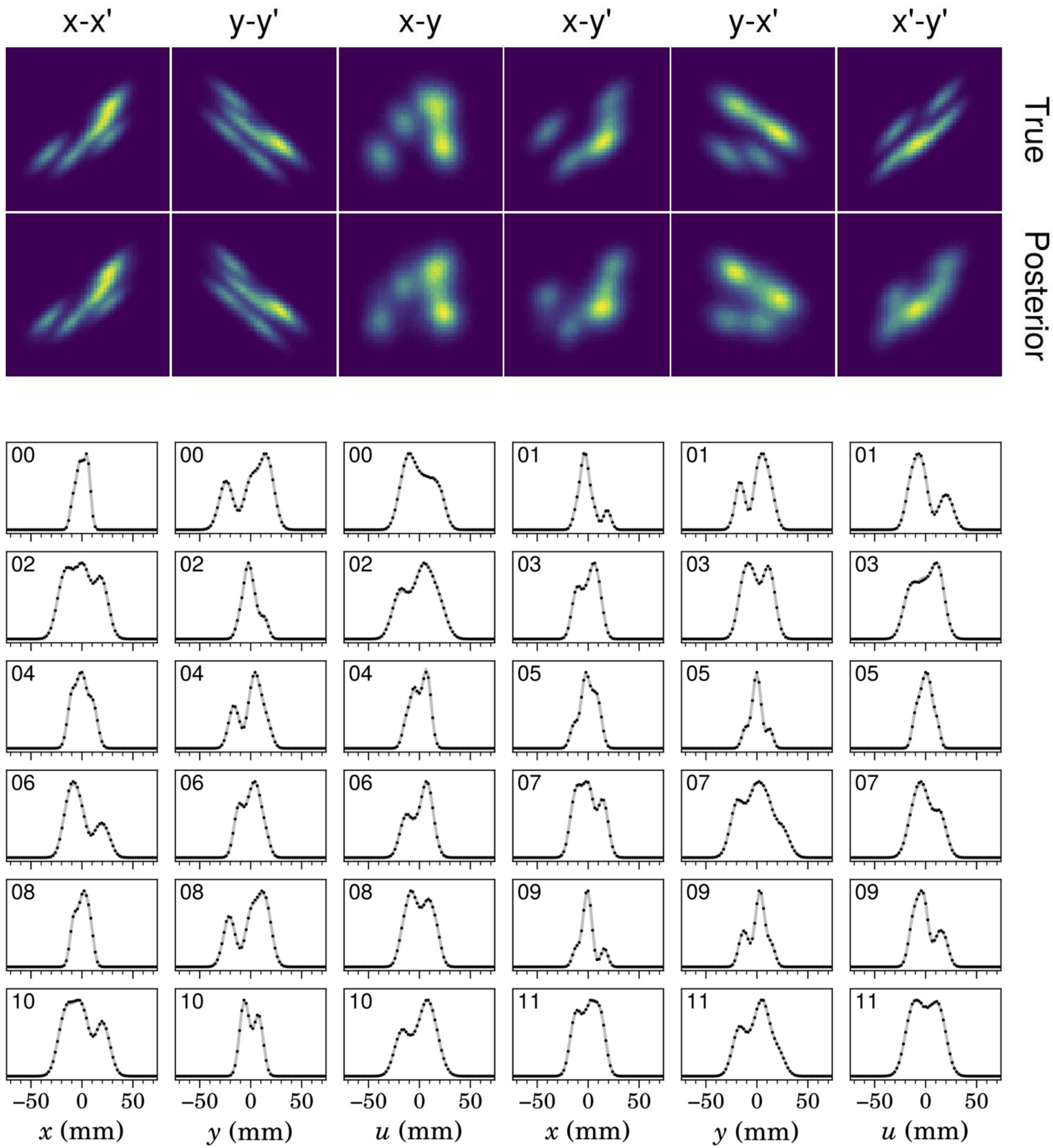
09
10
10
10
11
11
11
−50
0
50
x (mm)
y (mm)
u (mm)



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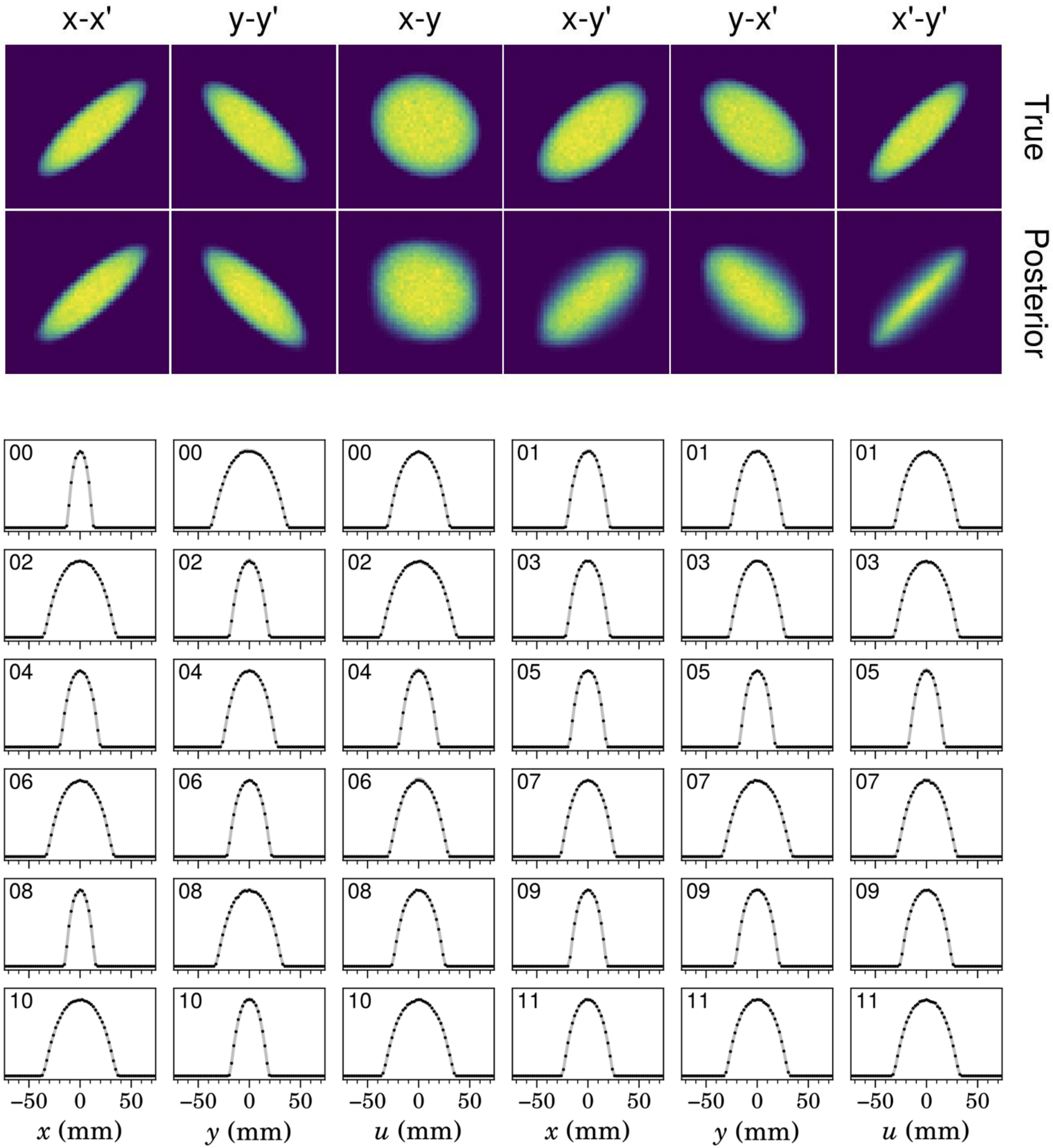
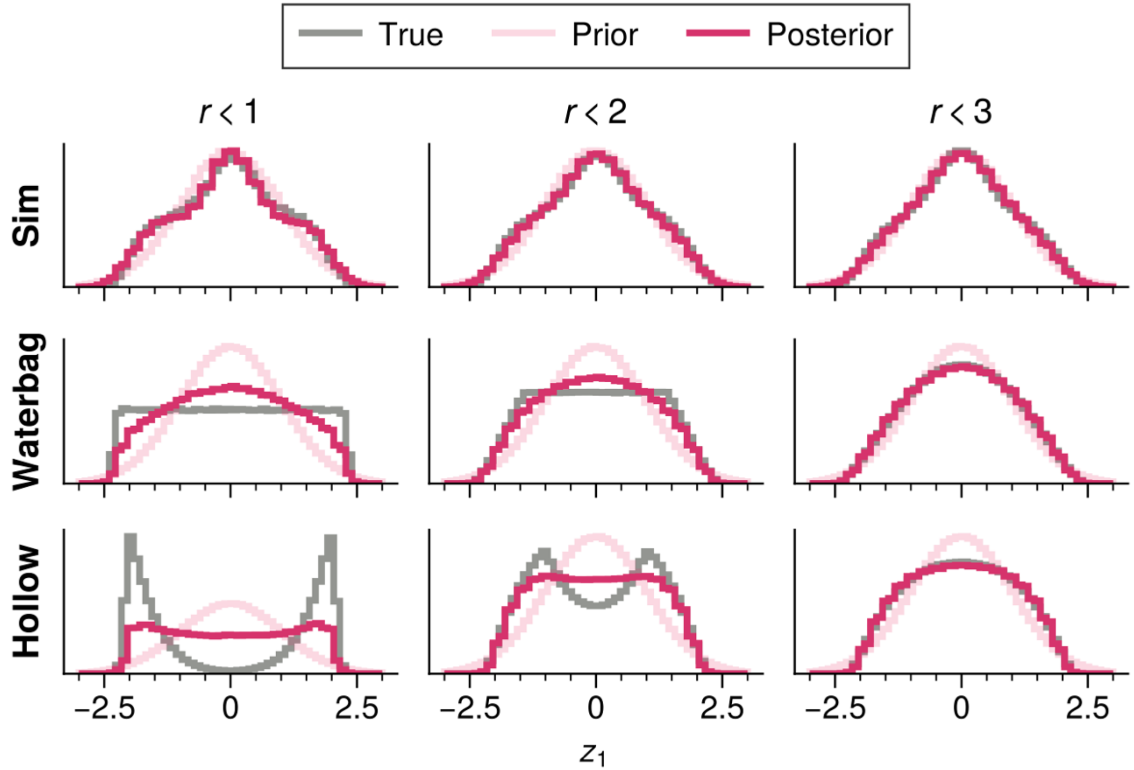

x-x'
y-y'
x-y
x-y'
y-x'
x'-y'
True
Posterior
00
01
02
03
04
05
06
07
08
09
10
11
−50
0
50
x (mm)
y (mm)
u (mm)

Hollow structures not fully recovered — results consistent with Gaussian prior distribution



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Summary

- 1D measurements can provide reasonable estimates of 4D phase space distribution in SNS RTBT.
- MENT provides important regularization of the solution.
- UQ is an important issue for real-world applications.
- Resources:
    - MENT repo: [https://github.com/austin-hoover/ment]
    - Code for this talk: [https://doi.org/10.5281/zenodo.21940431]
    - Email: hooveram@ornl.gov

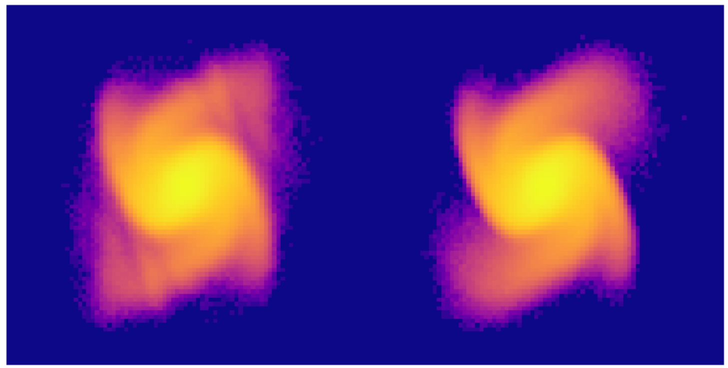
Future work

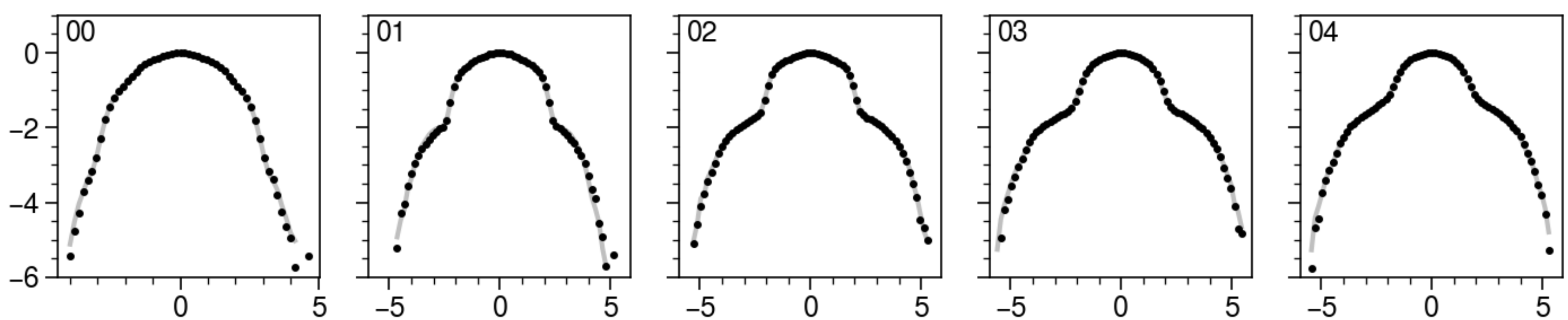
Research questions for phase space tomography in hadron linacs

- How should we quantify reconstruction **uncertainty** in the space of all possible distributions?
- What are the limits on 2D/4D **dynamic range**?
- Can we use nonlinear **space charge** to our advantage?
- Can we utilize **rf cavities**?
- Can we reduce number of measurements?
- Can we use coupled elements to avoid diagonal wires?

HDR 2D reconstructions?

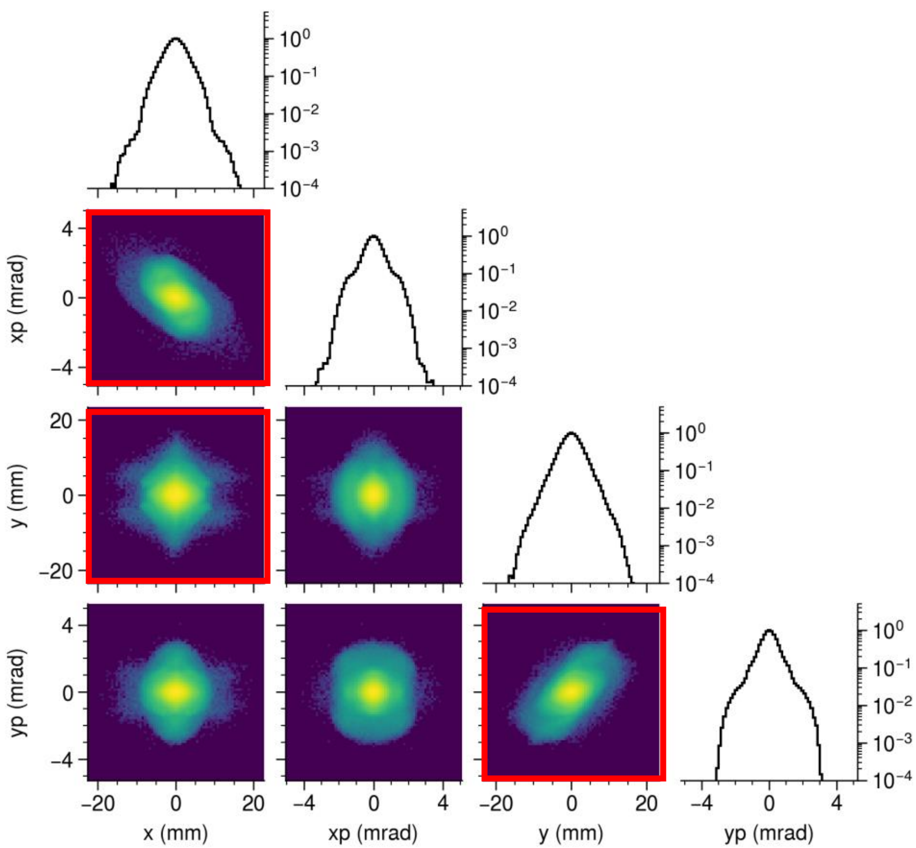
MENT TRUE



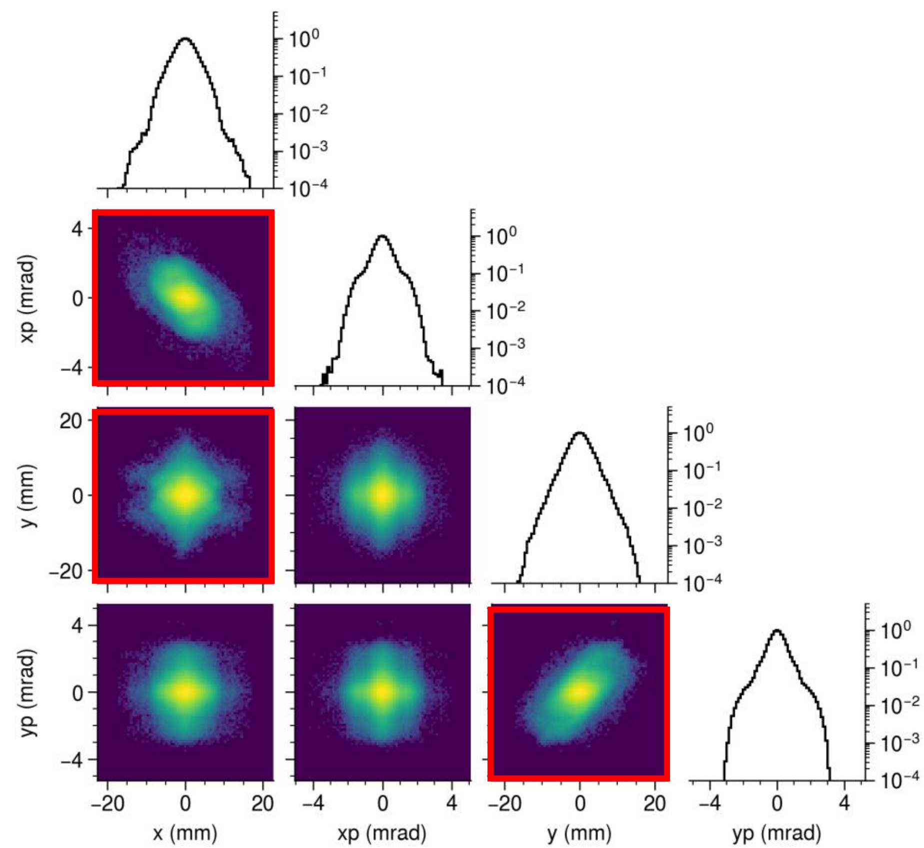


HDR 4D reconstructions?

TRUE



MENT

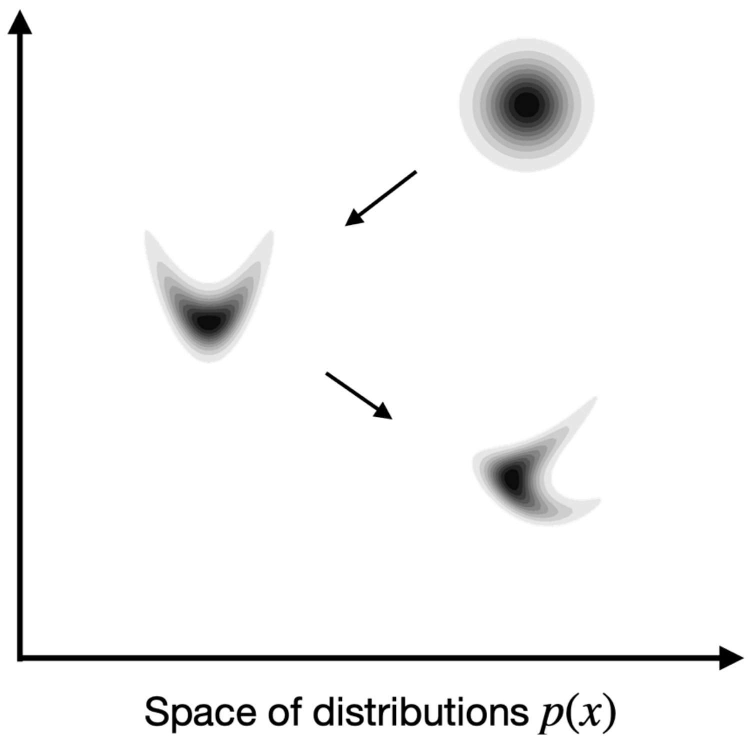


DATA

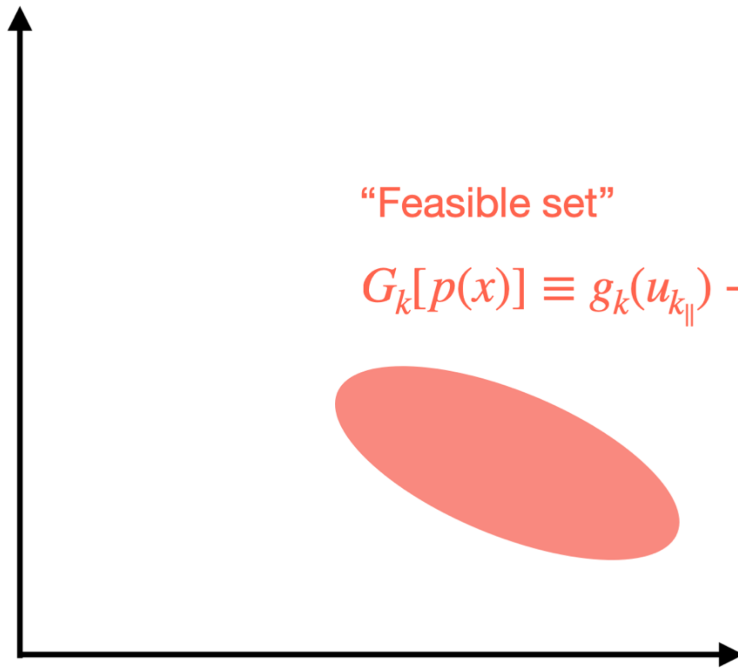
References

- Demonstration of a novel phase space painting method in a coupled lattice to mitigate space charge in high-intensity hadron beams. N. Evans, A. Hoover. T. Gorlov, and V. Morozov. Phys. Rev. Letters 136, 075002 (2026).
- N-dimensional maximum-entropy tomography via particle sampling. A. Hoover. Phys. Rev. Accel. Beams 28, L084601 (2025).
- Four-dimensional phase space tomography from one-dimensional measurements of a hadron beam. A. Hoover. Phys. Rev. Accel. Beams 27, 122802 (2024).
- High-dimensional maximum-entropy phase space tomography using normalizing flows. A. Hoover and J.C. Wong. Phys. Rev. Research 6.3, 033163 (2024).

Challenge 1: high-dimensional optimization



Challenge 2: no unique solution



Challenge 3: experimental design

- How many measurements?
- Which optics?

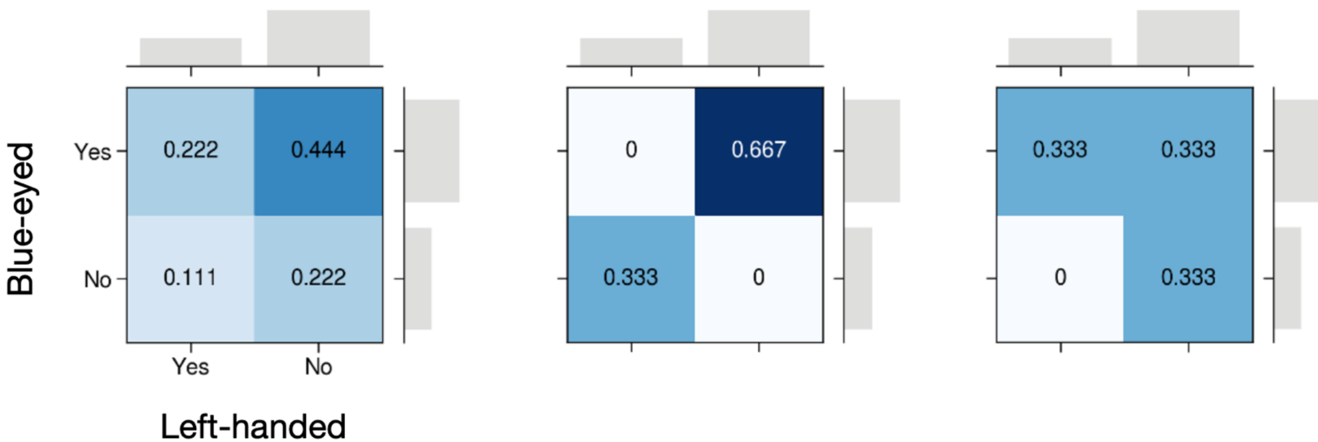

Blue-eyed
Yes
No
0.222
0.444
0.111
0.222
Yes
No
Left-handed
0
0.667
0.333
0
0.333
0.333
0
0.333

Table 1: Results for currently advertised functionals

Function	p (blue-eyes and left-handed)	Correlation
$-\sum p \log p$	$1/9 = 0.11111$	uncorrelated
$-\sum p^2$	$1/12 = 0.08333$	negative
$\sum \log p$	0.13013	positive
$\sum p^{1/2}$	0.12176	positive

MENT

Lagrangian:

$$\Psi = S[p(x), p_*(x)] + \sum_k \int G_k[p(x)] \lambda_k(u_{k_\parallel}) du_{k_\parallel}$$

Enforce $\delta\Psi = 0$ with respect to p and λ_k:

$$p(x) = p_*(x) \prod_k \exp\left[\lambda_k(u_{k_\parallel}(x))\right]$$

$$p(x) = p_*(x) \prod_k h_k(u_{k_\parallel}(x))$$

Prior

Positive functions defined on measurement axes

Posterior distribution for LLSQ system with uniform prior

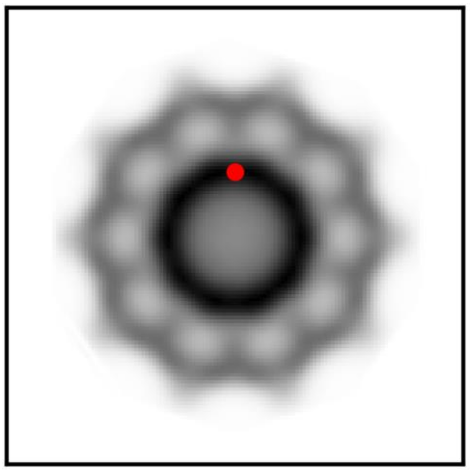
Posterior density $$P(\theta|\psi) \propto \exp\left[\frac{1}{2}\left(\theta - \hat{\theta}\right)^T \Sigma_\theta^{-1}\left(\theta - \hat{\theta}\right)\right]$$

Posterior mean $$\hat{\theta} = (A^T A)^{-1} A^T \psi$$

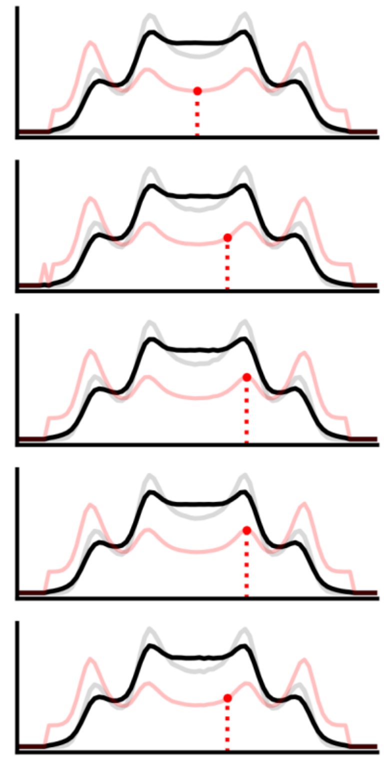
Posterior covariance $$\Sigma_\theta = \sigma^2 (A^T A)^{-1}$$

MENT solution is product of Lagrange multipliers

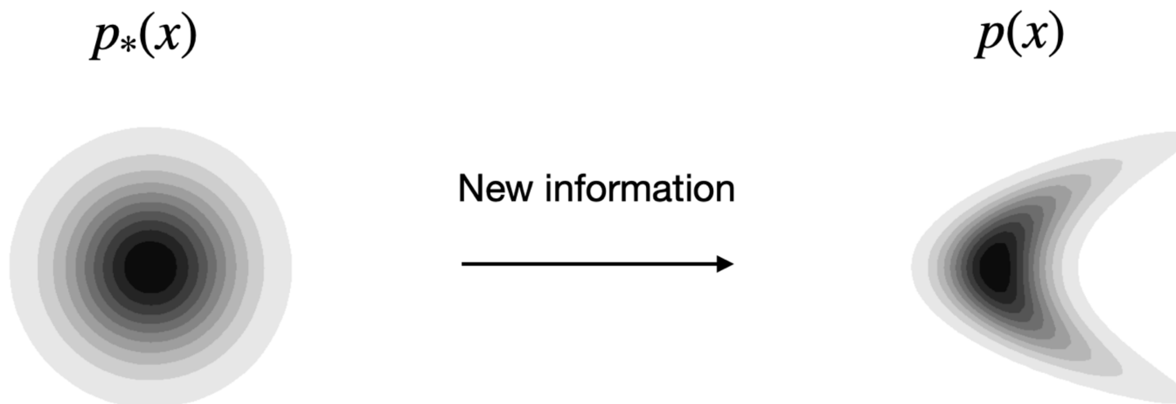
$$\rho(x) = \rho_*(x) \prod_k h_k(x)$$



$$\xrightarrow{\mathcal{M}_k(x)}$$



Entropy functional designed to enforce principle of minimal updating

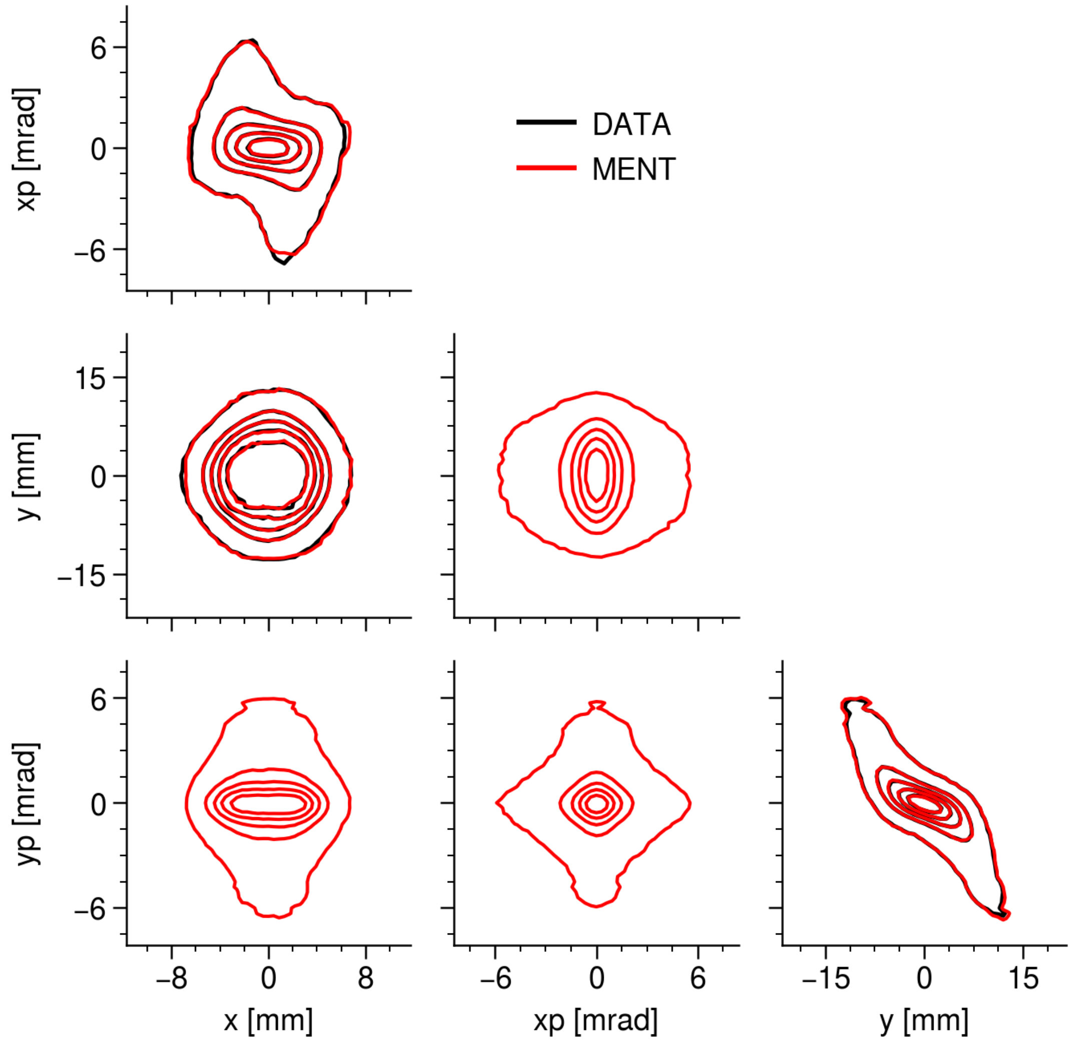


$$\boxed{S[p(x), p_*(x)] = -\int p(x) \log\left(\frac{p(x)}{p_*(x)}\right) dx}$$

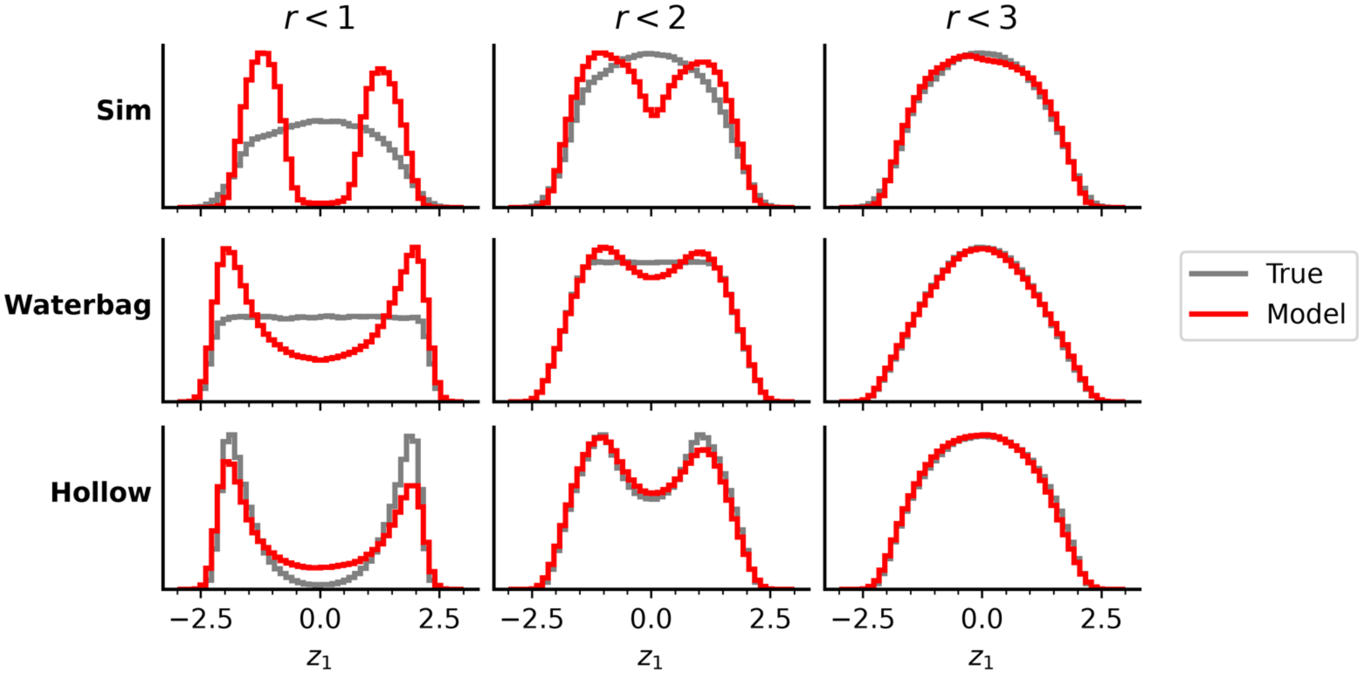
Entropy chosen to enforce principle of minimal updating

- In practice, solutions may be highly concentrated maximum-entropy distribution.
- The feasible set shrinks as more data are collected.
- This talk is not about foundations of entropy as an inference principle.

4:2 reconstruction in SNS BTF



Sliced views of GPSR reconstructions



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